7

Evapotranspiration

Evapotranspiration is a collective term for all the processes by which water in the liquid or solid phase at or near the earth's land surfaces becomes atmospheric water vapor. The term thus includes evaporation of liquid water from rivers and lakes, bare soil, and vegetative surfaces; evaporation from within the leaves of plants (**transpiration**); and sublimation from ice and snow surfaces.

Globally, about 62% of the precipitation that falls on the continents is evapotranspired, amounting to 72,000 km^3 yr^{-1} (Table 3-2). Of this, about 97% is evapotranspiration from land surfaces and 3% is open-water evaporation. Figures 3-23 and 3-24 show the latitudinal and global distributions of evapotranspiration, and Tables 3-3 and 3-4 give data on evapotranspiration from the continents and from major river basins. Note that evapotranspiration exceeds runoff in most of the river basins and on all the continents except Antarctica.

A quantitative understanding of evapotranspiration is of vital practical importance in several respects:

Understanding and predicting climate change requires the ability to model evapotranspiration, which is a major component of energy as well as water-vapor exchange between land surfaces and atmosphere. (See Figure 3-2.)

Over the long term, the difference between precipitation and evapotranspiration is the water available for direct human use and management. Thus quantitative assessments of water resources and the effects of climate and land-use change on those resources require a quantitative understanding of evapotranspiration.

Most of the water "lost" via evapotranspiration is used to grow the plants that form the base of the earth's land ecosystems, and understanding relations between evapotranspiration and ecosystem type is a requirement for predicting ecosystem response to climate change (Woodward 1987).

Much of the world's food supply is grown on irrigated land, and irrigation is one of the largest uses of water in the United States and many other countries. Efficient irrigation requires knowledge of crop water use (transpiration), so that water will be applied only as needed.

Evaporation has a significant influence on the yield of water-supply reservoirs, and thus on the economics of building reservoirs of various sizes.

The fraction of water falling in a given rainstorm that contributes to streamflow and to groundwater recharge is in large part determined by the "wetness" of the land; this wetness can only be assessed by determining the amount of evapotranspiration that has occurred since the previous storm.

Direct measurement of evapotranspiration is much more difficult and expensive than for precipitation and streamflow, and is usually impractical. Thus, in order to analyze these important scientific and practical issues, hydrologists have developed an array of methods that provide estimates of evapotranspiration based on measurements of more readily measured quantities. A major goal of this chapter is to show how the physics of evaporation and other basic principles such as the conservation of mass and energy are incorporated in these methods.

The basic physics of evaporation and atmospheric energy exchange are developed in Sections D.4 and D.6. These relations are involved in liquid-vapor and solid-vapor phase changes at all surfaces of hydrologic interest, including transpiring leaves, and this chapter begins by reviewing and extending those basic relations. Following this we define the basic types of evapotranspiration, and then focus on methods of estimation appropriate for each type in various situations.[1]

7.1 PHYSICS OF EVAPORATION AND TURBULENT ENERGY EXCHANGE

7.1.1 Evaporation

Evaporation is a diffusive process that follows Fick's first law [Table 2-1; Equations (D-26) and (D-38)], which can be written in finite-difference form as

$$E = K_E \cdot v_a \cdot (e_s - e_a), \tag{7-1}$$

where E is the evaporation rate [L T^{-1}], e_s and e_a are the vapor pressures of the evaporating surface and the overlying air, respectively [M L^{-1} T^{-2}], v_a is the wind speed [L T^{-1}], and K_E is a coefficient that reflects the efficiency of vertical transport of water vapor by the turbulent eddies of the wind [L T^2 M^{-1}].

K_E can be evaluated by dividing the right-hand side of Equation (D-42) by the latent heat of vaporization, λ_v, and the mass density of water, ρ_w. This gives an equation for evaporation rate in the same dimensions as Equation (7-1), from which we see that

$$K_E = \frac{0.622 \cdot \rho_a}{P \cdot \rho_w} \cdot \frac{k^2}{\left[\ln\left(\frac{z_m - z_d}{z_0}\right)\right]^2}$$

$$= \frac{0.622 \cdot \rho_a}{P \cdot \rho_w} \cdot \frac{1}{6.25 \cdot \left[\ln\left(\frac{z_m - z_d}{z_0}\right)\right]^2}, \tag{7-2}$$

where ρ_a is the density of air [M L^{-3}], P is the atmospheric pressure [M T^{-2} L^{-1}], $k = 0.4$ [1], z_m is the height at which wind speed and air vapor pressure are measured [L], z_d is the **zero-plane displacement** [L], and z_0 is the **roughness height** of the surface [L].

Equation (7-2) is derived from the logarithmic vertical distribution of wind speed [Equation (D-22)]. As explained in Section D.6, z_m is a fixed value for a given situation; z_d and z_0 depend on the roughness of the surface, which determines the intensity of the turbulent eddies at a given wind speed; and the density of air and pressure are approximately constants ($\rho_a = 1.220$ kg m^{-3}; $P = 101.3$ kPa at sea level). K_E can be adjusted for the stability condition of the air, which can be determined from the vertical gradients of wind speed and temperature as described in Section D.6.8 [Equations (D-53) and (D-55); Table D-5].

7.1.2 Vapor-Pressure Relations

The vapor pressure of an evaporating surface is equal to the saturation vapor pressure at the surface temperature, e_s^*, so

$$e_s = e_s^*, \tag{7-3}$$

where e_s^* is given to good approximation by Equation (D-7) with $T = T_s$:

$$e_s^* = 0.611 \cdot \exp\left(\frac{17.3 \cdot T_s}{T_s + 237.3}\right). \tag{7-4}$$

Here, vapor pressure is in kPa and temperature in °C. The vapor pressure in the air, e_a, depends on the relative humidity, W_a, as well as the air temperature, T_a:

$$e_a = W_a \cdot e_a^*, \tag{7-5}$$

where e_a^* is the saturation vapor pressure at the air temperature, given by Equation (D-7) with $T = T_a$ and W_a is expressed as a ratio.

Some approaches to estimating evapotranspiration make use of the slope of the relation between saturation vapor pressure and temperature, Δ. Its value can be found by taking the derivative of Equation (7-4):

[1] We concentrate here on the liquid-vapor transition; evaporation from snow was discussed in Section 5.4.2.

$$\Delta = \frac{de^*}{dT} = \frac{2508.3}{(T + 237.3)^2} \cdot \exp\left(\frac{17.3 \cdot T}{T + 237.3}\right), \quad \textbf{(7-6)}$$

where Δ is in kPa K^{-1} and T is in °C. Note that, like e^*, Δ increases exponentially with temperature.

7.1.3 Latent-Heat Exchange

Evaporation is always accompanied by a transfer of latent heat from the evaporating body into the air. This heat loss tends to produce a reduction of surface temperature, which may be completely or partially compensated by heat transfer to the surface from within the evaporating body or by radiative or sensible-heat transfer from the overlying air. The rate of latent-heat transfer, LE [E L^{-2} T^{-1}], is found simply by multiplying the evaporation rate by the latent heat of vaporization, λ_v, and the mass density of water, ρ_w:

$$LE = \rho_w \cdot \lambda_v \cdot E = \rho_w \cdot \lambda_v \cdot K_E \cdot v_a \cdot (e_s - e_a). \quad \textbf{(7-7)}$$

The latent heat of vaporization decreases as the temperature of the evaporating surface increases; this relation is given approximately by

$$\lambda_v = 2.50 - 2.36 \times 10^{-3} \cdot T, \quad \textbf{(7-8)}$$

where λ_v is in MJ kg^{-1} and T is in °C.

7.1.4 Sensible-Heat Exchange

The upward rate of sensible-heat exchange by turbulent transfer, H, is given in finite-difference form by Equation (D-52):

$$H = K_H \cdot v_a \cdot (T_s - T_a), \quad \textbf{(7-9)}$$

where, from Equation (D-49),

$$K_H \equiv c_a \cdot \rho_a \cdot \frac{k^2}{\left[\ln\left(\frac{z_m - z_d}{z_0}\right)\right]^2}$$

$$= c_a \cdot \rho_a \cdot \frac{1}{6.25 \cdot \left[\ln\left(\frac{z_m - z_d}{z_0}\right)\right]^2}. \quad \textbf{(7-10)}$$

Here $k = 0.4$, and c_a is the heat capacity of air ($c_a = 1.00 \times 10^{-3}$ MJ kg^{-1} K^{-1}), and the other symbols are as in Equation (7-2). As with K_E, K_H can be adjusted to account for non-neutral stability conditions [Equations (D-54) and (D-55) and Table D-5].

7.1.5 The Bowen Ratio, the Psychrometric Constant, and the Evaporative Fraction

In developing approaches to estimating evapotranspiration based on energy balances, we will see that it is sometimes useful to incorporate the ratio of sensible-heat exchange to latent-heat exchange. This quantity, originally formulated by Bowen (1926), is called the **Bowen ratio,** B:

$$B \equiv \frac{H}{LE}. \quad \textbf{(7-11)}$$

Combining Equations (D-42) and (D-49), we have

$$B = \frac{c_a \cdot \rho_a \cdot (T_s - T_a)}{0.622 \cdot \lambda_v \cdot (e_s - e_a)} = \gamma \cdot \frac{(T_s - T_a)}{(e_s - e_a)}. \quad \textbf{(7-12)}$$

Thus the Bowen ratio depends on the ratio of surface-air temperature difference to surface-air vapor-pressure difference, multiplied by a factor γ, where

$$\gamma = \frac{c_a \cdot P}{0.622 \cdot \lambda_v}. \quad \textbf{(7-13)}$$

The factor γ enters separately into some expressions for estimating evapotranspiration, and is called the **psychrometric constant.** However, it is not strictly a constant: Pressure is a function of elevation (Figure D-2) and varies slightly over time at a given location, and latent heat varies slightly with temperature [Equation (7-8)]. Using typical values of $c_a = 1.00 \times 10^{-3}$ MJ kg^{-1} K^{-1}, $P = 101.3$ kPa, and $\lambda_v = 2.47$ MJ kg^{-1}, we find $\gamma = 0.066$ kPa K^{-1}, and that value is commonly used. However, the decrease of pressure with elevation (Figure D-2) should be accounted for when calculating γ for applications at high elevations.

It has also proven useful in studies of regional evapotranspiration to define the **evaporative fraction,** EF, as the ratio of latent-heat exchange rate to total turbulent-heat exchange rate:

$$EF \equiv \frac{LE}{LE + H} = \frac{1}{B + 1}. \quad \textbf{(7-14)}$$

7.1.6 The Energy Balance

The general energy balance for an evapotranspiring body during a time period Δt can be written as

$$LE = K + L - G - H + A_w - \frac{\Delta Q}{\Delta t}, \quad \textbf{(7-15)}$$

where the first six terms represent average energy fluxes (energy per unit area of evaporating surface per unit time) via the following modes: evaporation, LE; net shortwave radiation input, K; net longwave radiation input, L; net output via conduction to the ground, G; net output of sensible-heat exchange with the atmosphere, H; net input associated with inflows and outflows of water (**water-advected energy**), A_w; and ΔQ is the change in the amount of heat stored in the body per unit area between the beginning and end of Δt.[2,3]

From Equation (7-15) we see that any evaporation occurring during Δt must be balanced by some combination of heat inputs from radiation or sensible heat from the atmosphere or ground, and/or a loss of heat energy (i.e., a temperature reduction) in the evaporating body.

In some situations, the atmospheric conditions above the evapotranspiring region are representative of an extensive area extending beyond the region, and there is no significant horizontal transport of energy by air movement to or from the area above the region (i.e., the water-atmosphere heat exchange is in approximate local equilibrium). When such equilibrium does not exist, horizontal air flows supply **air-advected energy** to the air overlying the region to maintain the energy balance.

7.2 CLASSIFICATION OF EVAPOTRANSPIRATION PROCESSES

The various methods for estimating evapotranspiration have been developed for specific surface and energy-exchange situations determined by the following conditions:

Type of surface: open water, bare soil, leaf or leaf canopy, a specific **reference crop** (usually a complete cover of well-watered grass, as discussed later), or land region (generally including vegetated surfaces, surface-water bodies, and areas of bare soil);

Availability of water: unlimited water available to evaporate, or water supply to the air may be limited because water vapor must pass through plant openings or soil pores;

Stored-energy use: ΔQ in Equation (7-15) may be significant, negligible, or nonexistent;

Water-advected energy use: A_w in Equation (7-15) may be significant, negligible, or nonexistent.

Table 7-1 shows how the various "types" of evapotranspiration are distinguished with respect to the above conditions, and the methods described in subsequent sections of this chapter are classified according to these types.

Additional considerations in choosing a method for use in a given circumstance are: (1) the purpose of the analysis (determination of the amount of evapotranspiration that has actually occurred in a given situation, incorporation in a hydrologic model, reservoir design, general water-resources assessment, etc.); (2) the available data (particular meteorological parameters measured and whether measurements were made at the area of interest or are estimated regional values); and (3) the time period of interest (hour, day, month, year; climatic average). We will indicate the applicability of the various methods with respect to these considerations.

7.3 FREE-WATER, LAKE, AND WETLAND EVAPORATION

In natural water bodies, water-advected heat and change in heat storage may play a significant role in the energy balance. The magnitude of these components in a particular case depends in large part on the area, volume, and residence time of water in the lake relative to the time period of the analysis. Because of the variable importance of these non-meteorologic factors in the energy balance, it is not generally possible to develop equations for predicting the evaporation for a particular lake from meteorologic data alone.[4]

[2] Where appropriate, a term for energy used in photosynthesis can be added to Equation (7-15). This use can amount to as much as 3% of $K + L$ for some crops (C.A. Federer, pers. comm.).

[3] Equation (7-15) is analogous to the energy-balance relation for a snowpack, Equation (5-26). Note, however, that the signs of H, LE, and G are reversed here because we are considering *outward* latent- and sensible-heat flows to be positive. Heat input due to rain [R in Equation (5-26)] is usually negligible in considering evaporation, but could be included in A_w in Equation (7-15).

[4] Advection and heat-storage effects are often considered to balance out for annual values. However, even annual balancing of advection and storage may not occur in lakes with residence times of more than one year.

TABLE 7-1
Classification of Types of Evapotranspiration

Evapotranspiration Type	Type of Surface	Availability of Water to Surface	Stored Energy Use	Water-Advected Energy Use
Free-water evaporation[a]	Open water	Unlimited	None	None
Lake evaporation	Open water	Unlimited	May be involved	May be involved
Bare-soil evaporation	Bare soil	Limited to unlimited	Negligible	None
Transpiration	Leaf or leaf canopy	Limited	Negligible	None
Interception loss	Leaf or leaf canopy	Unlimited	Negligible	None
Potential evapotranspiration	Reference crop[b]	Limited to air, unlimited to plants	None	None
Actual evapotranspiration	Land area[c]	Varies in space and time	Negligible	None

[a]Also called **potential evaporation.**

[b]Usually a complete ground cover of uniform short vegetation (e.g., grass); discussed further in Section 7.7.1.

[c]May include surface-water bodies and areas of bare soil.

In order to develop general methods for estimating evaporation from surface-water bodies, hydrometeorologists have formulated the theoretical concept of **free-water evaporation:** evaporation that would occur from an open-water surface in the absence of advection and changes in heat storage (Table 7-1) and which thus depends only on regionally continuous meteorologic or climatic conditions. **Lake evaporation** is determined by adjusting mapped or computed free-water evaporation to account for the advection and heat-storage effects in a given actual water body.

In the following discussion of methods, we will indicate how each approach can be applied to estimating free-water evaporation and/or actual lake evaporation.

7.3.1 Water-Balance Approach

Theoretical Basis

The water-balance approach involves applying the water-balance equation (see Section 2.5.1) to the water body of interest over a time period Δt and solving that equation for evaporation, E:

$$E = W + SW_{in} + GW_{in} - SW_{out} - GW_{out} - \Delta V. \quad \textbf{(7-16)}$$

Here, W is precipitation on the lake; SW_{in} and SW_{out} are the inflows and outflows of surface water, respectively; GW_{in} and GW_{out} are the inflows and outflows of ground water, respectively; ΔV is the change in the amount of water stored in the lake during Δt; and all quantities have dimensions of either volume or volume per unit lake area.

Practical Considerations

While this approach is simple in concept, it is generally far from simple in application. In practice we do not know the true values of the quantities in Equation (7-16) and must use measured or estimated values, which are subject to uncertainty. Our computation of E thus includes the net sum of all these measurement errors as well as the evaporation, as discussed in Section 2.5.2 and illustrated for watershed evapotranspiration in Example 2-1.

Measurement of all liquid-water inputs and outputs for a water body is generally a formidable task. All major streams entering the body and the outlet stream must be continuously gaged, and some method must be devised for estimating the amount of any non-channelized overland flow inputs. At best, ground-water flows are usually estimated from gradients observed in a few observation wells and assumptions about the saturated thickness and hydraulic conductivity of surrounding geologic formations, perhaps supplemented by scattered observations with seepage meters. (See Section 8.5.3.) If the lake is large and the surrounding topography irregular, it may be difficult to obtain precise measurements of precipitation. (See Sections 4.2 and 4.3.) Changes in storage can be estimated from careful observations of water levels only if one has good information on the lake's bathymetry, and if corrections are made for changes in water density.

TABLE 7-2
Range of Uncertainty in Precipitation and Streamflow Values Used in Computing Lake Water Balances[c]

Time Interval	Precipitation	Streamflow Inputs[b]	Streamflow Outputs
		General Range	
Daily	60–75	5–15 (50)	5 (15)
Monthly	10–25	5–15 (50)	5 (15)
Seasonal/annual	5–10	5–15 (30)	5 (15)
		Values for Typical Lake in Northern United States	
Monthly	(26)	(31)	(12)
Seasonal/annual	8 (17)	9 (23)	9 (12)

[a]Values are percentages of the true values. Those without parentheses are for "best" methodology; those in parentheses are "commonly used" methodology.

[b]Does not include overland flow.

[c]Based on analyses of Winter (1981).

Winter (1981) made a thorough analysis of the uncertainties in estimating lake water balances; even for the "readily" measurable components the results are somewhat discouraging (Table 7-2). For other components—overland flows and, especially, ground-water flows (see LaBaugh et al. 1997)—uncertainties on the order of 100% must generally be anticipated.

Applicability

The water-balance method is theoretically applicable in determining the amount of evaporation that has occurred in any water body during a given time period. In most situations, however, measurement problems preclude use of the method as an independent means of determination—especially where evaporation is a small fraction of the total outflow and less than the uncertainty in the overall balance (e.g., Figure 7-1). If reasonably accurate information on the significant balance components is available, the method can provide a rough—and in rare cases a quite accurate—check on evaporation estimated by other approaches. Table 7-2 indicates that the accuracy of the method should increase as Δt increases.

Because of the difficulty in finding situations where all the significant terms on the right side of Equation (7-16) can be measured with sufficient accuracy, the water-budget approach has only rarely been used to determine lake evaporation. One of the more successful applications of the approach was in an extensive comparison of methods for determining lake evaporation conducted by the U.S. Geological Survey in the early 1950s at Lake Hefner, OK (Harbeck et al. 1954). That lake was selected for the study because it was one of the few—out of more than 100 considered—for which a water balance could be computed with acceptable precision. Harbeck et al. (1954) concluded that daily evaporation calculated via the water balance at Lake Hefner was within 5% of the true value on 29% of the days, and within 10% on 62% of the days.

7.3.2 Mass-Transfer Approach

Theoretical Basis

The mass-transfer approach makes direct use of Equation (7-1), often written in the form

$$E = (b_0 + b_1 \cdot v_a) \cdot (e_s - e_a), \qquad \textbf{(7-17)}$$

where b_0 and b_1 are empirical constants that depend chiefly on the height at which wind speed and air vapor pressure are measured. Relations of the form of (7-17) were first formulated by the English chemist John Dalton in 1802, and are often called **Dalton-type equations.**

The theoretical analysis leading to Equations (7-1) and (7-2) indicates that $b_0 = 0$ and $b_1 = K_E$. Accepting this, we can evaluate b_1 by inserting appropriate numerical values for the quantities in Equation (7-2). Brutsaert (1982) suggested that $z_0 = 2.30 \times 10^{-4}$ m and $z_d = 0$ m over typical water surfaces,[5] and using these values along with standard values for air and water properties and a measurement height of 2 m, we find that

$$E = 1.26 \times 10^{-3} \cdot v_a \cdot (e_s - e_a), \qquad \textbf{(7-18a)}$$

where E is in m day^{-1}, v_a is in m s^{-1}, and vapor pressures are in kPa. When v_a is in km day^{-1}, the relation becomes

$$E = 1.46 \times 10^{-5} \cdot v_a \cdot (e_s - e_a). \qquad \textbf{(7-18b)}$$

Practical Considerations

Use of Equation (7-18) requires measurement of wind speed; surface vapor pressure is calculated from surface temperature via Equation (7-4) and air vapor pressure from air temperature and rela-

[5] The values of z_0 and z_d increase as v_a increases due to the effects of waves.

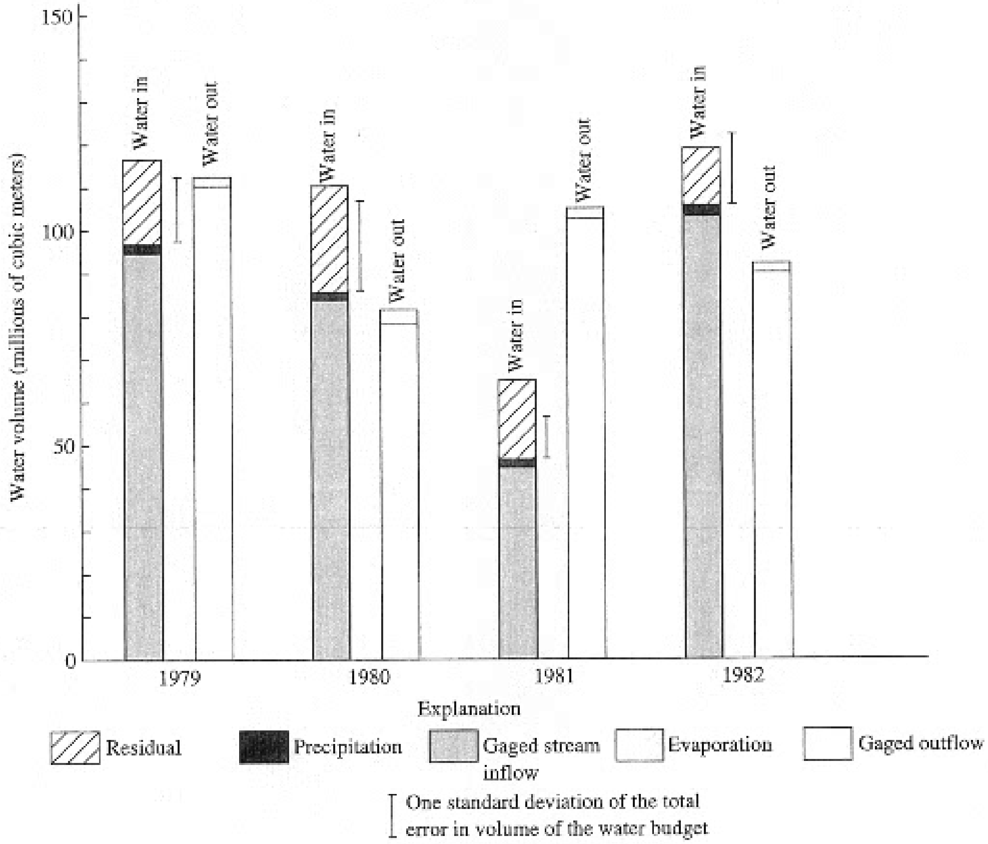


FIGURE 7-1
Water balance for the Williams Fork Reservoir, CO, for four years. The vertical bars represent the uncertainty in the overall budget (standard deviation of total error); note that this is several times larger than annual evaporation. From Labaugh (1984), used with permission of the American Geophysical Union.

tive humidity measurements via Equations (7-4) and (7-5). The value of K_E depends on the height at which wind speed is measured [Equation (7-2)], so this height must be noted (the 2-m height is standard). Clearly all measured values should be representative of the entire water body, which may be difficult to accomplish using shore-based observation stations (Sene et al. 1991). In fact, in a comparative study, Winter et al. (1995) concluded that "the mass transfer equation is acceptable only if a raft station is used so that water temperature and wind-speed data can be obtained near the center of the lake." The air vapor pressure is usually taken to be the value that is unaffected by the lake, and thus should be measured at several meters height or on the upwind shore.

Many studies have been done to verify the appropriate values for b_0 and b_1 and—because of variations in lake size, measurement height, instrument location, atmospheric stability, precision with which evaporation rate could be determined by other approaches, and time scale—each study has obtained a different set of values. The Lake Hefner project (Harbeck et al. 1954) and a study by Ficke (1972) in Indiana were among the most detailed and careful of these studies because accurate estimates of daily evaporation could be independently obtained via the water balance. In both cases, v_a is the wind speed at 2-m elevation and e_a represents the vapor pressure of the air unmodified by passage over the lake. Both studies confirmed that $b_0 = 0$, and Ficke (1972) found K_E almost identical to the value in Equation (7-18) while Harbeck et al. (1954) found K_E about 15% larger. A later study of Lake Erie found K_E very close to Harbeck's Lake Hefner value (Derecki 1976).

It appears that much of the variability in K_E found in various studies is a function of lake area. Harbeck (1962) found the empirical relation

$$K_E = 1.69 \times 10^{-5} \cdot A_L^{-0.05}, \qquad \textbf{(7-19)}$$

where K_E is in m km^{-1} kPa^{-1} [i.e., for use in Equation (7-18b)] and A_L is in km^2 (Figure 7-2). There are two reasons for this area effect: (1) since water surfaces are smoother than land surfaces, the efficiency of turbulent eddies in the vertical transport of water vapor decreases as the wind travels longer distances over a lake; and (2) the value of e_a will increase with downwind distance as evaporation occurs, decreasing the effective vapor-pressure difference over the lake compared with the value measured anywhere except on the downwind shore.

The value of K_E is also affected by atmospheric stability. When the water surface is warmer than the air, the air in contact with the surface is warmed by conduction and tends to rise. This tends to induce instability, so that there is turbulent transport of water vapor (and heat) away from the surface even in the absence of wind. This process, called **free convection,** is most common in situations where the water body has been artificially heated, as in a cooling pond or a river reach receiving cooling water from a power plant. However, it also occurs in large lakes because of thermal inertia: Derecki (1981) found considerable seasonal variation in K_E for Lake Superior, with values much lower than those given by Equations (7-18) and (7-19) in the summer when the lake is colder than the air, and higher in the fall and winter, when the lake is warmer.

One approach to account for the effect of atmospheric stability on K_E is to include a stability correction in the formulation of Equation (7-2), as described in Section D.6.8 [Equations (D-53) and (D-55)]. Alternatively, when $T_s > T_a$ the effects of free convection can be modeled by assuming that b_0 in Equation (7-17) has a positive value that increases with the temperature difference between the surface and the air. In a comparative study, Rasmussen et al. (1995) found that the following equation gave the best results:

$$E = \frac{[2.33 \cdot (T_s - T_a)^{1/3} + 2.68 \cdot v_a] \cdot (e_s - e_a)}{\lambda_v},$$
$$T_s > T_a. \qquad \textbf{(7-20)}$$

In this equation, E is evaporation rate in mm day^{-1}; T_s and T_a are the surface and air temperatures, respectively, in °C; v_a is wind speed in m s^{-1}; e_s and e_a are surface and air vapor pressures, respectively, in kPa, and λ_v is latent heat of vaporization in MJ kg^{-1} [Equation (7-8)].

Applicability

It must be emphasized that the various versions of the mass-transfer equation give the *instantaneous rate* of evaporation under given *instantaneous* values of wind speed and vapor pressure. Because the vapor pressures used in the equation are determined from measured temperatures via the nonlinear relation of Equation (7-4), and because wind speed and vapor-pressure differences may be correlated, one cannot assume that a mass-transfer equation will give the correct *time-averaged* rate of evaporation when time-averaged temperatures and wind speeds are used as independent variables. Using Lake Hefner data, Jobson (1972) determined the errors introduced in calculating average evaporation from averaged temperatures and wind speeds for averaging periods of 3 hr, 1 day, and 1 month. The results, shown in Table 7-3, indicate that little error is introduced for averaging periods up to 1 day, but that a bias is introduced with monthly averaging.

Thus the mass-transfer approach is potentially useful for determining the amount of free-water or lake evaporation that has occurred during a given time period during which the independent variables v_a, T_s, T_a, and W_a have been measured at a representative location. Observations of T_s may also be available from satellites that remotely sense thermal radiation (Croley 1989). It appears safe to use the approach when the variables are averaged over periods of up to 1 day.

The method can also be used to predict, forecast, or model free-water or lake evaporation based on predicted, forecast, or modeled values of the independent variables. It is particularly useful for modeling artificially heated water bodies, for which surface temperatures can usually be readily calculated. However, it is difficult to apply the method for naturally heated water bodies, because there are virtually no climatic data on water-surface temperatures, and modeling such temperatures is difficult. This turns out to be a serious limitation in the practical usefulness of the mass-transfer approach.

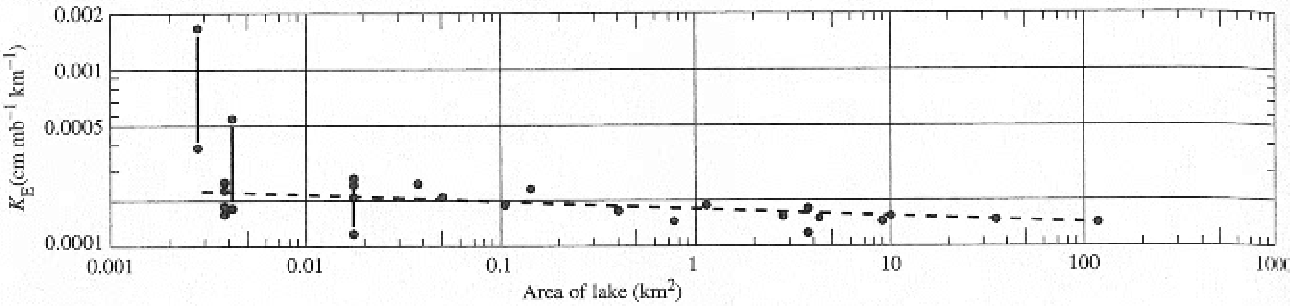


FIGURE 7-2
Value of mass-transfer coefficient, K_E, as a function of lake area. Data are for the southwestern United States, Indiana, and western Canada. The dashed line is Equation (7-19). After Dunne and Leopold (1978).

EXAMPLE 7-1

The data shown in Table 7-4 were collected at Lake Hefner, OK, on 12 July 1951. The lake's area is 9.4 km^2. Compute the evaporation via Equation (7-18).

Solution Substituting the lake's area into Equation (7-18) gives

$$K_E = 1.51 \times 10^{-5} \text{ m km}^{-1} \text{ kPa}^{-1}.$$

Equation (7-4) gives

$$e_s^* = 3.56 \text{ kPa}$$

and

$$e_a^* = 3.62 \text{ kPa},$$

and Equation (7-5) gives

$$e_a = 0.69 \cdot 3.62 \text{ kPa} = 2.50 \text{ kPa}.$$

Converting the wind speed to km day^{-1} yields

$$5.81 \text{ m s}^{-1} \cdot \frac{86{,}400 \text{ s}}{1 \text{ day}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} = 502 \text{ km day}^{-1}.$$

Substituting these values in Equation (7-18) gives

$$E = (1.51 \times 10^{-5} \text{ m km}^{-1} \text{ kPa}^{-1}) \cdot (502 \text{ km day}^{-1}) \cdot (3.56 \text{ kPa} - 2.50 \text{ kPa}) = 0.00803 \text{ m day}^{-1} = 8.03 \text{ mm day}^{-1}.$$

Note that this value is considerably larger than that determined by the water-budget approach for that day.

TABLE 7-3
Errors in Computing Time-Averaged Evaporation Using Time-Averaged Temperatures and Wind Speeds

	Averaging Period		
	3 hr	1 day	1 month
Median % error	0	0	+4
% of time periods with less than 5% error	97	79	—
% of time periods with less than 10% error	>99	93	—

Data from Jobson (1972), based on Lake Hefner studies.

7.3.3 Eddy-Correlation Approach

Theoretical Basis

As described in Section D.6.9 [Equations (D-56)–(D-58)], the rate of upward movement of water vapor near the surface (i.e., the local evaporation rate) is proportional to the time average of the product of the instantaneous fluctuations of vertical air movement, u_a', and of absolute humidity, q', around their respective mean values:

$$E = \frac{\rho_a}{\rho_w} \cdot \overline{u_a' \cdot q'}, \tag{7-21}$$

where the overbar denotes time averaging.

If sensors capable of accurately recording and integrating high-frequency (on the order of 10 s^{-1}) fluctuations in humidity and vertical velocity are used, eddy-correlation measurements are often considered to be measurements of the "true" evaporation rates because the method has a sound theoretical foundation and requires no assumptions about parameter values, the shape of the velocity profile, or atmospheric stability.

TABLE 7-4
Daily Values Measured during Evaporation Studies at Lake Hefner, OK, 12 July 1951

Daily Average						
T_a (°C)	T_s (°C)	W_a	P (kPa)	v_a(cm s^{-1})	T_{span} (°C)	v_{pan}(cm s^{-1})
27.2	26.9	0.69	97.3	581	27.5	279

Daily Total				
K_{in} (MJ m^{-2} day^{-1})	Albedo	L_{in} (MJ m^{-2} day^{-1})	Water-Budget Evaporation (cm)	Class-A Pan Evaporation (cm)
30.6	0.052	34.4	0.558	1.24

From U.S. Geological Survey (1954).

Practical Considerations

The eddy-correlation approach can give measurements representative of limited (~ 1 ha) areas upwind of the sensors, with accuracies of about 10% for time periods as short as 0.5 hr (Stannard and Rosenberry 1991). However, the stringent instrumentation requirements make the approach suitable only for research applications.

7.3.4 Energy-Balance Approach

Theoretical Basis

The energy-balance equation for an evaporating water body was given as Equation (7-15). Dividing that equation by the latent heat of vaporization, λ_v, and the density of water, ρ_w, yields

$$E = \frac{K + L - G - H + A_w - \Delta Q/\Delta t}{\rho_w \cdot \lambda_v}, \quad \textbf{(7-22)}$$

where E has dimensions [L T^{-1}].

The energy-balance approach to determining the average rate of evaporation over a time period Δt thus involves measuring or otherwise determining the rates of energy input and output by the various modes, along with the change in energy stored in the water body during Δt, and solving Equation (7-22).

Equation (7-22) can be written in a different and often more useful form by using the Bowen ratio [Equation (7-12)], so that the sensible heat-loss rate, H, is replaced by

$$H = B \cdot LE = B \cdot \rho_w \cdot \lambda_v \cdot E. \quad \textbf{(7-23)}$$

For a water body, energy can be advected in as precipitation and ground-water and surface-water inflows, and advected out by ground-water and surface-water outflows; all this advected energy is included in A_w. Then using Equation (7-23) and solving Equation (7-22) for E yields a modified energy-balance equation:

$$E = \frac{K + L - G + A_w - \Delta Q/\Delta t}{\rho_w \cdot \lambda_v \cdot (1 + B)}. \quad \textbf{(7-24)}$$

Practical Considerations

As with the water balance, the energy-balance approach requires precise determination of all the non-negligible quantities in the basic equation, and all measurement errors are included in the final computation of evaporation. However, in some situations the measurement problems for determining the energy balance are less severe than for the water balance because (1) it is possible to eliminate some of the terms in the energy-balance equation and (2) it may be possible to use regional climatic data to estimate the radiation components. We now consider how values for the modified energy-balance relation can be determined.

Shortwave Radiation Net incoming shortwave, or solar, radiation, K, is given by

$$K = K_{in} \cdot (1 - a), \quad \textbf{(7-25)}$$

where K_{in} is incoming solar radiation and a is the albedo (reflectivity) of the water surface.

As discussed in Section 5.4.2, incoming and reflected solar radiation can be measured with pyranometers, but this is routinely done at only scattered locations. More commonly, K_{in} for a lake is estimated from the clear-sky solar radiation, K_{cs}, and the fraction of sky covered by cloud, C, where K_{cs} is determined from the latitude and time of year using the relations given in Section E.1. An empirical relation that has been used in evaporation modeling (see Croley 1989) is

$$K_{in} = [0.355 + 0.68 \cdot (1 - C)] \cdot K_{cs}. \quad \textbf{(7-26)}$$

However, manual observations of C are no longer available in the United States because human weather observers are being replaced by automated stations. Because of this, Lindsey and Farnsworth (1997) recommended that, where K_{in} is not directly measured, it is best determined from gridded estimates of solar radiation developed from Geostationary Operational Environmental Satellite (GOES) imagery. Useful estimates of K_{in} can also be developed using daily maximum and minimum temperature and precipitation data (Hunt et al. 1998).

Koberg (1964) presented an empirical relation giving the albedo of a water surface as a function of K_{in}:

$$a = 0.127 \cdot \exp(-0.0258 \cdot K_{in}), \quad \textbf{(7-27)}$$

where K_{in} is in MJ m^{-2} day^{-1}; other studies have assumed a constant value in the range $0.05 \leq a \leq 0.10$.

Longwave Radiation Net longwave radiation input, L, is equal to incoming atmospheric longwave radiation flux, L_{at}, minus the portion of L_{at} reflected and the radiation flux emitted by the water surface, L_w. Since the longwave reflectivity of the surface equals $1 - \varepsilon_w$, where ε_w is the emissivity of the water surface, it follows that

$$L = L_{at} - (1 - \varepsilon_w) \cdot L_{at} - L_w = \varepsilon_w \cdot L_{at} - L_w. \quad \textbf{(7-28)}$$

Again, instruments exist that can measure longwave radiation fluxes, but these are even less common than pyranometers. Thus L_{at} and L_w are usually calculated from relations based on the Stefan-Boltzmann Equation [Equation (D-1)]. The reasoning used in Equations (5-35)–(5-37) for a snow surface can be applied to a water surface, yielding

$$L = \varepsilon_w \cdot \varepsilon_{at} \cdot \sigma \cdot (T_a + 273.2)^4 - \varepsilon_w \cdot \sigma \cdot (T_s + 273.2)^4, \quad \textbf{(7-29)}$$

where ε_{at} is the effective emissivity of the atmosphere, σ is the Stefan-Boltzmann constant ($\sigma = 4.90 \times 10^{-9}$ MJ m^{-2} day^{-1} K^{-4}), and the temperatures are in °C. The value of ε_w is 0.97 (Table D-1).

As discussed in Section 5.4.2, ε_{at} is largely a function of humidity and cloud cover, and can be estimated via empirical relations such as

$$\varepsilon_{at} = 1.72 \cdot \left(\frac{e_a}{T_a + 273.2}\right)^{1/7} \cdot (1 + 0.22 \cdot C^2), \quad \textbf{(7-30)}$$

where e_a is in kPa, T_a is in °C, and C is cloud-cover fraction (Kustas et al. 1994).

Conduction to Ground In most situations, the heat exchange by conduction between a lake and the underlying sediments is negligible.

Water-Advected Energy Net water-advected energy is found from

$$A_w = c_w \cdot \rho_w \cdot (w \cdot T_a + SW_{in} \cdot T_{swin} - SW_{out} \cdot T_{swout} + GW_{in} \cdot T_{gwin} - GW_{out} \cdot T_{gwout}), \quad \textbf{(7-31)}$$

where ρ_w is the mass density of water, c_w is the specific heat of water, w is average precipitation rate, SW and GW represent surface-water and groundwater inflows and outflows (per subscript) expressed as volumes per time per unit lake area, and the Ts represent temperatures of the respective inflows and outflows.

Change in Stored Energy The change in energy storage in a lake is found from the volumes and average temperatures of the lake water at the beginning and end of Δt:

$$\Delta Q = \frac{c_w \cdot \rho_w}{A_L} \cdot (V_2 \cdot T_{L2} - V_1 \cdot T_{L1}). \quad \textbf{(7-32)}$$

Here, V is lake volume, T_L is average lake temperature, the subscripts 1 and 2 designate values at the beginning and end of Δt, respectively, and A_L is lake area.

Evaluation of ΔQ thus involves the same considerations and difficulties as determining ΔV in the water-balance equation [Equation (7-16)], plus the problem of computing an average lake temperature. These difficulties can be minimized by choosing Δt so that the term in parentheses in Equation (7-32) is likely to be small. In most situations this will be true if $\Delta t = 1$ yr; in some climates lakes become isothermal at the temperature of maximum density (3.98 °C) twice a year, so those times can be used to bound Δt. It may be possible to relate average lake temperature to water-surface temperature (Rosenberry et al. 1993), which can be remotely sensed; however, the relation between the two must be established for each lake, and at least for deep lakes will vary seasonally (Figure 7-3a).

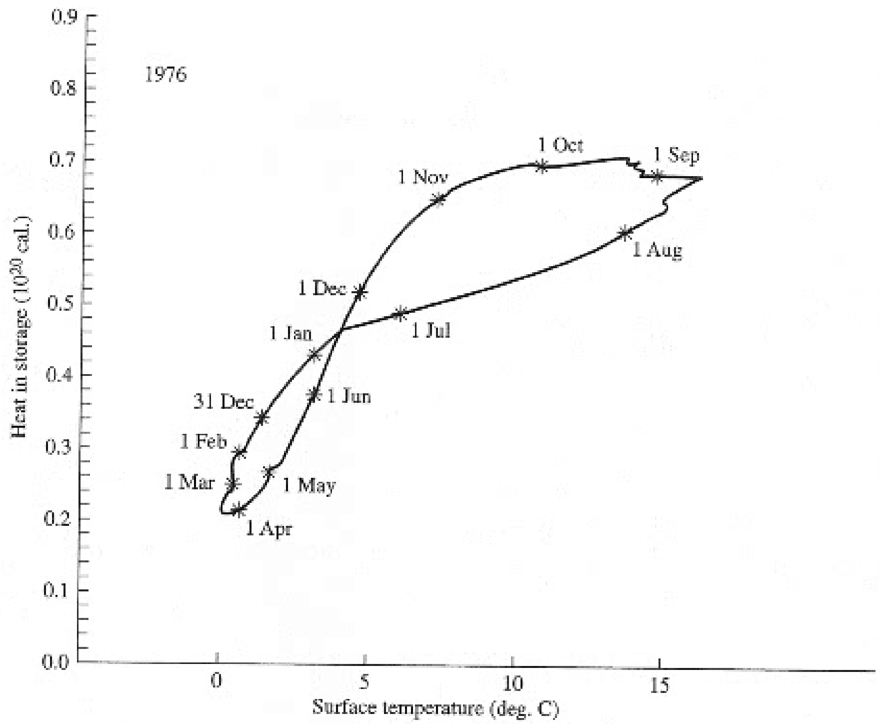


FIGURE 7-3
(a) Variation of total heat storage with surface temperature in Lake Superior in 1976. (b) Annual cycles of air temperature, humidity, wind speed, and evaporation in Lake Superior, 1975–1977. From Croley (1992), used with permission of the American Geophysical Union.

Heat storage significantly influences the timing of evaporation in large, deep lakes (Croley 1992). For example, it reaches a maximum in the fall in Lake Superior (Figure 7-3a), shifting the annual peak of evaporation such that it is out of phase with the annual cycles of air temperature, water temperature, and humidity (Figure 7-3b).

The Bowen Ratio The principal advantage of using the Bowen ratio in the energy balance is to eliminate the need for wind data, which would be required if sensible-heat exchange were separately evaluated. However, one still needs data for surface and air temperatures and relative humidity, so the same considerations for time averaging apply as for the mass-transfer approach.

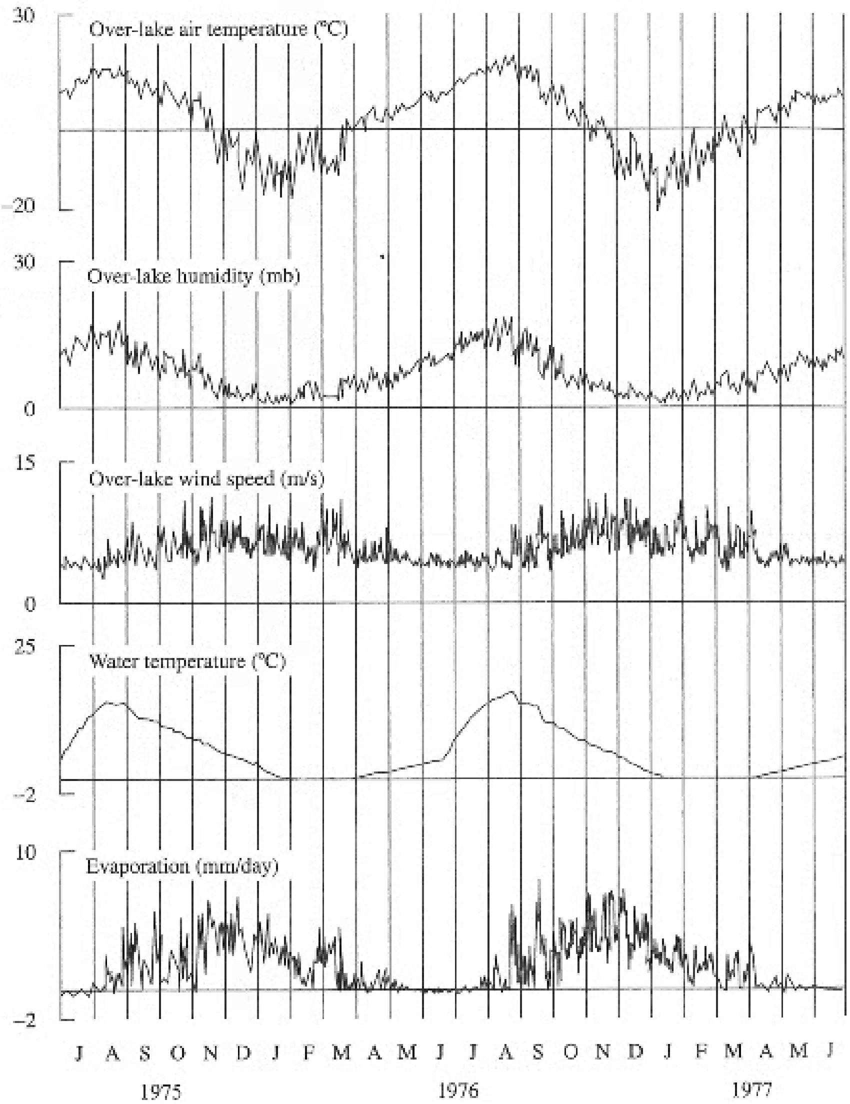
Applicability

Use of Equation (7-24) to determine actual evaporation for a lake involves many of the same kinds of measurement difficulties as for the water-balance approach. L. J. Anderson, who conducted the elaborate energy-balance studies at Lake Hefner, concluded that "the energy-budget equation, when applied to periods greater than 7 days, will result in a maximum accuracy approaching ± 5 per cent of the mean . . . evaporation, providing all terms in the energy budget have been evaluated with the utmost accuracy, particularly change in energy storage" (Harbeck et al. 1954, p. 117). He found that considerable error in evaluating the change in energy storage may occur if the method is applied to periods of less than seven days. Rosenberry et al. (1993) found that the energy-budget approach gave the best estimates of lake evaporation for a small lake in Minnesota and investigated the effects of using various instrumentation and approaches to determining energy-budget components.

As with the mass-transfer method, it is difficult to apply the energy-budget approach in the prediction, forecasting, or modeling mode because of the requirement for water-surface temperatures. However, Hostetler and Bartlein (1990) have developed a model that simulates the vertical distribution of lake temperatures through time coupled to energy-balance computations, and this appears promising as an approach to estimating the effects of past and future climate change on lake evaporation.

In estimating free-water evaporation, Equation (7-24) can be simplified because the most troublesome terms, A_w and $\Delta Q/\Delta t$, are small; this is also generally true in calculating annual evaporation for

FIGURE 7-3
(Continued)



lakes if their residence times do not greatly exceed one year. However, even for these cases there is the need for water-surface temperature. As we will see in Section 7.3.5, the most useful applications of the energy-balance approach are in combination with the mass-transfer method, whereby one can eliminate the need for surface data in estimating free-water evaporation.

EXAMPLE 7-2

Estimate the evaporation from Lake Hefner on 12 July 1951 using the energy-balance approach with the data in Table 7-4.

Solution Data indicate that water-advected energy and change in heat storage were negligible over this date (U.S. Geological Survey 1954). Net shortwave radiation, K, is

$$K = K_{in} \cdot (1 - a) = (30.6 \text{ MJ m}^{-2} \text{ day}^{-1}) \cdot (1 - 0.052)$$
$$= 29.0 \text{ MJ m}^{-2} \text{ day}^{-1}.$$

Incoming longwave radiation, L_{at}, was measured, so net longwave radiation is found as

$$L = L_{at} - 0.03 \cdot L_{at} - 0.97 \cdot (4.90 \times 10^{-9}) \cdot (T_s + 273.15)^4,$$

and substituting appropriate values from Table 7-4 gives $L = -5.23$ MJ m^{-2} day^{-1}. Thus

$$K + L = 23.8 \text{ MJ m}^{-2} \text{ day}^{-1}.$$

From Equation (7-8),

$$\lambda_v = 2.50 - (2.36 \times 10^{-3}) \cdot 26.9 = 2.44 \text{ MJ kg}^{-1}.$$

With the vapor pressures calculated in Example 7-1, the Bowen ratio, B, is

$$B = \frac{(1.00 \times 10^{-3} \text{ MJ kg}^{-1} \text{ K}^{-1}) \cdot (97.3 \text{ kPa}) \cdot (26.9^\circ\text{C} - 27.2^\circ\text{C})}{0.622 \cdot (2.44 \text{ MJ kg}^{-1}) \cdot (3.56 \text{ kPa} - 2.50 \text{ kPa})} = -0.0183.$$

We can now substitute into Equation (7-24) as follows:

$$E = \frac{23.8 \text{ MJ m}^{-2} \text{ day}^{-1}}{(1000 \text{ kg m}^{-3}) \cdot (2.44 \text{ MJ kg}^{-1}) \cdot (1 - 0.0183)} = 0.00959 \text{ m day}^{-1} = 9.59 \text{ mm day}^{-1}.$$

This value is about 1.6 mm day^{-1} larger than that estimated via mass transfer (Example 7-1), and both estimates are considerably larger than determined from the water balance.

7.3.5 Penman or Combination Approach

Theoretical Basis

Following the steps shown in Box 7-1, Penman (1948) was the first to show that the mass-transfer and energy-balance approaches could be combined to arrive at an evaporation equation that did not require surface-temperature data. Van Bavel (1966) generalized Penman's original development by replacing an empirical wind function with Equation (7-2), resulting in the following theoretically sound and dimensionally homogeneous relation:

BOX 7-1

Derivation of the Penman Combination Equation

Neglecting ground-heat conduction, water-advected energy, and change in energy storage, Equation (7-22) becomes

$$E = \frac{K + L - H}{\rho_w \cdot \lambda_v}. \quad \text{(7B1-1)}$$

The sensible-heat transfer flux is given by Equation (7-9):

$$H = K_H \cdot v_a \cdot (T_s - T_a). \quad \text{(7B1-2)}$$

The slope of the saturation-vapor vs. temperature curve at the air temperature can be approximated as

$$\Delta = \frac{e_s^* - e_a^*}{T_s - T_a}, \quad \text{(7B1-3)}$$

from which we obtain

$$T_s - T_a = \frac{e_s^* - e_a^*}{\Delta}. \quad \text{(7B1-4)}$$

Equation (7B1-4) can now be substituted into (7B1-2):

$$H = \frac{K_H \cdot v_a}{\Delta} \cdot (e_s^* - e_a^*). \quad \text{(7B1-5)}$$

This relation remains true if e_a is added and subtracted from each of the terms in brackets:

$$H = \frac{K_H \cdot v_a}{\Delta} \cdot (e_s^* - e_a) - \frac{K_H \cdot v_a}{\Delta} \cdot (e_a^* - e_a). \quad \text{(7B1-6)}$$

Making use of Equation (7-3), we rearrange Equation (7-1) to give

$$e_s^* - e_a = \frac{E}{K_E \cdot v_a}. \quad \text{(7B1-7)}$$

Substituting (7B1-7) into (7B1-6) yields

$$H = \frac{K_H \cdot v_a}{\Delta} \cdot \frac{E}{K_E \cdot v_a} - \frac{K_H \cdot v_a}{\Delta} \cdot (e_a^* - e_a). \quad \text{(7B1-8)}$$

Now Equation (7B1-8) can be substituted into (7B1-1) and the result solved for E:

$$E = \frac{K + L + (K_H \cdot v_a/\Delta) \cdot (e_a^* - e_a)}{\rho_w \cdot \lambda_v + (K_H/K_E) \cdot \Delta}. \quad \text{(7B1-9)}$$

From the definitions of K_H [Equation (7-10)], K_E [Equation (7-2)], and γ [Equation (7-13)], we see that

$$K_H = \gamma \cdot \rho_w \cdot \lambda_v \cdot K_E, \quad \text{(7B1-10)}$$

and substituting this relation into (7B1-9), multiplying the numerator and denominator by Δ, and making use of Equation (7-5) yields Equation (7-33).

$$E = \frac{\Delta \cdot (K + L) + \gamma \cdot K_E \cdot \rho_w \cdot \lambda_v \cdot v_a \cdot e_a^* \cdot (1 - W_a)}{\rho_w \cdot \lambda_v \cdot (\Delta + \gamma)}. \tag{7-33}$$

Equation (7-33) assumes no heat exchange with the ground,[6] no water-advected energy, and no change in heat storage, and makes use of one approximation [Equation (7B1-3)]. This relation has become known as the **Penman equation** or **combination equation.**

Note that the essence of the Penman equation can be represented as

$$E = \frac{\Delta \cdot \text{net radiation} + \gamma \cdot \text{"mass transfer"}}{\Delta + \gamma}; \tag{7-34}$$

that is, evaporation rate is a weighted sum of a rate due to net radiation and a rate due to mass transfer. Note, however, that the mass-transfer relation now depends on the difference between the actual vapor pressure and the saturation vapor pressure at the *air* temperature, rather than at the water-surface temperature. The weighting coefficients are given by the slope of the saturation-vapor-pressure vs. temperature curve at the air temperature, Δ, [Equation (7-6)], and the psychrometric constant, γ, which depends on atmospheric pressure [Equation (7-13)].

While Penman (1948) intended his approach to eliminate the need for surface-temperature data, such data are in fact required to evaluate the longwave energy exchange, L [Equation (7-29)], except in the rare instances where measured values are available. Kohler and Parmele (1967) showed that another approximation could be made to avoid this problem with little error and arrived at the following modifications of the terms in Equation (7-33):

Replace L with L', where

$$L' \equiv \varepsilon_w \cdot L_{at} - \varepsilon_w \cdot \sigma \cdot (T_a + 273.2)^4; \tag{7-35}$$

replace γ with γ', where

$$\gamma' \equiv \gamma + \frac{4 \cdot \varepsilon_w \cdot \sigma \cdot (T_a + 273.2)^3}{K_E \cdot \rho_w \cdot \lambda_v \cdot v_a}. \tag{7-36}$$

As noted, Penman's development did not include water-advected energy or changes in energy storage, and hence is applicable only for free-water evaporation. Kohler and Parmele (1967) also developed a method whereby the effects of water-advected energy and changes in heat storage can be incorporated in the combination approach to provide a more generalized estimate of open-water evaporation. They reasoned that lake evaporation, E_L, is related to Penman evaporation, E_P, as

$$E_L = E_P + \alpha_{KP} \cdot \left(A_w - \frac{\Delta Q}{\Delta t} \right), \tag{7-37}$$

where A_w and ΔQ are given by Equations (7-31) and (7-32), respectively, and α_{KP} is the fraction of the net addition of energy from advection and storage that was used in evaporation during Δt. Since the total net addition ($A_w - \Delta Q/\Delta t$) is allocated among evaporation, sensible-heat transfer, and emitted radiation, it follows that

$$\alpha_{KP} = \frac{\Delta}{\Delta + \gamma + 4 \cdot \varepsilon_w \cdot \sigma \cdot (T_w + 273.2)^3 / (\rho_w \cdot \lambda_v \cdot K_E \cdot v_a)}. \tag{7-38}$$

Practical Considerations

The combination approach as originally developed for free-water evaporation requires representative data on net shortwave radiation, net longwave radiation, wind speed, air temperature, and relative humidity. As modified by Equations (7-35)–(7-36), it requires data on incoming and reflected shortwave radiation, incoming longwave radiation, wind speed, air temperature, and relative humidity. Since incoming longwave flux can be estimated from

$$L_{at} = \varepsilon_{at} \cdot \sigma \cdot (T_a + 273.2)^4, \tag{7-39}$$

with ε_{at} estimated from relations like Equation (7-30), the data requirements reduce to incident shortwave radiation [which can be estimated from Equations (7-26) and (7-27)], wind speed, air temperature, and relative humidity.

Where humidity data are lacking, one can assume for non-arid regions that the dew-point temperature equals the daily minimum temperature (Gentilli 1955; Bristow 1992) and use that temperature in Equation (7-4) to estimate e_a. Linacre (1993) developed an even more simplified empirical version of the Penman equation that does not require direct measurement of radiation or humidity.

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[6] If it is significant, the ground-heat conduction term can be included by replacing $(K + L)$ with $(K + L - G)$.

Note that Kohler and Parmele's (1967) generalization of the combination approach to give lake evaporation via Equations (7-37) and (7-38) reintroduces the need for lake water-surface temperature and the flow and temperature data needed to evaluate A_w and ΔQ.

Applicability

Many studies have shown that evaporation estimates made with the combination approach compare well with those determined by other methods. For example, Van Bavel (1966) showed that free-water evaporation calculated with Equation (7-33) compared closely with actual evaporation from a shallow pan on an hourly basis (Figure 7-4), and the Penman method performed well in comparative studies of lake (Winter et al. 1995) and wetland (Souch et al. 1996) evaporation. Kohler and Parmele (1967) tested their generalized version of the combination approach against evaporation determined by water-balance methods at Lake Hefner and other lakes, and found good agreement when evaporation was calculated daily and summed to use with weekly to monthly values of advection and storage.

Because it gives satisfactory results, because it has a theoretical foundation, and because it requires meteorological inputs that are widely available or can be reasonably well estimated from available data, the combination method has become the "standard" hydrological method for determining free-water evaporation. One of its major advantages is that it eliminates the need for surface-temperature data, so that it can be readily used both in determining evaporation from measured data and in the predictive or modeling context.

Van Bavel (1966) showed that Equation (7-33) can be used to give reliable estimates of daily evaporation using measured total daily radiation and average daily values of temperature, humidity, and wind speed. As with the mass-transfer approach, it is probably unwise to expect such reliability if averages for monthly or longer periods are used.

It must be emphasized that, as with all approaches to determining evaporation, calculated

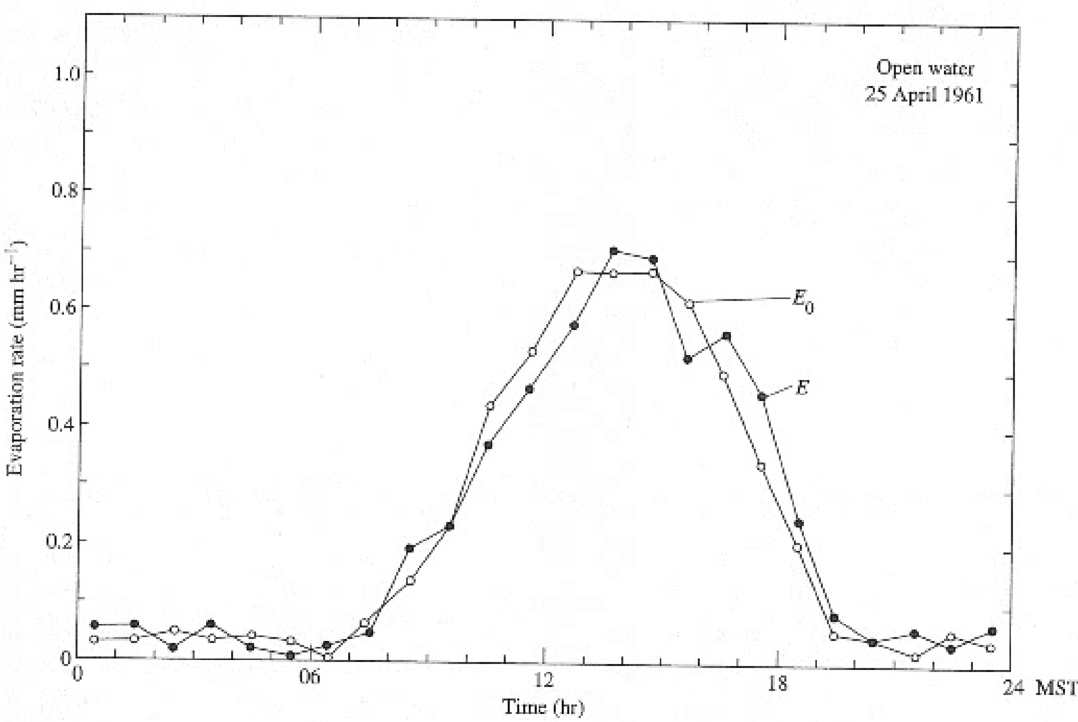

FIGURE 7-4
Comparison of hourly evaporation rates determined from measurements in a shallow pan (E) with those computed via the combination approach [E_0, Equation (7-33)] at Tempe, AZ. From Van Bavel (1966), used with permission of the American Geophysical Union.

values are only valid to the degree that the input data are correct and representative of the evaporating water body. In particular, Van Bavel (1966) found that significant errors can be introduced when empirical radiation relations [e.g., Equations (7-26)–(7-30)] are used in place of measured values.

EXAMPLE 7-3

Estimate the evaporation from Lake Hefner on 12 July 1951 using the combination approach with the data in Table 7-4.

Solution The values of Δ and γ are found from Equations (7-6) and (7-13), respectively:

$$\Delta = 0.212\text{ kPa K}^{-1} \quad \text{and} \quad \gamma = 0.0645\text{ kPa K}^{-1}.$$

All other values needed for Equation (7-33) are available from Table 7-4 and Examples 7-1 and 7-2. Since K_E was calculated in m km^{-1} kPa^{-1}, v_a must be in km day^{-1} to give consistent units. We then obtain

$$E = \frac{0.212 \cdot 23.8 + 0.0645 \cdot (1.51 \times 10^{-5}) \cdot 1000 \cdot 2.44 \cdot 502 \cdot 3.62 \cdot (1 - .69)}{1000 \cdot 2.44 \cdot (0.212 + 0.0645)}$$
$$= 0.00947\text{ m day}^{-1} = 9.47\text{ mm day}^{-1}.$$

This value is quite close to that calculated from the energy balance.

It is also of interest to compare this result with that obtained from the combination approach using the modifications of Kohler and Parmele (1967). To do this, we replace L with L', from Equation (7-35):

$$L' = 0.97 \cdot 34.4 - 0.97 \cdot (4.90 \times 10^{-9}) \cdot (27.2 + 273.15)^4 = -5.38\text{ MJ m}^{-2}\text{ day}^{-1}.$$

This gives a net radiation term of

$$29.0 - 5.38 = 23.6\text{ MJ m}^{-2}\text{ day}^{-1}.$$

Next we replace γ with γ' from Equation (7-36):

$$\gamma' = 0.0645 + \frac{4 \cdot 0.97 \cdot (4.90 \times 10^{-9}) \cdot (27.2 + 273.2)^3}{(1.51 \times 10^{-5}) \cdot 1000 \cdot 2.44 \cdot 502}$$
$$= 0.0924\text{ kPa K}^{-1}.$$

Substituting these quantities into the Penman Equation yields

$$E = \frac{0.212 \cdot 23.6 + 0.0924 \cdot (1.51 \times 10^{-5}) \cdot 1000 \cdot 2.44 \cdot 502 \cdot 3.62 \cdot (1 - .69)}{1000 \cdot 2.44 \cdot (0.212 + 0.0924)}$$
$$= 0.00934\text{ m day}^{-1} = 9.34\text{ mm day}^{-1},$$

which is quite close to the value calculated via Equation (7-33).

Thus the values calculated from the combination method for this date are fairly close to that obtained from the energy balance, higher than given via mass transfer, and substantially higher than that determined from the water balance.

7.3.6 Pan-Evaporation Approach

Theoretical Basis

A direct approach to determining free-water evaporation is to expose a cylindrical pan of liquid water to the atmosphere and to solve the following simplified water-balance equation for a convenient time period, Δt (usually one day):

$$E = W - [V_2 - V_1]. \quad \textbf{(7-40)}$$

Here, W is precipitation during Δt and V_1 and V_2 are the storages at the beginning and end of Δt, respectively. Precipitation is measured in an adjacent non-recording rain gage; the storage volumes are determined by measuring the water level in a small **stilling well** in the pan with a high-precision micrometer called a **hook gage.** The water surface is maintained a few centimeters below the pan rim by adding measured amounts of water as necessary.

Pans on land are placed in clearings suitable for rain gages (see Section 4.2.2), surrounded by a fence to prevent animals from drinking, and may be sunken so that the water surface is approximately in the same plane as the ground surface or raised a standard height above ground. For special studies of lake evaporation, pans are sometimes placed in the center of a floating platform with dimensions large enough to insure stability and prevent water splashing in, again with the surface either at or above the lake surface.

Practical Considerations

Several different standard types of pans are used by different countries and agencies. Shaw (1988) described the types used in Great Britain, the Soviet Union, and the United States, and Figure 7-5 shows the standard "Class-A" pan used by the U.S. National Weather Service (NWS) and in Canada.

An evaporation pan differs from a lake in having far less heat-storage capacity, in lacking surface- or ground-water inputs or outputs, and, with raised

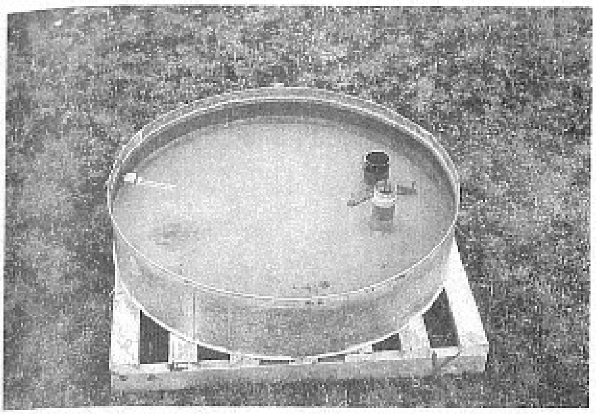

FIGURE 7-5
Standard U.S. National Weather Service "Class-A" evaporation pan. The pan is 4 ft (1.22 m) in diameter by 10 in. (25.4 cm) high and set on a low platform. The stilling well in which water level is measured is on the right, and a floating thermometer is on the left. Typically an anemometer is installed next to the pan and a rain gage must be located nearby. Photo by author.

pans like the Class-A, in having sides exposed to the air and sun. These differences significantly affect the energy balance, elevating the warm-season average temperature and vapor pressure of the water surface of a pan relative to that of a nearby lake. The ratio of lake evaporation to pan evaporation is called the **pan coefficient**; its annual average over the United States is about 0.7. Within the evaporation season, heat-storage effects cause water temperatures in lakes to be generally lower than those of pans in the spring, and higher in the fall. Thus pan coefficients follow the same pattern, with values lower than the seasonal average in the spring and higher in the fall (e.g., Figure 7-6).

The Class-A pan is usually installed with an anemometer and a floating maximum–minimum thermometer, providing data on average daily wind speed and average water-surface temperature. Kohler et al. (1955) developed the following empirical equation to use these data along with air temperature to account for the energy exchange through the sides of a pan and thereby adjust daily pan evaporation to daily free-water evaporation:

$$E_{fw} = 0.7 \cdot [E_{pan} \pm 0.064 \cdot P \cdot \alpha_{pan} \cdot (0.37 + 0.00255 \cdot v_{pan}) \cdot |T_{span} - T_a|^{0.88}]. \quad \textbf{(7-41)}$$

In this equation, E_{fw} and E_{pan} are daily free-water and pan evaporation, respectively, in mm day^{-1}, P is atmospheric pressure in kPa, v_{pan} is the average wind speed at a height of 15 cm above the pan in km day^{-1}, T_{span} is the water-surface temperature in the pan, temperatures are in °C, and the operation following E_{pan} is + when $T_{span} > T_a$ and – when $T_{span} < T_a$. The factor α_{pan} is the proportion of energy exchanged through the sides of the pan that is used for, or lost from, evaporation; it can be estimated as

$$\alpha_{pan} = 0.34 + 0.0117 \cdot T_{span} - (3.5 \times 10^{-7}) \cdot (T_{span} + 17.8)^3 + 0.0135 \cdot v_{pan}^{0.36}, \quad \textbf{(7-42)}$$

using the same units as in Equation (7-41) (Linsley et al. 1982).

EXAMPLE 7-4

Adjust the measured Class-A pan evaporation at Lake Hefner to give an estimate of the free-water evaporation on 12 July 1951. Data are given in Table 7-4.

Solution First we convert v_{pan} from m s^{-1} to km day^{-1} as in Example 7-1, and find v_{pan} = 241 km day^{-1}. Next compute α_{pan} via Equation (7-42):

$$\alpha_{pan} = 0.34 + 0.0117 \cdot 27.5 - (3.5 \times 10^{-7}) \cdot (27.5 + 17.8)^3 + 0.0135 \cdot 241^{0.36} = 0.726.$$

Substituting this value into Equation (7-41) yields

$$E_{fw} = 0.7 \cdot [12.4 + 0.064 \cdot 97.3 \cdot 0.726 \cdot (0.37 + 0.00255 \cdot 241) \cdot |27.5 - 27.2|^{0.88}] = 9.76 \text{ mm day}^{-1},$$

which is close to the value given by the combination method.

Because inputs and outputs of energy through the sides of a pan balance out over the course of a year, adjustments via Equation (7-41) and (7-42) are not needed for annual values.

Applicability

Pan evaporation data provide a useful basis for understanding the regional climatology of seasonal or annual free-water evaporation: The NWS publishes

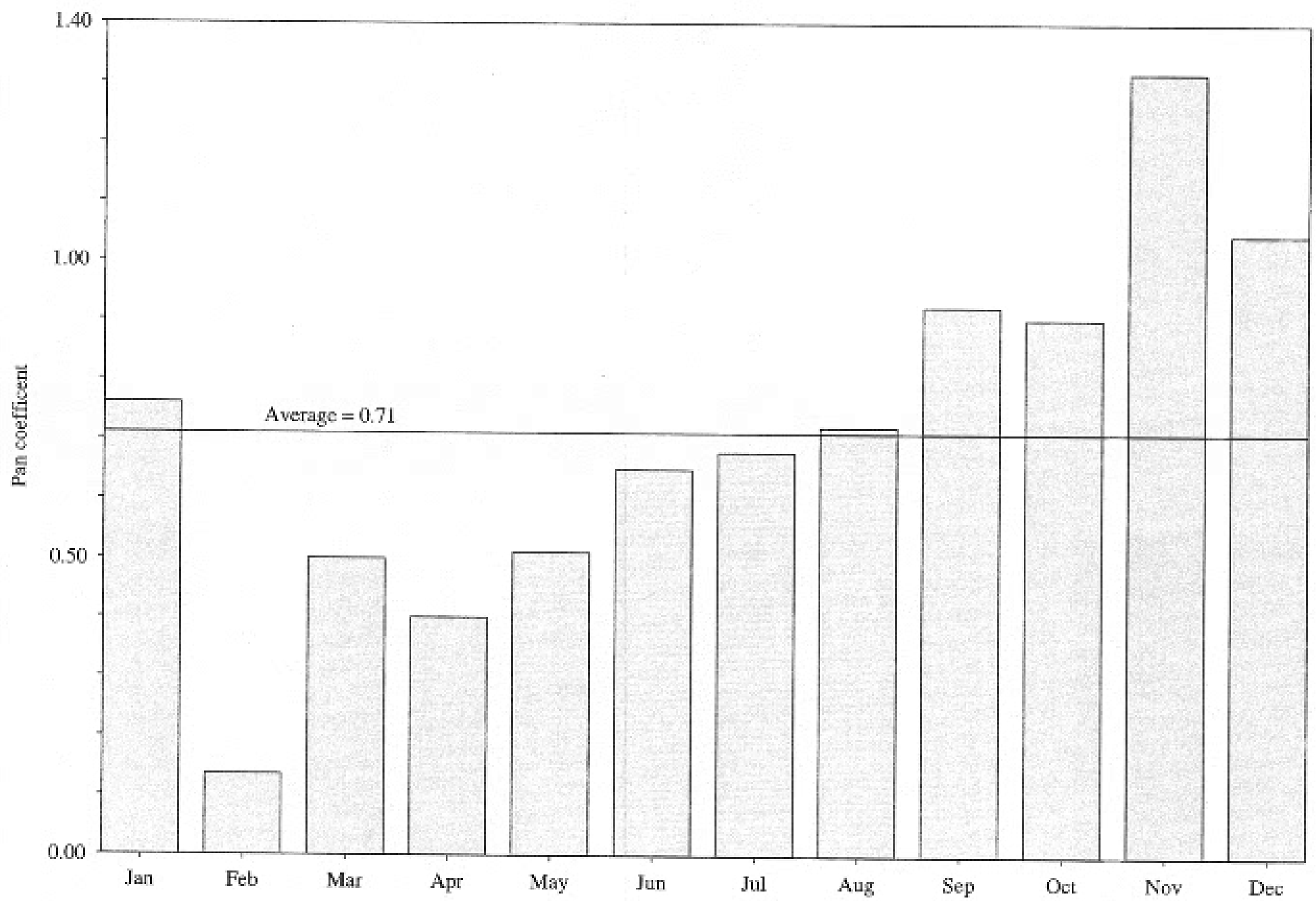

FIGURE 7-6
Monthly Class-A pan coefficients at Lake Hefner, OK, June 1950–May 1951. Data from Harbeck et al. (1954).