University of Colorado
Department of Civil, Environmental and Architectural Engineering
CVEN 5454 (Quantitative Methods)
Homework #1
Due September 19, 2025
Topics: Probability concepts – Chapter 2 of the Text and Chapters 2 of the first two
Spring books

Probability basics, De Morgan's laws, Addition Rule, Conditional Probability, Total Probability, Bayes Theorem and Counting Events

Problems 1-5 from "A Modern Introduction to Probability and Statistics – Springer Book"– Chapter 2

1. 2.4 [write out the steps clearly]
2. 2.14 [Monty Hall problem]
3. 2.15 [write out the steps clearly to deriving the final equation]
4. 2.18
5. 2.19

6. In any given year, the winter in a Midwest city can be cold (C) and wet (W). On average, 50% of the winters in this city are cold and 30% of the winters are wet. Moreover, 40% of the cold winters are also wet. An unpleasant winter (U) is one when the weather is either cold or wet or both.
(a) Are the events C and W statistically independent? Justify.
(b) What is the probability of an unpleasant winter in any given year?
(c) What is the probability that the winter in any given year will be cold but not wet?
(d) If the winter each year is indeed unpleasant, what is the probability that it will be both cold and wet?

7. A community is concerned about the supply of energy for the coming winter. Suppose there are three major sources of energy for the community, namely electrical power, natural gas and oil. Let E, G, and O denote, respectively, the shortages of these sources of energy for next winter. Also, it is estimated that the respective probabilities of these shortages are as follows:
P(E) = 0.15; P(G) = 0.1; and P(O)=0.2
Furthermore, if there is a shortage of oil, the probability that there will be a shortage of electrical energy will be doubled. The shortage of gas may be assumed to be statistically independent of shortages in oil and electricity.
(a) What is the probability that there will be shortages in all the three sources of energy next winter?
(b) What is the probability that there will be shortages in at least one of the following sources next winter: gas and electricity?
(c) If there is a shortage of electricity next winter, what is the probability that there will also be shortages in both gas and oil?
(d) What is the probability that at least two of the three sources of energy will be in short supply next winter?

8. A dam is proposed to be built in a seismically active area as shown below.
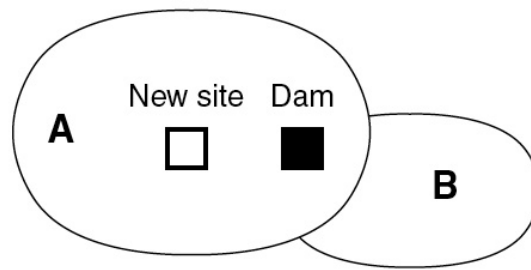
Two regions, A and B, can be identified in the vicinity such that earthquakes in either area could cause damage to the proposed dam. Earthquakes occur independently between regions A and B. suppose the annual probabilities of earthquake occurrences in regions A and B are 0.01 and 0.02, respectively. Moreover, the chance of two or more earthquakes occurring annually in each region is negligible.

   (a) What is the probability of an earthquake occurring in the vicinity of the dam in any year?

   (b) If an earthquake occurred in A (but not in B), the likelihood of damage to the dam is 0.3; however, if an earthquake occurred in B (but not in A), the likelihood of damage is only 0.1. Furthermore, if earthquakes occurred in both regions, the dam would have a 50-50 chance of damage. What is the probability that the dam will be damaged in any given year?

   (c) Suppose the dam can be relocated close to the center of region A, such that earthquakes in region B will not cause any damage to the dam. However, the likelihood of damage due to an earthquake in region A will increase to 0.4. Should the dam be sited in this new location if the objective is to minimize the probability of incurring damage? Substantiate your answer.

   (d) Would the decision in part (c) change if the new site is also susceptible to: (i) a landslide caused by severe rainstorms with an annual probability of 0.002, and (ii) a 0.001 annual probability of subsidence due to poor supporting sub-soil structures? Explain. Assume that the dam will be damaged during landslide or subsidence. Also, the events of damage caused by earthquakes, landslides, and subsidence are statistically independent.

Problems 9-11 are from the "Text" – Chapter 2.
9. 2.183
10. 2.202
11. 2.187

Problems 12-14 from Applied Probability and Statistics – Springer Book – Chapter 2
12. Problem 20
13. Problem 24
14. Problem 7

15. How large must a class be to make the probability of finding two people with the same birthday at least 50%? Also graph the class size versus the desired probability.