AREN 2110
In-class exercise – work in pairs
20 extra credit points

Solution
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Names

A steam power plant (heat engine) is designed to operate with the boiler pressure (where heat is transferred into the cycle) at a constant 10 MPa and the condenser pressure (where heat is rejected) at a constant 20 kPa. After heat rejection, water enters the pump as saturated liquid. After heat addition in the boiler, steam enters the turbine as saturated vapor. Assume the entire cycle is reversible and that the pump and turbine devices are adiabatic.

Step	Device	Process	Inlet T (°C)	Inlet P (kPa)	Inlet h (kJ/kg)
1→2	Pump	Isentropic pressure increase	60.06	20	251.42
2→3	Boiler	Isobaric heat transfer	60.06	10,000	261.4
3→4	Turbine	Isentropic expansion	311.0	10,000	2725.3
4→1	Condenser	Isobaric heat transfer	60.06	20	1845.44

Use property values from the Table:

T (°C)	P (kPa)	s_f (kJ/kg-K)	s_{fg} (kJ/kg-K)	s_g (kJ/kg-K)	h_f (kJ/kg)	h_{fg} (kJ/kg)	h_g (kJ/kg)
45.81	10	0.6492	7.4996	8.1488	191.81	2392.1	2583.9
60.06	20	0.8320	7.0752	7.9073	251.42	2357.5	2608.9
311.00	10000	3.3603	2.2556	5.6159	1407.8	1317.6	2725.5

Assume the specific volume of water entering the pump is 0.001 m^3/kg

a. Determine the ratio of the work produced by the turbine to the work required by the pump.
b. What is the efficiency of the power plant?
c. Draw the process on the T-s diagram on page 2.
d. If the furnace (HTR) supplying heat to the boiler is at 500 °C and the river (LTR) is at an average temperature of 20 °C, what is the entropy generated in the surroundings (kJ/kg-K)?
e. In winter the low temperature reservoir (a river) is colder and therefore the condenser can reject heat at 10 kPa. Will the winter efficiency be higher of lower than the design efficiency? Explain without calculations – hint: use the T-s diagram.

In-class problem Solution

a) $-w_p = h_2 - h_1 = v(P_2 - P_1)$

$$-w_p = 0.001\,\frac{m^3}{kg}(10{,}000 - 20\,kPa) = 9.98\,kJ/kg$$

$h_1 = 251.42\,kJ/kg$ (table), $h_2 = -w_p + h_1 = 9.98 + 251.42 = \underline{261.4\,\frac{kJ}{kg}}$

$-w_T = h_4 - h_3$, $h_3 = h_g$ @ 10 MPa (table) $= \underline{2725.3}\,\frac{kJ}{kg}$

$s_4 = s_3 = s_g$ @ 10 MPa $= 5.6159\,kJ/kgK$

isentropic expansion produces mixture @ (4) $s_f < s_4 < s_g$ @ 20 kPa

$$x_4 = \frac{5.6159 - 0.832}{7.0752} = 0.676$$

$$h_4 = 0.676(2357.5) + 251.42 = \underline{1845.44}\,kJ/kg$$

$$-w_T = 1845.44 - 2725.3 = -880.06\,kJ/kg$$

$$w_T = 880.06\,kJ/kg$$

$$\frac{w_T}{w_p} = \frac{880.06}{9.98} = \boxed{88.2}$$

b. $\eta = \frac{|w_T| - |w_p|}{q_H} = \frac{880.06 - 9.98}{h_3 - h_2} = \frac{870.68}{2725.5 - 261.4} = \boxed{0.35}$

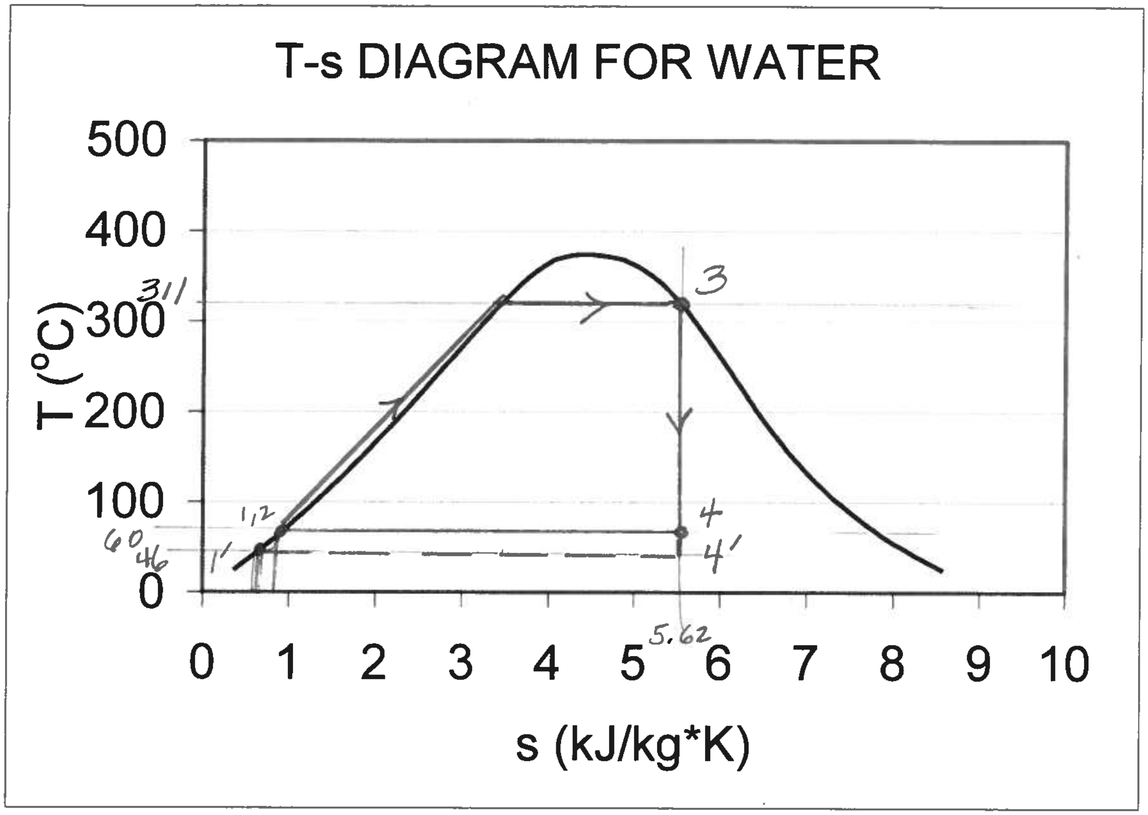
c. (on diagram)

d. $S_{gen} = -\sum_k \left(\frac{q}{T}\right)_k$ for cycle, $\Delta s = 0$

$$S_{gen} = -\frac{q_H}{T_H} - \frac{q_L}{T_L} = -\frac{(h_3 - h_2)}{T_H} - \frac{(h_1 - h_4)}{T_L}$$

$$S_{gen} = -\frac{(2725.5 - 261.4)}{773} - \frac{(251.42 - 1845.44)}{293} = -3.19 - (-5.44)$$

$$S_{gen} = \boxed{2.25\,\frac{kJ}{kgK}}$$



e) $\eta_{winter} > \eta_{design}$ since

Area 1'-2'-3-4' > Area 1-2-3-4

$\oint T ds = q_{net} = w_{net}$

$w_{net}' > w_{net}$

and

Area under 1'-2'-3 $\simeq$ Area under 1-2-3

$q_H' \simeq q_H$