

University of Colorado
Department of Civil, Environmental and Architectural Engineering
CVEN 5454 Quantitative Methods

Homework #5 Due by December 9, 2025

Topics: Regression Chapters 11, 12, Helsel and Hirsch Chapters 10,11,12 and 15.
Bivariate and conditional PDFs – Kernel density estimators and Copulas. Using R-
commands for these will make it easier!

1. Bivariate PDFs – Copula/Kernel Density Estimators

For the Lees Ferry monthly flow

- (i) Plot the bivariate Normal PDF (X = Mar flow and Y = May flow)
- (ii) Plot the conditional PDF for Y , for $X = 0.6\text{MaF}$ and $X = 0.8\text{MaF}$
- (iii) Repeat (i) and (ii) using Kernel density estimators
- (iv) **Optional:** Use Copula to obtain the bivariate PDF/CDF.

2. Correlation/Mutual Information (MI)

Select May flow and one of the monthly flows (Jan, Feb, Mar or Apr)

For this pair compute

- (a) Correlation coefficient
- (b) Kendall's tau
- (c) Spearman's rank correlation.

Perform significance test on all of them, report the p-value and summarize.

- (d) Compute the Mutual Information (MI) using Kernel Density Estimators and its significance. Compare with the correlation coefficients above.

3. Simple Regression / Robust regression/trend line

Consider the data in Problem 12.4 of Helsel and Hirsch. To this data perform the following.

- (a) Fit a robust or nonparametric line – i.e., Kendall-Theil line between Calcium (dependent variable) and %Carbon-14 (independent variable).
- (b) Also fit a linear regression line and show them with the scatterplot of the data.
- (b) Perform model diagnostics for these two models along with a significance test on their slopes and compare them.

4. Multivariate Regression

Problem 11.1 of Helsel and Hirsch. Obtain the *best model* for LOGTN. To this end, the following steps need to be performed. You can get the excel file with the data from (<https://www.sciencebase.gov/catalog/item/5bf30260e4b045bfcae0c205>)

- (a) Scatterplot LOGTN with all the potential predictor variables and apply a default smoother in R to smooth the scatterplot and comment on the relationships you see (linear, nonlinear etc.).
- (b) Using PRESS or AIC or BIC to obtain the best model and perform model diagnostics including ANOVA. (As **optional exploration** try all three to see if the best model differs).
- (c) Plot the historical and modeled values with 95% confidence interval along with 1:1 line for visual inspection of the model performance.
- (d) Estimate the skill of the model by repeatedly dropping 10% of points - drop 10% of points, fit the best model to the rest and predict the dropped points; compute skill measures – R and RMSE; repeat. Boxplot the skill measures.
- (e) Comment on the model shortcomings