University of Colorado
Department of Civil, Environmental and Architectural Engineering
CVEN 5454 Quantitative Methods
Homework #3
Due November 11th, 2025

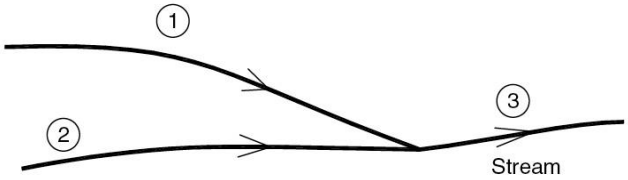
Topics: Monte Carlo, Kernel density, Functions of Random Variables, Distributions of sample statistics, Parameter estimation – point and Bayesian – Chapters 5,7. If you use R-commands to solve any of the problems other than 7, include them in your solution.

Joint Probability and functions of Random Variables

1. Problem 5-1
2. Problem 5-76
3. Problem 5-79

4. The flows in two tributaries 1 and 2 combine to form stream 3 as shown in the figure below. Suppose the concentration of pollutants in tributary 1 is X, which is normally distributed as N(mean=20,sd=4) parts per unit volume (ppv), whereas that in tributary 2 is Y which is N(mean=15,sd=3) puv. The flow rate in tributary 1 is 600 cubic feet per sec (cfs) and that in 2 is 400 cfs. Hence, the concentration of pollutants in the stream is

$$Z = (600X + 400Y)/(600+400) = 0.6X + 0.4Y$$



Pollution occurs if pollutant concentration exceeds 20 puv.

(a) What is the probability that at least one of the two tributaries will be polluted?
(b) Assume X and Y are statistically independent. Determine the probability of a polluted stream.
(c) Suppose the same industrial plant dumps the pollutants into the two tributaries, hence X and Y are correlated, say with rho=0.8. What is the probability of a polluted stream?

5. Solve Problem 4 via Monte Carlo.
6. Problem 5-65
7. Suppose an engineering variable Y is an exponential function of a random variable X as follows:

$$Y = e^X$$

And X is normally distributed as N(mean=2,sd=0.4). Derive the PDF of Y

8. Verify Problem 7 by Monte Carlo in R. The steps are (i) Generate an ensemble of x using $f(x)$ given in the problem by Monte-Carlo (ii) Compute corresponding values of y, (iii) Plot an histogram of y and overlay it with PDF from Kernel density estimator and (iv) Overlay the PDF $f(y)$ that you derived in Problem 7.

9. Problem 5-91

Point Estimation, MLE, Bayesian Estimation and Distribution of Sample Statistics
10. Problem 7-13
11. Problem 7-64
12. Problem 7-32
13. Problem 7-20

14. Problem 7-35
15. Problem 7-43 (Optional)
16. Problem 7-47