

it may induce droplet formation and growth, it cannot induce the importation of water vapor necessary to sustain the process of precipitation.

## D.6 PHYSICS OF TURBULENT TRANSFER NEAR THE GROUND

### D.6.1 Planetary Boundary Layer

The **planetary boundary layer** is the lowest layer of the atmosphere in which the winds, which are induced by horizontal pressure gradients, are affected by the frictional resistance of the surface. The thickness of this layer varies in space and time from a few tens of meters to one or two kilometers, depending on the topography and roughness of the surface, the wind velocity, and the rate of heating or cooling of the surface (Peixoto and Oort 1992). The frictional resistance produces turbulent eddies,

which are irregular and chaotic motions with vertical components (Figure D-9). These vertical components are the means by which momentum, sensible heat, and water vapor and its accompanying latent heat are exchanged between the atmosphere and the land surface.

### D.6.2 Turbulent Velocity Fluctuations

Because of turbulent eddies, the instantaneous horizontal wind velocity at any level,  $v_a$ , fluctuates in time, and we can separate the instantaneous velocity into a time-averaged component,  $\bar{v}_a$ , and a deviation from that average,  $v_a'$ , caused by the eddies:

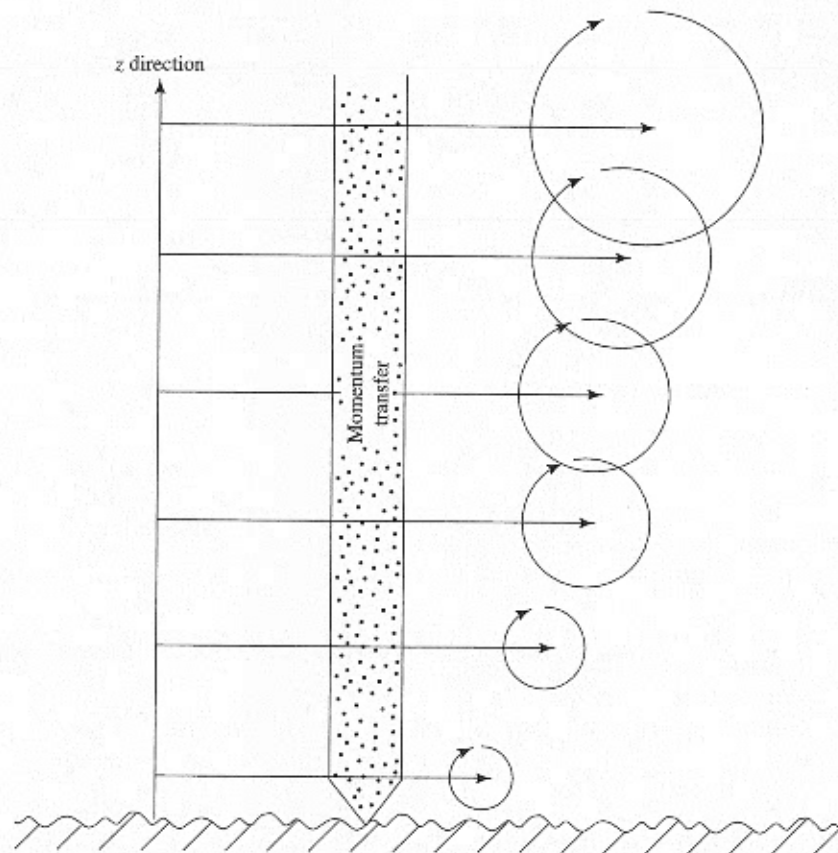
$$v_a = \bar{v}_a + v_a' \quad (\text{D-15})$$

Note that  $v_a'$  may be positive or negative and the overbar denotes time-averaging. By definition, the time-averaged value of the fluctuations equals zero, so

$$\overline{v_a'} = 0 \quad (\text{D-16})$$

FIGURE D-9

Conceptual diagram of the process of momentum transfer by turbulent diffusion. Friction caused by surface roughness reduces average velocities (straight arrows) near the surface and produces turbulent eddies (circular arrows), resulting in a net downward transfer of momentum. The vertical component of the eddies moves heat and water vapor upward or downward, depending on the directions of temperature and vapor-pressure gradients.



and

$$\overline{v_a + v_a'} = \overline{v_a} + \overline{v_a'} = \overline{v_a}. \quad (\text{D-17})$$

The wind speed discussed in this text is the value averaged over a period appropriate for defining the "prevailing conditions" for the analysis (usually one hour to one day). To simplify the notation, we use the symbol  $v_a$  rather than  $\overline{v_a}$  to denote this time average.

Under prevailing conditions producing a given average horizontal wind speed at a given level, turbulent eddies are also reflected in a time-varying vertical air velocity, designated  $w_a$ , that can be similarly separated into a time-averaged component,  $\overline{w_a}$ , and an instantaneous deviation from that average,  $w_a'$ :

$$w_a = \overline{w_a} + w_a'. \quad (\text{D-18})$$

However, there is no net vertical air movement, so the time average of both components is zero and we have

$$\overline{w_a} = 0 \quad (\text{D-19})$$

and

$$\overline{w_a'} = 0. \quad (\text{D-20})$$

Because friction due to the surface causes the average wind speed to increase upward in the planetary boundary layer (Section D.6.3), a positive (negative) vertical velocity deviation,  $w_a'$ , reduces (increases) the horizontal velocity at the new level and produces a negative (positive) fluctuation in the horizontal velocity,  $v_a'$ . The intensity of turbulence can be characterized by a quantity called the **friction velocity**,  $u_*$ , defined as

$$u_* = (-v_a' \cdot w_a')^{1/2}, \quad (\text{D-21})$$

where the minus sign is required because simultaneous values of  $w_a'$  and  $v_a'$  have opposite signs.

### D.6.3 Vertical Distribution of Wind Velocity

Measurements in the planetary boundary layer show that the vertical distribution of wind can be well represented by a logarithmic relation that has been developed from theoretical considerations:

$$v_a = \frac{1}{k} \cdot u_* \cdot \ln \left( \frac{z - z_d}{z_0} \right), \quad z > z_d + z_0. \quad (\text{D-22})$$

In this equation,  $v_a$  is the time-averaged velocity at height  $z$  above the ground surface,  $[L T^{-1}]$ ;  $u_*$  is the

friction velocity  $[L T^{-1}]$ ; the height  $z_d$  is called the **zero-plane displacement**  $[L]$ ; the height  $z_0$  is called the **roughness height**,  $[L]$  (Figure D-10); and  $k$  is a dimensionless constant. Note that  $v_a = 0$  at  $z = z_d + z_0$ , which is the effective or nominal surface level.

Equation (D-22) is known as the **Prandtl-von Karman Universal Velocity-Distribution** for turbulent flows. Experimental data have shown that  $k = 0.4$  and that the values of  $z_d$  and  $z_0$  are approximately proportional to the average height of the vegetation or other roughness elements covering the ground surface [Equations (7-50) and (7-51)].

### D.6.4 Diffusion

**Diffusion** is the process by which constituents of a fluid, such as its momentum, heat content, or a dissolved or suspended constituent, are transferred from one position to another within the fluid. Such transfers occur whenever there are differences in concentrations of the constituent in different parts of the fluid. The rate of transfer of a constituent  $X$  in the direction  $z$  is directly proportional to the gradient of concentration of  $X$  in the  $z$  direction; that is,

$$F_z(X) = -D_X \cdot \frac{dC(X)}{dz}, \quad (\text{D-23})$$

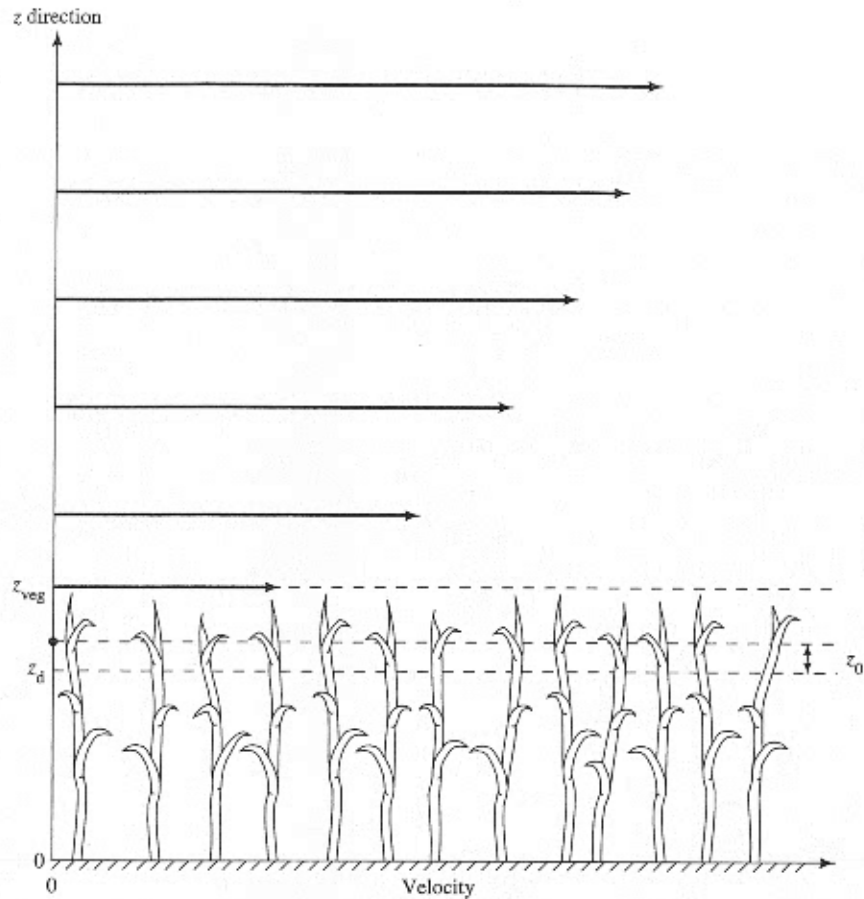
where  $F_z(X)$  is the rate of transfer of  $X$  in direction  $z$  per unit area per unit time (called the **flux** of  $X$ ),  $C(X)$  is the concentration of  $X$ , and  $D_X$  is called the **diffusivity** of  $X$  in the fluid (Figure D-11). Equation (D-23) is the mathematical expression of Fick's First Law of Diffusion (Table 2-1).

The minus sign in Equation (D-23) indicates that  $X$  always moves from regions where its concentration is higher to regions where its concentration is lower. Diffusivity always has dimensions  $[L^2 T^{-1}]$ , while the dimensions of the other quantities in Equation (D-23) depend on the nature of the property  $X$ . As will be developed later, the value of the diffusivity depends on (1) the nature of the property  $X$  and (2) the physical mechanism by which the property is transferred.

We have already seen that water vapor is transferred between the surface and the air whenever there is a difference in the vapor pressure between the surface and the overlying air [Equation (D-12)] and that a transfer of latent heat always accompanies the vapor transfer [Equations (D-13) and (D-14)]. A second mode of non-radiant heat transfer occurs

FIGURE D-10

Vertical distribution of wind velocity over a vegetative surface of height  $z_{veg}$ . The profile follows the logarithmic relation of Equation (D-22). The zero-plane displacement,  $z_d$ , is about  $0.7 \cdot z_{veg}$ , and the roughness height,  $z_0$ , is about  $0.1 \cdot z_{veg}$ . Note that Equation (D-22) gives  $v_a = 0$  when  $z = z_d + z_0$ .



in the form of **sensible heat**, that is, heat energy that can be directly sensed via measurement of the temperature and application of Equation (B-19). Thus, sensible-heat transfer occurs whenever there is a temperature difference between the surface and the air; the relation that is analogous to Equation (D-12) is

$$H \propto T_s - T_a, \quad (\text{D-24})$$

where  $H$  is the rate of sensible-heat transfer from surface to air per unit area of surface [ $\text{E L}^{-2} \text{T}^{-1}$ ], and  $T_s$  and  $T_a$  are the surface and air temperatures, respectively. When  $T_s < T_a$ ,  $H$  is negative and sensible heat moves from the air to the surface.

Equation (D-23) is the basis for a quantitative understanding of the processes that transfer water vapor and latent and sensible heat between the surface and the overlying atmosphere. However, several steps are required to combine that equation with Equations (D-12), (D-13), and (D-24) to produce a useful quantitative formulation of these processes.

We will be concerned here only with diffusion in the vertical direction, and  $z$  will henceforth represent the height above the ground surface [L].

Since the absolute humidity  $\rho_v$  is the concentration of water vapor in the air, the diffusion equation for water vapor, ( $V$ ), is

$$F_z(V) = -D_v \cdot \frac{d\rho_v}{dz}, \quad (\text{D-25})$$

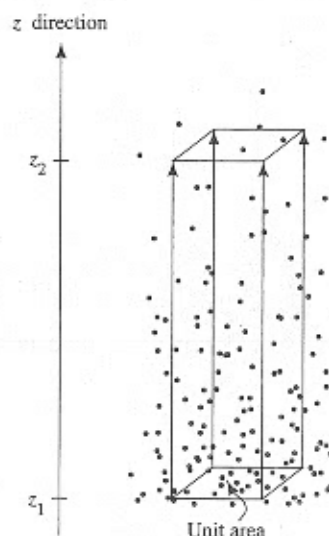
where  $D_v$  is the diffusivity of water vapor. Since latent heat and water vapor are directly coupled [Equation (D-13)], the diffusion equation for latent heat is

$$F_z(LE) = -D_v \cdot \lambda_v \cdot \frac{d\rho_v}{dz}. \quad (\text{D-26})$$

From Equation (B-19), the concentration of sensible heat,  $h$  [ $\text{E L}^{-3}$ ], in air can be expressed as

$$h = \rho_a \cdot c_a \cdot (T_a - T_b), \quad (\text{D-27})$$



**FIGURE D-11**

Conceptual diagram of the general diffusion process. Concentration of dots indicate concentration of quantity  $X$ . More of the property  $X$  moves from  $z_1$  to  $z_2$  in a given time when the diffusivity,  $D_x$ , is higher.

where  $c_a$  is the heat capacity of air (at constant pressure),  $T_a$  is the air temperature, and  $T_b$  is an arbitrary base temperature, usually taken as  $0^\circ\text{C}$ . Thus Equation (D-23) is written for the diffusion of sensible heat,  $H$ , as

$$F_z(H) = -D_H \cdot \frac{d(\rho_a \cdot c_a \cdot T_a)}{dz}, \quad (\text{D-28})$$

where  $D_H$  is the diffusivity of sensible heat in turbulent air.

To evaluate  $D_V$  and  $D_H$ , we must understand the mechanism of diffusion in the planetary boundary layer. As noted earlier, diffusion is effected by the bulk vertical movement of air in turbulent eddies, which carries latent heat (in the form of water vapor) and sensible heat vertically in the directions dictated by Equations (D-12) and (D-24) (Figure D-9). These eddies are also the mechanism by which the frictional drag of the surface is transmitted into the air flow to reduce the wind velocity; this process can also be viewed as a downward diffusion of momentum (mass times velocity).

Thus, in order to quantitatively express the effectiveness of turbulent eddies in energy and water-vapor transfer, we must examine the vertical transfer of momentum. Theory and field measurements indicate that when the actual lapse rate

equals the adiabatic lapse rate (called "neutral" conditions), the diffusivities of momentum, water vapor, and heat are equal. Thus we develop the basic heat- and vapor-transfer relations for neutral conditions, and then show how these can be modified to represent transfers in stable and unstable conditions.

### D.6.5 Momentum Transfer

Like latent and sensible heat, momentum transfer is also governed by Equation (D-23). Recall that momentum equals mass times velocity, so that the concentration of momentum (momentum per unit volume) at any level equals the mass density of the air times the velocity,  $\rho_a \cdot v_a$ . Thus Equation (D-23) becomes

$$F_z(M) = -D_M \cdot \frac{d(\rho_a \cdot v_a)}{dz}, \quad (\text{D-29})$$

where  $D_M$  is the diffusivity of momentum in turbulent air.

In the lowest levels of the atmosphere,  $\rho_a$  can be considered constant at the prevailing air temperature, so Equation (D-29) can be simplified to

$$F_z(M) = -D_M \cdot \rho_a \cdot \frac{dv_a}{dz}. \quad (\text{D-30})$$

Velocity always increases with height because frictional drag slows air movement near the ground [Equation (D-22)]. Therefore,  $dv_a/dz$  is always positive. Thus  $F_z(M)$  is always negative, reflecting the fact that momentum is being transferred downward via turbulent eddies from where velocities are higher to where they are lower.

$F_z(M)$  has the dimensions of a force per unit area [ $\text{F L}^{-2}$ ] and physically represents the horizontal shear stress due to differences of wind velocity at adjacent levels. Because momentum is conserved, this stress does not vary with height.

It can be shown that shear stress is directly proportional to the square of the friction velocity:

$$F_z(M) = -\rho_a \cdot u_*^2. \quad (\text{D-31})$$

Thus we can combine Equations (D-30) and (D-31) and solve for the diffusivity of momentum,  $D_M$ :

$$D_M = \frac{u_*^2}{dv_a/dz}. \quad (\text{D-32})$$

To evaluate the denominator of Equation (D-32), we see from Equation (D-22) that

Su  
32Th  
pc  
me  
edth  
ve  
su  
me  
folTh  
(D

to

Th  
forD.1  
Th  
tio  
(DTh  
be

[Se

$$\frac{dv_a}{dz} = \frac{u_*}{k \cdot (z - z_d)} \quad (\text{D-33})$$

Substitution of Equation (D-33) into Equation (D-32) then gives

$$D_M = k \cdot u_* \cdot (z - z_d) \quad (\text{D-34})$$

Thus the diffusivity of momentum increases in proportion to distance above the zero-plane displacement height and in proportion to the velocity of the eddies, which is characterized by  $u_*$ .

Without elaborate instrumentation to record the rapid fluctuations of horizontal and vertical velocity,  $u_*$  and hence  $D_M$  cannot be directly measured. However, we can evaluate  $D_M$  from measurements of wind velocity,  $v_m$ , at some height,  $z_m$ , as follows. From Equation (D-22),

$$u_* = \frac{k \cdot v_m}{\ln\left(\frac{z_m - z_d}{z_0}\right)} \quad (\text{D-35})$$

This value can then be substituted into Equation (D-34) to give

$$D_M = \frac{k^2 \cdot v_m \cdot (z_m - z_d)}{\ln\left(\frac{z_m - z_d}{z_0}\right)} \quad (\text{D-36})$$

We can also use Equations (D-33) and (D-34) to show that

$$D_M = k^2 \cdot (z - z_d)^2 \cdot \frac{dv_a}{dz} \quad (\text{D-37})$$

This relation will be useful in developing relations for calculating latent- and sensible-heat transfer.

### D.6.6 Latent-Heat Transfer

The upward flux of water vapor equals the evaporation rate,  $E$ , and this equivalence along with Equation (D-8b) can be used to rewrite Equation (D-26) as

$$\rho_w \cdot E = -D_V \cdot \frac{0.622 \cdot \rho_a}{P} \cdot \frac{de}{dz} \quad (\text{D-38})$$

The upward latent-heat transfer rate,  $LE$ , can then be found directly as

$$LE = \lambda_v \cdot \rho_w \cdot E = -D_V \cdot \lambda_v \cdot \frac{0.622 \cdot \rho_a}{P} \cdot \frac{de}{dz} \quad (\text{D-39})$$

[See Equation (D-26).]

As noted earlier, vapor and latent heat are transported vertically by the same turbulent eddies that are involved in momentum transfer. Thus, in order to develop a useful relation for calculating the rate of latent-heat transfer, we need to combine the relations that apply to the vertical velocity profile and the transfer of momentum with the diffusion equation for latent heat [Equation (D-39)].

As noted,  $D_V = D_M$  under neutral lapse-rate conditions. Thus we can replace  $D_V$  in Equation (D-39) with Equation (D-37) to give

$$LE = -k^2 \cdot (z - z_d)^2 \cdot \lambda_v \cdot \frac{0.622 \cdot \rho_a}{P} \cdot \frac{de}{dz} \cdot \frac{dv_a}{dz} \quad (\text{D-40})$$

Assuming a logarithmic profile for wind velocity and vapor pressure and integrating between two observational heights  $z_1$  and  $z_2$ , for  $z_2 > z_1 \geq (z_d + z_0)$ , then yields

$$LE = -\lambda_v \cdot \frac{0.622 \cdot \rho_a}{P} \cdot \frac{k^2}{\left[\ln\left(\frac{z_2 - z_d}{z_1 - z_d}\right)\right]^2} \cdot (v_2 - v_1) \cdot (e_2 - e_1), \quad (\text{D-41})$$

where  $v_i = v_a(z_i)$  and  $e_i = e(z_i)$ .

Equation (D-41) can be used for determining latent-heat transfer rate using observations of wind speed and vapor pressure at two heights,  $z_1$  and  $z_2$ . If we take the lower height as that of the nominal surface (i.e.,  $z_1 = z_d + z_0$ ,  $v_1 = 0$ , and  $e_1 = e_s$ ) and use the subscript  $a$  in place of 2, Equation (D-41) becomes

$$LE = -\lambda_v \cdot \frac{0.622 \cdot \rho_a}{P} \cdot \frac{k^2}{\left[\ln\left(\frac{z_a - z_d}{z_0}\right)\right]^2} \cdot v_a \cdot (e_a - e_s), \quad (\text{D-42})$$

which requires wind-speed measurement at only one level.

As noted,  $k = 0.4$ . At typical near-surface conditions, we can assume that  $\lambda_v = 2.47 \text{ MJ kg}^{-1}$ ,  $\rho_a = 1.24 \text{ kg m}^{-3}$ , and  $P = 101.3 \text{ kPa}$  and define a **bulk latent-heat-transfer coefficient**,  $K_{LE}$ , as

$$K_{LE} \equiv \frac{3.01 \times 10^{-3}}{\left[\ln\left(\frac{z_a - z_d}{z_0}\right)\right]^2}, \quad (\text{D-43})$$

where  $K_{LE}$  has units of  $\text{MJ m}^{-3} \text{ kPa}^{-1}$  and depends only on the measurement height and the nature of

the surface. For a given experimental situation,  $z_w$ ,  $z_\phi$ , and  $z_0$  will be fixed. Then we can write Equation (D-41) as

$$LE = K_{LE} \cdot (v_2 - v_1) \cdot (e_1 - e_2). \quad (\text{D-44})$$

and Equation (D-42) as

$$LE = K_{LE} \cdot v_a \cdot (e_s - e_a). \quad (\text{D-45})$$

(Note that the minus signs have been eliminated by reversing the vapor pressures.)

Equation (D-45) shows that the upward rate of latent-heat transport depends on the product of the wind speed and the difference between the vapor pressure at the surface and the vapor pressure in the overlying air.

Note that Equations (D-44) and (D-45) give the evaporation rate,  $E$ , if their right-hand sides are divided by the latent heat,  $\lambda_v$ . In this form, these equations are used to estimate evaporation from snowpacks [Equations (5-45) and (5-46)] and open-water surfaces [Equation (7-17)], and evapotranspiration from vegetated surfaces [Equation (7-76)]. Thus Equations (D-44) and (D-45) are more precise versions of Dalton's Law [Equation (D-12)].

### D.6.7 Sensible-Heat Transfer

Again assuming that density and specific heat are essentially constant under prevailing conditions, we can simplify the diffusion equation for sensible heat [Equation (D-28)] to

$$F_z(H) = H = -D_H \cdot \rho_a \cdot c_a \cdot \frac{dT_a}{dz}. \quad (\text{D-46})$$

If  $D_H = D_M$ , we can again substitute Equation (D-37) into Equation (D-46) to give

$$H = -k^2 \cdot \rho_a \cdot c_a \cdot (z - z_d)^2 \cdot \frac{dv_a}{dz} \cdot \frac{dT_a}{dz}. \quad (\text{D-47})$$

Integrating between levels  $z_1$  and  $z_2$  yields

$$H = -\rho_a \cdot c_a \cdot \frac{k^2}{\left[ \ln \left( \frac{z_2 - z_d}{z_1 - z_d} \right) \right]^2} \cdot (v_2 - v_1) \cdot (T_2 - T_1). \quad (\text{D-48})$$

Again, we can take level 1 as the nominal surface and write

$$H = -\rho_a \cdot c_a \cdot \frac{k^2}{\left[ \ln \left( \frac{z_a - z_d}{z_0} \right) \right]^2} \cdot v_a \cdot (T_a - T_s). \quad (\text{D-49})$$

Equations (D-48) and (D-49) are exactly analogous to Equations (D-41) and (D-42) for latent-heat transfer. And, as before, we can assume that  $\rho_a = 1.24 \text{ kg m}^{-3}$  and  $c_a = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$  and define a bulk sensible-heat transfer coefficient  $K_H$  as

$$K_H = \frac{1.99 \times 10^{-4}}{\left[ \ln \left( \frac{z_a - z_d}{z_0} \right) \right]^2}, \quad (\text{D-50})$$

with units  $\text{MJ m}^{-3} \text{ K}^{-1}$ . As with  $K_{LE}$ ,  $K_H$  depends only on the measurement height and the nature of the surface, and for a given experimental situation,  $z_w$ ,  $z_\phi$ , and  $z_0$  will be fixed. Then we can write Equations (D-48) as

$$H = K_H \cdot (v_2 - v_1) \cdot (T_1 - T_2) \quad (\text{D-51})$$

and Equation (D-49) as

$$H = K_H \cdot v_a \cdot (T_s - T_a). \quad (\text{D-52})$$

(Again, the minus signs have been eliminated by reversing the surface and air temperatures.)

Equation (D-52) shows that the upward rate of sensible-heat transport depends on the product of the wind speed and the difference between the temperature at the surface and the temperature in the overlying air.

### D.6.8 Effects of Atmospheric Stability on Heat and Vapor Transfer

Figure D-12 shows unstable, neutral, and stable lapse rates near the ground. When a parcel of air is transported upward in a turbulent eddy, it cools adiabatically. Thus if the actual lapse rate is steeper than adiabatic (unstable), the air in the eddy is warmer and, hence, less dense than the surrounding air and will continue to rise due to buoyancy, enhancing vertical transport of heat. If the actual lapse rate is less than the adiabatic (stable), the air in the eddy will be cooler and denser than the surroundings and will sink toward the surface, reducing vertical transport.

Under neutral conditions, the diffusivities of water vapor and sensible heat are identical to the

diff  
D<sub>H</sub>  
are  
tie:  
cal  
duc  
car  
ble  
D<sub>V</sub>  
cal  
is s  
vec  
lap  
sup  
situ  
sur

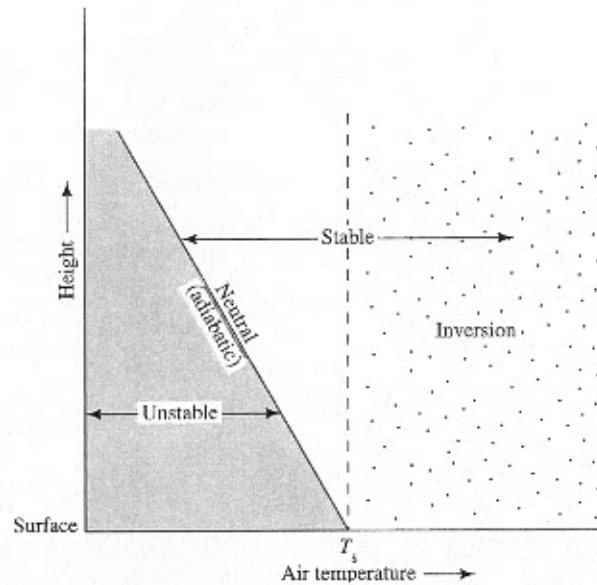
ity  
the  
17)  
He  
co  
fol  
Cli

wa  
anc  
ger  
tra



## LEAP

stable lapse rates near the surface. Air at surface is at surface temperature,  $T_s$ . When air moves upward in a turbulent eddy, it cools along the adiabatic line. If the actual gradient (lapse rate) is steeper than the adiabatic gradient (i.e., is in the unstable region), vertical turbulent motion is enhanced by buoyancy effects. If the actual gradient is less steep than the adiabatic gradient (i.e., is in the stable region), vertical turbulent motion is suppressed by buoyancy effects. Inversions are stable gradients in which the air temperature increases with height.



diffusivity of momentum (i.e.,  $D_V/D_M = 1$  and  $D_H/D_M = 1$ ), because the same turbulent eddies are responsible for the transport of all three quantities. However, under unstable conditions, the vertical movement of eddies is enhanced beyond that due to the wind velocity, and there can be significant vertical transport of water vapor and/or sensible heat but little transport of momentum, so  $D_V/D_M > 1$  and  $D_H/D_M > 1$ . These conditions typically occur when wind speed is low and the surface is strongly heated by the sun, inducing strong convection. (See Section 4.1.2.) Conversely, when the lapse rate near the ground is stable, turbulence is suppressed and  $D_V/D_M < 1$  and  $D_H/D_M < 1$ . This situation is typical when warm air overlies a cold surface, such as a snowpack.

In estimating evaporation, the effect of instability can be accounted for by using a value of  $b_o > 0$  in the empirical mass-transfer equation [Equation (7-17)] or by using a relation such as Equation (7-20). Here we present more general approaches to accounting for the effects of non-neutral lapse rates, following the discussion by Anderson (1976) and Cline (1997).

Stability-correction factors for momentum, water vapor, and sensible heat, designated  $\Phi_M$ ,  $\Phi_V$ , and  $\Phi_H$ , respectively, can be incorporated into the general equations for latent- and sensible-heat transfer as follows:

Latent heat and evaporation [Equation (D-41)]:

$$LE = -\lambda_v \cdot \frac{0.622 \cdot \rho_a}{P \cdot \Phi_M \cdot \Phi_V} \cdot \frac{k^2}{\left[ \ln \left( \frac{z_2 - z_d}{z_1 - z_d} \right) \right]^2} \cdot (v_2 - v_1) \cdot (e_2 - e_1). \quad (\text{D-53})$$

Sensible heat [Equation (D-48)]:

$$H = -\frac{\rho_a \cdot c_a}{\Phi_M \cdot \Phi_H} \cdot \frac{k^2}{\left[ \ln \left( \frac{z_2 - z_d}{z_1 - z_d} \right) \right]^2} \cdot (v_2 - v_1) \cdot (T_2 - T_1). \quad (\text{D-54})$$

The  $\Phi$  factors are related to the stability condition of the atmosphere, which is characterized by the dimensionless **Richardson number**,  $Ri$ , given by

$$Ri = \frac{2 \cdot g \cdot (z_2 - z_1) \cdot (T_2 - T_1)}{(T_2 + T_1 + 2 \cdot 273.2) \cdot (v_2 - v_1)^2}, \quad (\text{D-55})$$

where the subscripts again refer to measurements at two heights,  $z_2 > z_1 \geq z_d + z_0$ . Neutral conditions exist when  $Ri = 0$ , stable conditions when  $Ri < 0$ , and unstable conditions when  $Ri > 0$ . To determine the  $\Phi$ -factor values, first calculate  $Ri$ , then use the relations given in Table D-5.

### D.6.9 Eddy Correlation

We have seen that vertical transfer of momentum, energy, and water vapor in the planetary boundary layer is effected by turbulent eddies. Using notation

TABLE D-5

Formulas for Stability Factors ( $\Phi_M$ ,  $\Phi_v$ , and  $\Phi_H$ ) for Computing Latent- and Sensible-Heat Transfer as Functions of Richardson number ( $Ri$ ). See Equations (D-53)–(D-55). From Cline (1997).

Stability Factor	$Ri < -0.03$	$-0.03 \leq Ri \leq 0$	$0 < Ri < 0.19$
$\Phi_M$	$(1 - 18 \cdot Ri)^{-1/4}$	$(1 - 18 \cdot Ri)^{-1/4}$	$(1 - 5.2 \cdot Ri)^{-1}$
$\Phi_v, \Phi_H$	$1.3 \cdot (1 - 18 \cdot Ri)^{-1/4}$	$(1 - 18 \cdot Ri)^{-1/4}$	$(1 - 5.2 \cdot Ri)^{-1}$

analogous to that of Equation (D-15) for velocity, we can also express the instantaneous values of specific humidity,  $q$ , and temperature,  $T_a$ , at any level as sums of a time-average value (denoted by the overbar) and an instantaneous fluctuation from the average (denoted by the prime). We then have

$$q = \bar{q} + q' \quad (\text{D-56})$$

and

$$T_a = \bar{T}_a + T_a', \quad (\text{D-57})$$

where  $\bar{q}' = \bar{T}_a' = 0$ .

If positive (negative) fluctuations of the vertical velocity,  $u_a'$ , are accompanied by positive (negative) values of specific humidity,  $q'$ , then  $u_a'$  and  $q'$  are positively correlated, water vapor is being transferred upward, and evaporation is occurring. Under these conditions, the time averaged product  $\overline{u_a' \cdot q'}$  is greater than 0.

Conversely, if positive (negative) velocity fluctuations are accompanied by negative (positive)

humidity fluctuations, then  $u_a'$  and  $q'$  are negatively correlated, water vapor is being transferred downward, and condensation is occurring. Under these conditions,  $\overline{u_a' \cdot q'}$  is less than 0.

Thus, if the simultaneous fluctuations  $u_a'$  and  $q'$  can be measured, the evaporation rate  $E$  [ $\text{L T}^{-1}$ ] can be computed as

$$E = \frac{\rho_a}{\rho_w} \cdot \overline{u_a' \cdot q'}. \quad (\text{D-58})$$

Analogous reasoning can be used for temperature, so that upward sensible-heat transfer rate,  $H$ , can be computed from simultaneous measurement of  $T_a'$  and  $u_a'$  as

$$H = \rho_a \cdot c_a \cdot \overline{u_a' \cdot T_a'}. \quad (\text{D-59})$$

Equations (D-58) and (D-59) are the basis for the eddy-correlation method of measuring sensible- and latent-heat transfer and evaporation (Sections 7.3.3 and 7.8.3).