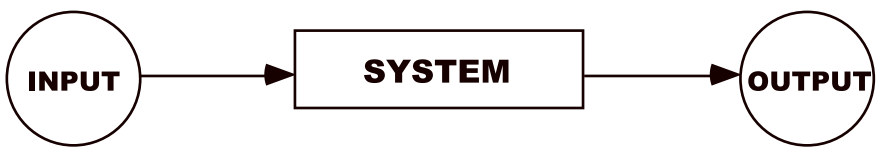
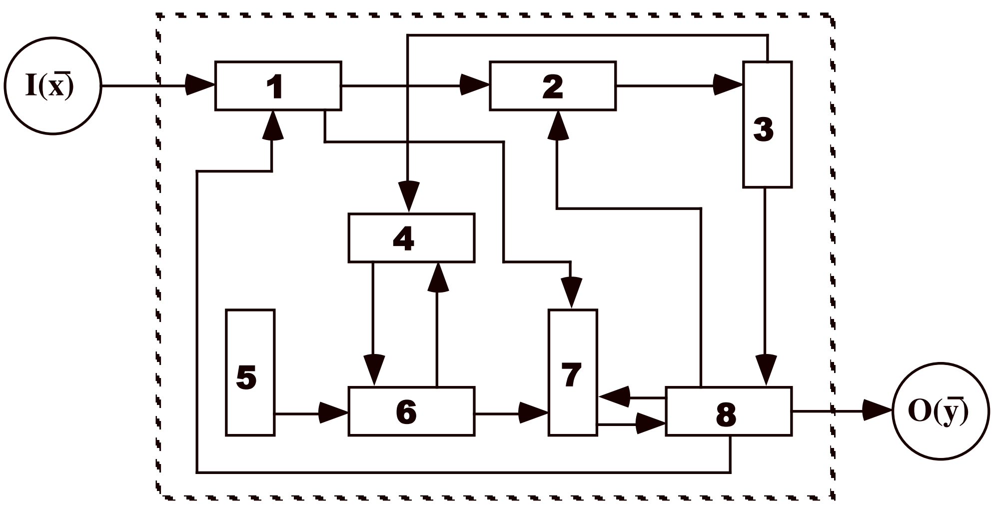
1. WATER SYSTEM OPTIMIZATION: CONCEPTS AND METHODS

1.1 SYSTEMS DEFINITIONS

Engineering project design and optimization can be effectively approached using concepts of systems analysis. A system can be thought of as a set of components or processes that transform resource inputs into product (goods and services) outputs. The basic concept of a system is represented in Figure 1.1.



(a)



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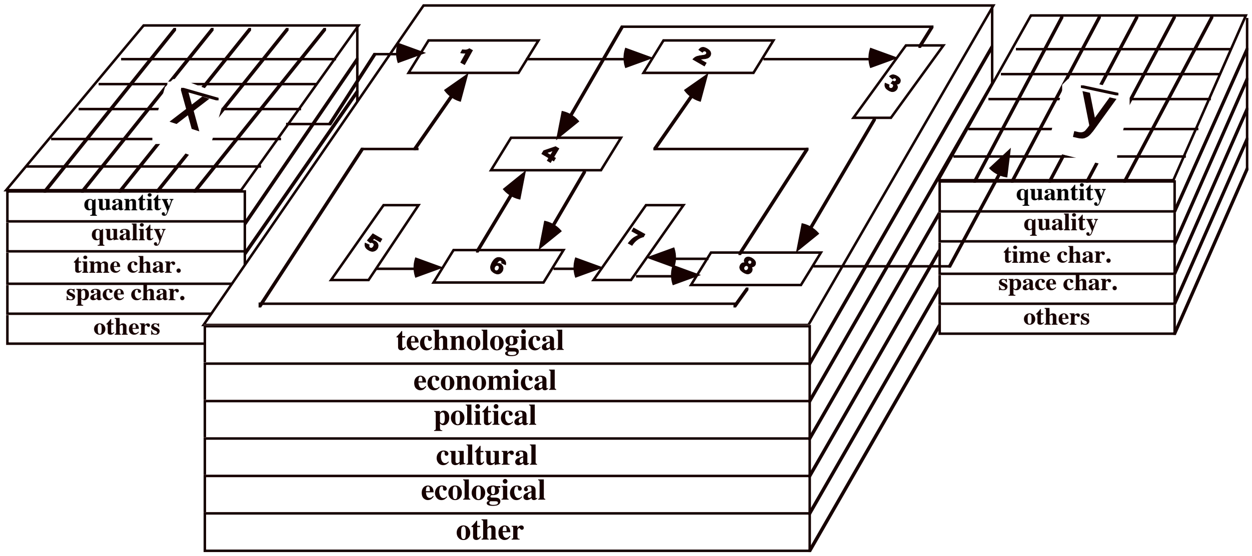


(c)

Figure 1.1: Representation of a System

In Figure 1.1b, the system is defined by a boundary which separates those components that are an interrelated part of the system from those outside which are part of the "environment". Determining the boundary depends on the physical system, the technological and spatial elements and the assumptions and the purposes for which the analysis is being conducted (see Figure 1.1c). For example, in a water resources system, the analyst must decide which hydrologic basin and water sources, dams, reservoir, and conveyance systems, and service areas and water uses to include in the "system".

The inputs define the flow of resource into the system and the outputs and products from the system. A system often has several subsystems. In the more detailed representation of Figure 1.2, the inputs include controllable or decision variables, which represent design choices that are open to the engineer. Assigning values to controllable variables establishes an alternative.

The outputs describe the performance of the system or its consequences upon the environment. They indicate the effects of applying design and planning decisions via the input variables and are evaluated against system objectives and criteria in order to assess the worth of the respective alternatives in terms of time, reliability, costs or other appropriate units.

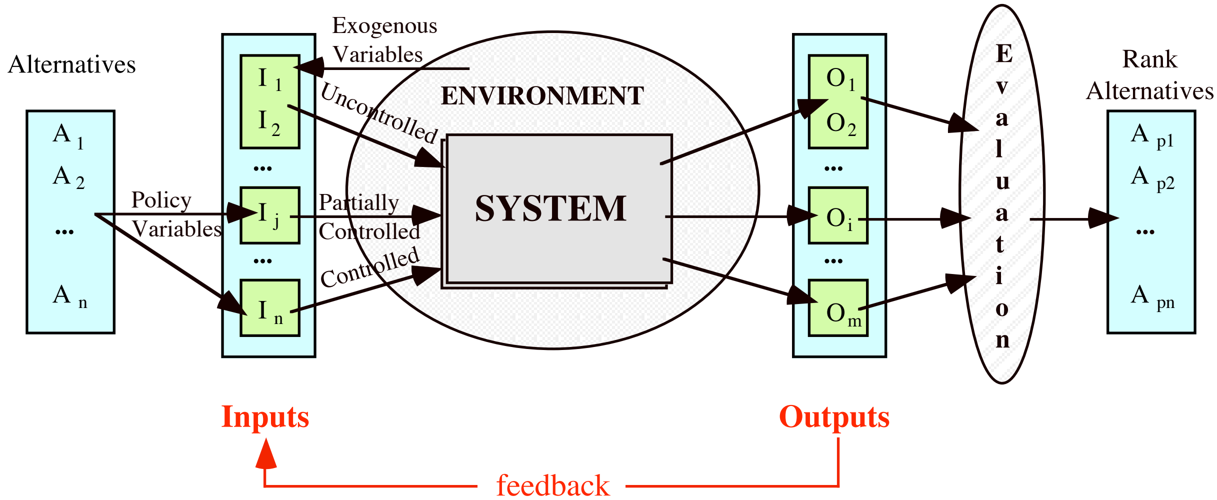


Figure 1.2: Detailed Representation of a System

1.2 WATER RESOURCES SYSTEMS DESCRIPTIONS

Water resources systems modeling may be treated at various levels of specificity as illustrated by Figure 1.3. If the design is concerned with local water supply planning, then the system boundary would include the key elements shown by Problem 1 in Figure 1.3. If basin-wide multipurpose planning or operation is of concern, the system boundary must be expanded to include the kinds of elements shown in Problem 2. The engineer might be interested in statewide allocation of water among basins and uses as illustrated by Problem 3. As a further example, the specific elements and interconnections of a multipurpose basin are further depicted in the system block diagram of Figure 1.4. This type of diagram is useful in constructing the mathematical optimization or simulation models for the system. Table 1.1 summarizes many of the relevant input, outputs, decision variables, and system constraints and components of water resources systems.

1.3 MATHEMATICAL MODELS OF SYSTEMS: OVERVIEW AND CONCEPTS

Figure 1.5 is a representation of a “modeling space”, with each face of the cube representing an important dimension of quantitative models. Depending on whether variable relationships are probabilistic or deterministic, static or dynamic, and linear or nonlinear (as represented by the faces of the cube) various analytical techniques (the corners) are required to handle them.

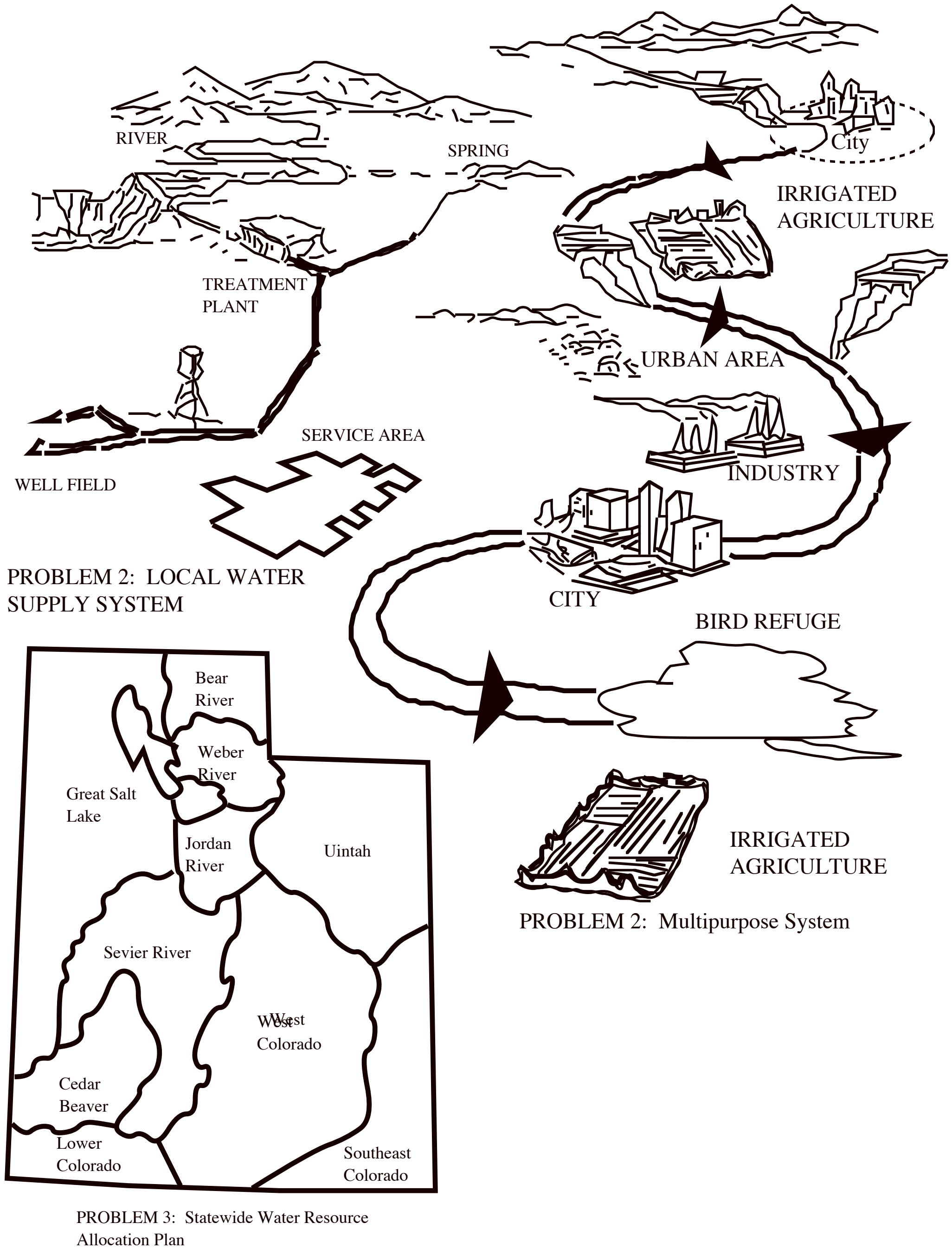


Figure 1.3: Levels of Specificity in Water Resources Systems Modeling

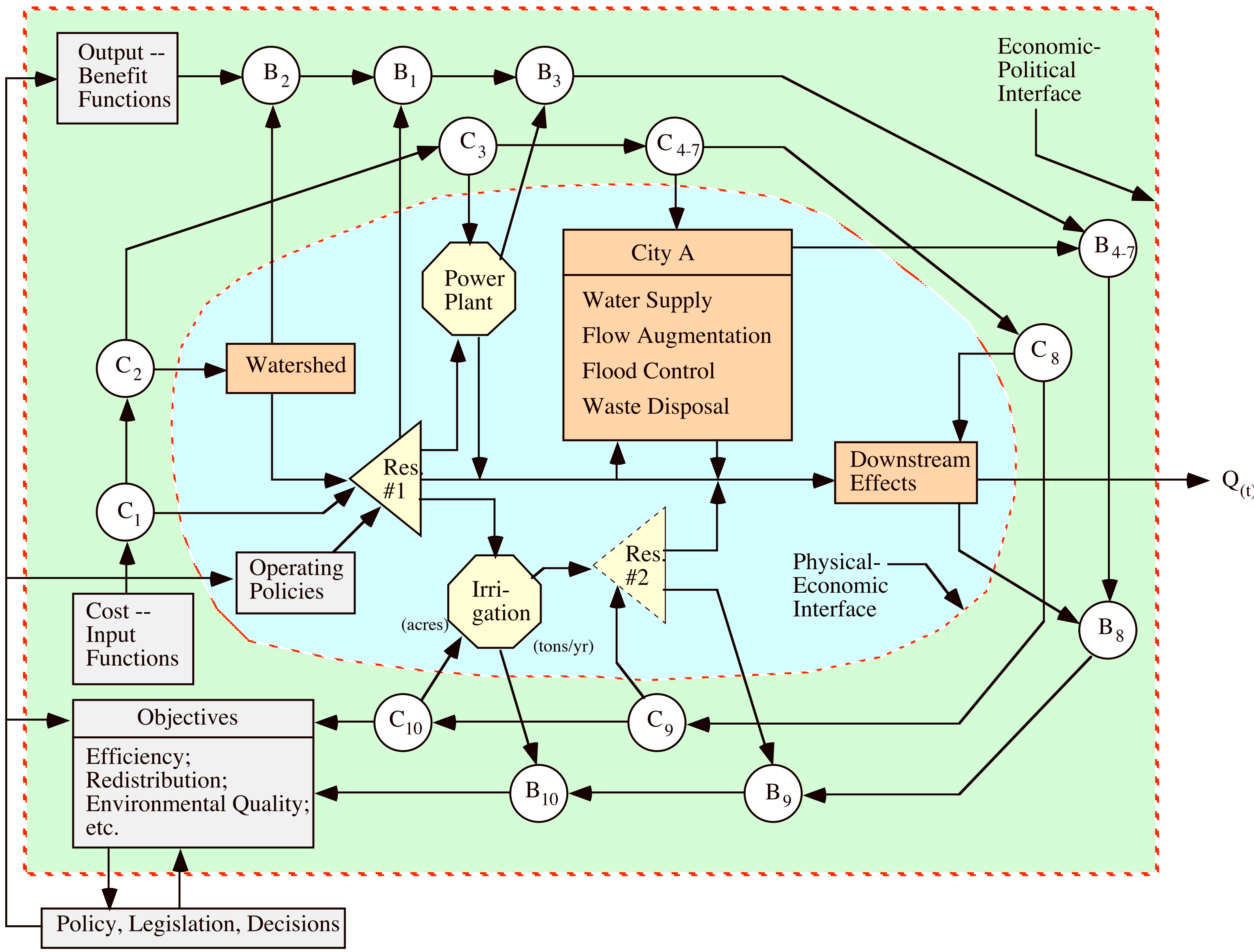


Figure 1.4: Hierarchy of Systems and Systems Functions

Table 1.1: Elements of Water Resources Systems

Inputs to Water Resources Systems:

A. Water sources
  1. Surface sources: for example, surface water flow, sedimentation, or salt load, precipitation
  2. Underground sources
  3. Imported sources: for example, desalting water, imported water
  4. Reuse and recycling: for example, treated water from treatment plant, recycling water in irrigation

B. Other natural resources
  1. Land
  2. Minerals, etc.

C. Economic resources

Outputs of Water Resources Systems:

A. Water allocation to user sectors
  1. Municipal
  2. Agriculture
  3. Industry
  4. Hydroelectric power
  5. Flood control
  6. Navigation
  7. Recreation
  8. Fish and wildlife habitats

B. Quantity and quality of the water resource system
  1. Flow of the stream
  2. Quality of stream

System Decision Variables:

A. Management and planning
  1. Operating strategies
  2. Land use zoning
  3. Regional coordination and allocation policy
  4. Number and location of treatment plants
  5. Sequence of treatments and treatment level achieved

B. Investment policy
  1. Budget allocation to various subsystems
  2. Timing of investment: for example, stages of development, interest rate
  3. Taxing and subsidy strategies

Constraints on Systems Performance:

1. Economic constraints: for example, budget, B/C ratio
2. Political constraints: for example, tradeoff between regions
3. Law: for example, water rights
4. Physical and technology constraints: for example, probability of water availability
5. Standards: system output may have to meet certain standards: for example, effluent standards from wastewater treatment plants

System Physical and Engineering Components:

A. Planning and management system components
  1. Dam and control structures
  2. Levees and other protecting structures
  3. Distribution or collection systems comprised of (a) canals, (b) pipes, (c) pumping stations and other control structures
  4. Treatment plants

B. Descriptive system components
  1. Physical properties of stream: for example, roughness, slope
  2. Biochemical properties of stream: for example, rate of aeration, rate of self-regeneration
  3. Chemical properties of stream: for example, hardness, pH

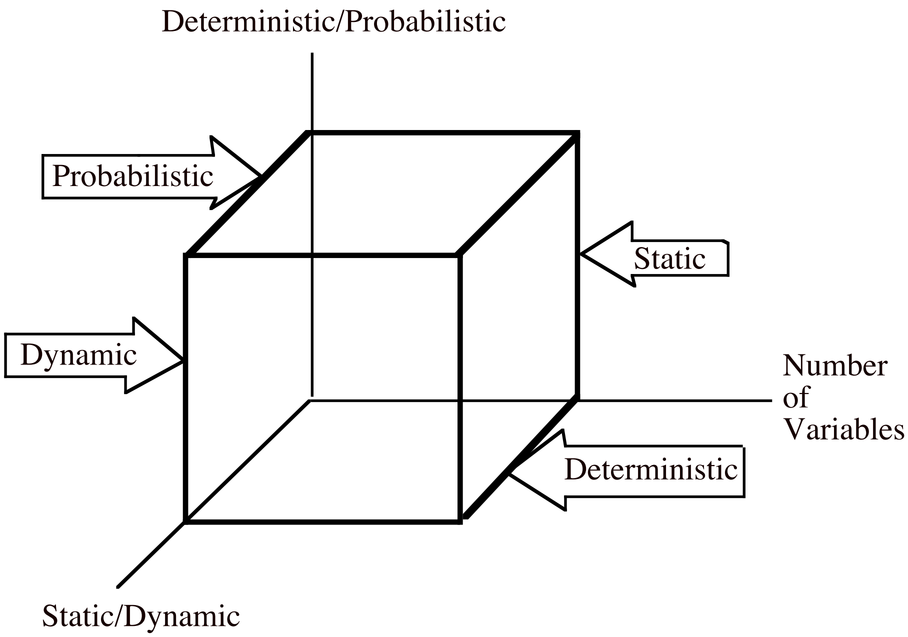


Figure 1.5: Modeling Space (Cube)

Broadly speaking the purpose of modeling may be either predictive or prescriptive. Predictive models of systems are constructed to clarify the internal structure of a system and predict its behavior or response to input variables. On the other hand, prescriptive models strive not only to reproduce the behavior of the system itself, but also to evaluate the consequences of design alternatives according to predetermined measures of performance.

Identifying the model structure for predictive or prescriptive models must be based either on formal theory or some very strong plausibility arguments. Systems models cannot be devised by simply using statistical manipulations of data and information to determine variable interactions. Moreover, all the information relevant to the system may not be quantifiable as numerical data. Hence, systems modeling techniques may be quantitative and nonquantitative or both.

Table 1.2 provides a general classification of modeling methods and techniques useful in systems analysis. The entries in the Table are classified under the heading of predictive or prescriptive models according to the theoretical basis for model construction. Table 1.3 relates the models to the modeling cubic dimensions shown in Figure 1.5.

This overview of course cannot provide detailed descriptions of the various modeling approaches. Whole textbooks are devoted to these subjects. Instead, this discussion simply tries to provide a basic classification of the techniques in order to understand where optimization fit among the various methods.

Table 1.2: An Overview of Approaches to Systems Modeling

	Application or Use	
Problem Type	Predictive	Prescriptive
Quantitative		
Deterministic	Systems Transformations algebraic equations, differential equations, state variable formulations, input-output analysis	Optimization Procedures Classical Optimization Theory; differential calculus, lagrangians, optimal control theory
	Networks graph theory	Networks CPM and PERT
Stochastic	Stochastic Processes inventory theory, queuing theory, Markov processes	Decision Analysis Statistical (Bayesian) decision theory, game theory
	Statistical Models regression analysis, component and factor analysis, stepwise multiple regression, discriminant analysis, econometric analysis	
Simulation	Deterministic and stochastic model components and models	Monte Carlo methods, search techniques for dominant solutions
Non-quantitative	Verbal Models scenarios, survey research	Verbal Models Delphi inquiries
	People Models role playing	People Models operational gaming

Table 1.3: Types of Models in "Modeling Space"

Type of Model or Solution Approach	Time Variance		Random Variance		State of Domain Variance		Functional Variance	
	Static	Dynamic	Deterministic	Probabilistic	Single Variate	Multivariate	Linear	Nonlinear
Systems Transformation								
Linear, nonlinear systems								
1^{st} order diff. equations		X	X		X			
state variables		X	X			X		
Input-output analysis	X		X			X		
Optimization								
Classical Optimization								
Differential Calculus	X		X			X		X
Lagrange Multipliers	X		X			X		X
Optimal Control Theory								
Mathematical Programming								
linear programming	X		X			X	X	
nonlinear programming	X		X			X		X
integer programming	X					X	X	
dynamic programming	X	X				X		X
stochastic programming	X			X		X		X
genetic programming	X	X		X		X		X
Stochastic								
Inventory				X				
Queuing		X		X				
Markov		X		X				
Multivariate	X			X				
Networks								
Graph Theory								
CPM and PERT								
Simulation								

1.4 A GENERAL MODEL OF SYSTEM OPTIMIZATION

1.4.1 System Design and Optimization

Engineering design problems can be mathematically described by three functions associated with each of the design factors: the physical processes (design or production function), the resource function's costs, and the output's or product's values (benefit functions) (see Figure 1.6). The definition of each function is derived from different sources.

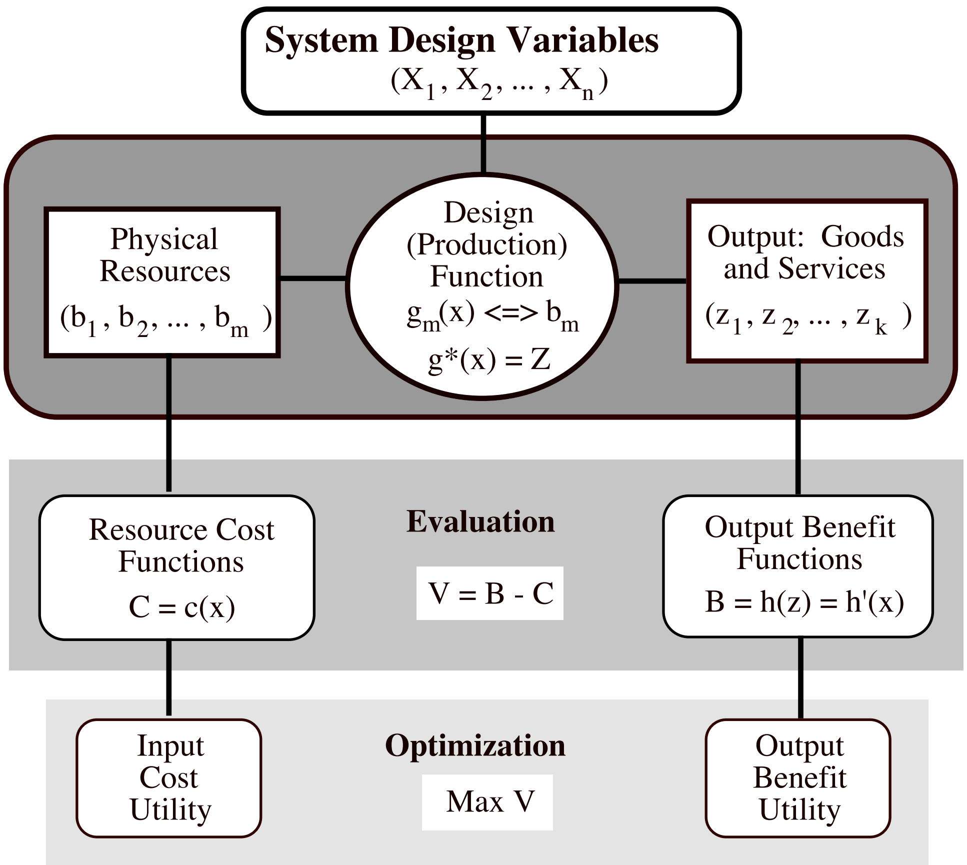


Figure 1.6: Model of Systems Design and Optimization

The design function, based on the physical nature of the system without regard to value, describes the maximum product that can be obtained from the input of any given set of resources. The resource cost function is usually defined by the economic market value of those resources. The product valuation function may be determined either by a market or, in the case of social benefits such as conservation--which often do not have a market--by a political process. The first design step is to model the production process, the physical process for transforming resources into products.

From a general mathematical description of the systems optimization problem can be stated as follows:

The objective is to maximize the net value of output, or:

$$V = \text{Benefit} - \text{Cost} \qquad \text{...[1.1]}$$

This is also expressed as Profit = Revenue - Cost, or

$$P = R - C \qquad \text{...[1.2]}$$

The benefit (revenue) and cost functions are functions of a set of control design variables, x_1, x_2, ..., x_n, and can be represented as nonlinear and linear mathematical functions. A general mathematical description for maximizing net value is:

$$\text{Objective function: Maximize } Z = f(\mathbf{x}) \qquad \text{...[1.3]}$$

Subject to Design Function and Constraint set:

$$\mathbf{g}(\mathbf{x}) \leq,=,\geq \mathbf{b} \qquad \text{...[1.4]}$$

$$\mathbf{x} \geq 0 \text{ (nonnegativity condition)} \qquad \text{...[1.5]}$$

where: Z is a measure of effectiveness $(Z = B - C) = f(\mathbf{x})$

$\mathbf{x}$ is a vector of n control design variables, $x_1, x_2, ..., x_n$

$f(\mathbf{x}) = f(x_1, x_2, ..., x_n)$ is a function of control variables

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} g_1(\mathbf{x}) \\ g_2(\mathbf{x}) \\ \cdots \\ g_m(\mathbf{x}) \end{bmatrix}$$ is a vector of m design and/or constraint equations of $\mathbf{x}$

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_m \end{bmatrix}$$ is a resource constraint vector

1.4.2 The Design Function

The design and constraint functions represent the transformation of resource inputs to project outputs or products. They are the core model, and represent the physical system design alternatives.

Specifically, the system design (production) function is the mathematical description of the output that can be obtained from any given set of resources. A general design function may be expressed as:

$$g_i(x_1, x_2, ..., x_n) \leq,=,\geq b_i \qquad ...[1.6]$$

Money and value are not part of the expression; $g_i(\mathbf{x})$ is in units of production output, and the x_j represent physical rather than monetary resources. For example, $g(\mathbf{x})$ could be the maximum product for given quantities of land (x_1) and water (x_2). Other examples of design functions that can be represented as an algebraic equation are the power output from a hydro-generating station as a function of rainfall intensity and duration over a drainage basin, or the amount of BOD removal in a wastewater treatment system as a function of influent BOD concentration and detention time in treatment. The design (production) function is a relationship between physical quantities alone.

Thus $g(\mathbf{x})$ may be a series of physical relationships which are based on theoretical knowledge (hydraulics, mass balance, etc.) or may be statistically based on probability distributions, regression function, etc. These are no different than the functions engineers have always used in traditional problem solving. There is a difference in how they are related to the formal way in which the equations or inequalities are written so that a solution algorithm (usually a separate piece of computer software) can be used to generate all possible outputs of interest including the "best" or optimal solution. This contrasts with traditional approaches where usually only one or a few solutions are produced. A great advantage of modeling is the ease with which "what if" question can be asked to explore alternate assumptions on resource limits or production efficiency with no further mental effort once the system is described mathematically.

Each point on the design function represents the maximum output that can be obtained for any given set of resources. The function, therefore, dominates any lesser amount of product that would be obtained from a wasteful or technologically poor use of these resources. The production function is thus, by definition, the locus of all technically efficient combinations of resources.

The significance of the design function can be simply illustrated in Figure 1.7. The amount of a crop that might be produced on a given parcel of land, x, is g(x) tons. This point on the design production function dominates other feasible outputs, such as A and B, that might be achieved with the same land, x, if, for example, the land were not all cultivated. Conversely, it is infeasible to produce any more than g(x) tons with a quantity of land equal to x. The point $(x^*, g^*(x))$ is at the edge of feasible and infeasible amounts of production for x. The production function can thus be conveniently visualized as the boundary between the feasible and infeasible regions in the input-output space.

A design function can relate any number of resource inputs to one or several product outputs. Graphically, it is difficult to visualize whenever there are more than two inputs--since our usual perception is limited to three dimensions--but its functional significance remains the same.

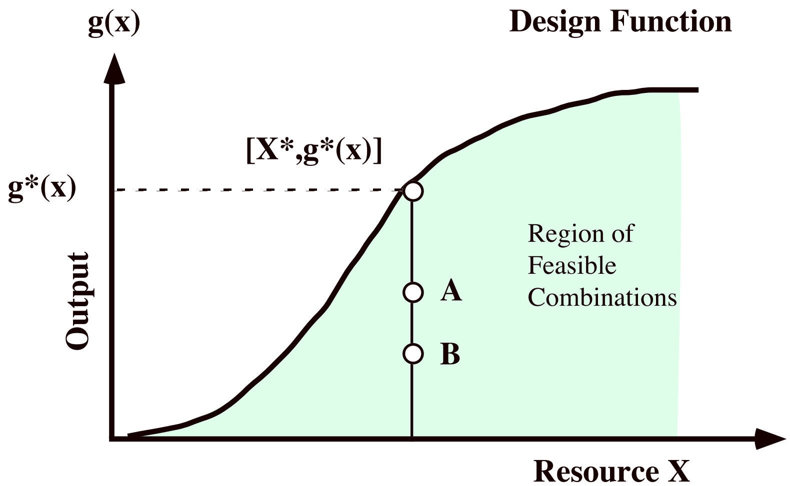


Figure 1.7: Example of Design Function

The concept of the design function is general: it does not necessarily have any particular form and cannot always be written as an algebraic expression or system of equations. Sometimes the maximum product for any set of resources may be simply tabulated. Whether the design function is determined by formula, by detailed design, or by complex simulation methods, its meaning is the same. It represents the limit on what can be achieved with available technology and a given set of resources.

1.4.3 Evaluation Models and Design Optimization

In a design analysis, evaluation of alternatives may have to be conducted at several different levels of decision. This is depicted in the generalized model of design evaluation in Figure 8. The lowest level of evaluation is in terms of system performance.

Performance or effectiveness analysis simply looks at the capability of the physical system to meet the specified needs or requirements; for example, the ability of a structure to carry the design loads, or a water system to maintain a certain flow and pressure, or a production process to turn out a given quantity and quality of project.

Significance analysis relates the quantitative measure of an output to its qualitative value. To use the economist's language, it is the process of describing the utility function for the particular outputs. These processes describes the degree to which individuals or society as a whole places positive value or negative value on, or are indifferent to, project outputs.

A Generalized Model of Decision Evaluation

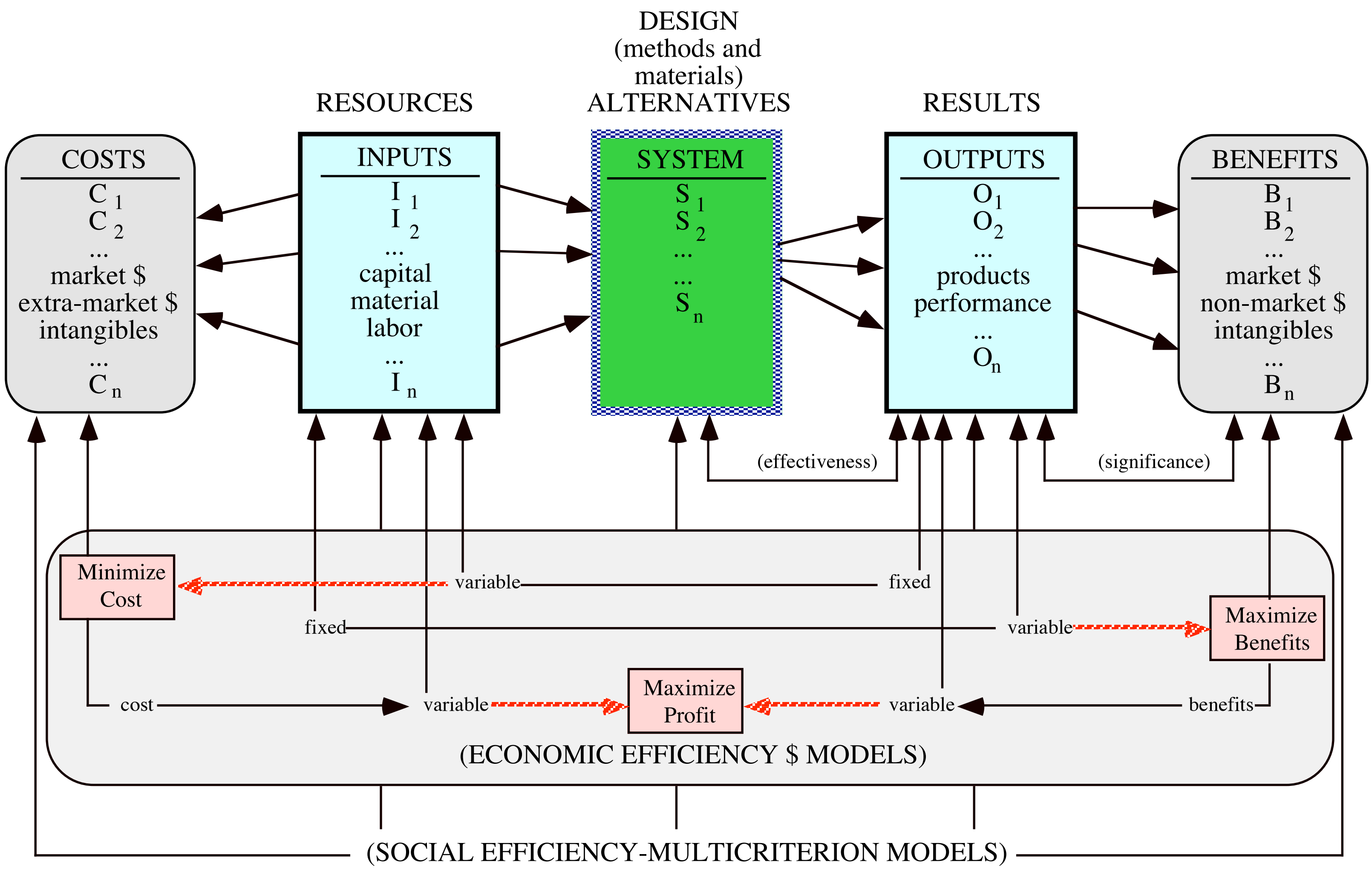


Figure 1.8: System Performance and Optimization

Cost Effectiveness: In cases where project costs are monetary but benefits are measured in some other unit, then cost-effectiveness analysis can be used for single criterion evaluation. For example, assume that the objective is to reduce sedimentation of a reservoir resulting from present watershed development. A systems model is constructed and three possible alternatives are tested to determine the effectiveness in reduction of sediment level as a function of the designs and associated costs. Then either a level of effectiveness must be specified and then the cost minimized for that level, or the limit on cost specified and the effectiveness maximized. For example in Figure 1.9 if the cost cannot exceed C_2 and reduction of sediment to level E_1 is all that is required for the reservoir uses then Alternative 2 operated at cost C_1 is the best choice. On the other hand, if the goal were to reach some minimum level of effectiveness E_2 regardless of the cost then Alternative 3 at cost C_2 would be the choice. However, some flexibility should be allowed in the analysis for if C_3 is a reasonable cost to pay, then by only a slight increase in costs to C_3 large gains in effectiveness can be achieved with Alternative 1. In fact, the approach of setting costs at the place where slope of the cost-effectiveness curve flattens is a judicious one since little is gained by further expenditures past that point.

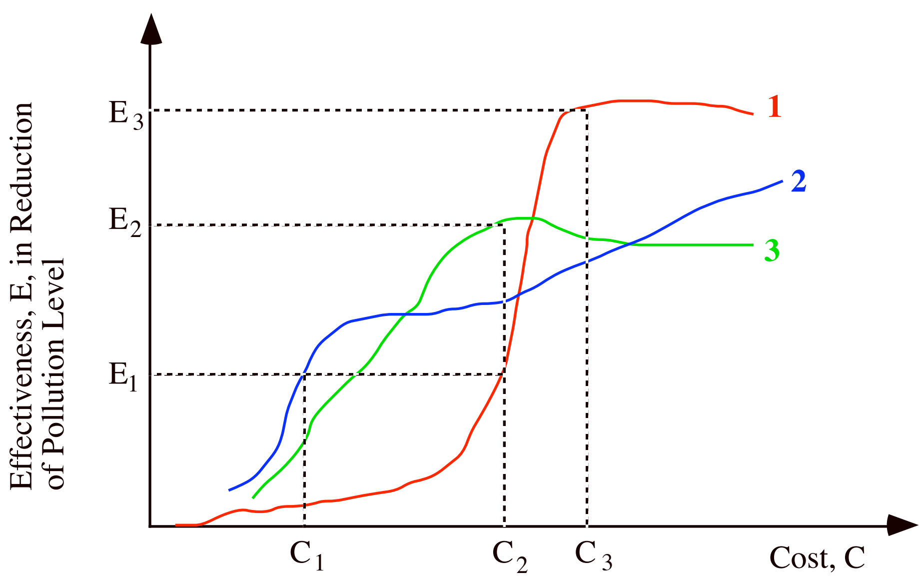


Figure 1.9: Cost-Effectiveness Curves of Reservoir Sediment Control Measures

Economic Benefit Analysis: If consequences of alternatives can be valued on an economic (monetary) scale, then the preferred alternative is the one which produces the largest net economic benefit (revenues minus costs). With both benefits and costs measured in monetary units, evaluations can be performed using the tools of engineering economic analysis (Grant et al., 1987). Since investment in project facilities and the returns from project operation occur over long periods of time, to correctly evaluate both present and future benefits and costs they must be compared at the same point in time. One method to accomplish this is by "discounting" the costs and benefits using an appropriate interest rate applied over the useful project life to obtain the net present value of project outputs. This general procedure stated in equation form is as follows:

If the question involves the present value of a single future quantity (C), the equation is:

$$PV = \frac{C}{(1 + r)^t} \qquad ...[1.7]$$

where r is the interest rate per time period, and t is the number of time periods.

If the present value of a uniform series of future amounts is needed, the equation is:

$$PV = C\,\frac{(1 + r)^t - 1}{r\,(1 + r)^t} \qquad ...[1.8]$$

Also, if rather than present value, one is doing the reverse--that is, a capital recovery analysis, the factor needed is the reciprocal of the factor in the second equation. Therefore, the capital recovery factor is:

$$RC = \frac{r\,(1 + r)^t}{(1 + r)^t - 1} \qquad ...[1.9]$$

These relationships are all that are needed for the vast majority of the engineering economics analyses that one encounters, including those in this course.

Besides comparison of present worth of benefits minus present worth of costs, engineering economic analysis may also be formulated as a benefit cost ratio, equivalent uniform annual costs (benefits), and rate of return including incremental rates of return. These methods are presented and discussed by various writers (Grant et al., 1987; Winfrey, 1969; DeGarmo et al., 1984; Howes, 1971; and others). All of the methods when correctly applied will give equivalent answers. The principal difficulties in benefit cost studies are the selection of an appropriate time period and discount rate, since the results of the analysis are often sensitive to these factors.

<u>Social Efficiency (Multiple Criteria Analysis)</u>. Project alternatives that have several noncommensurate outputs involving both market and non-market values require multiple criteria comparisons in evaluation. Insight into this very important problem of trade-offs between the pluses and minuses of noncommensurate project consequences is provided the theory of welfare economics. To illustrate the problem, consider the joint optimization of two objectives corresponding to the outputs, $O_1 = f(\mathbf{x})$ and $O_2 = g(\mathbf{x})$, where **x** is a set of input levels associated with a range of alternatives. O_1 and O_2 are plotted in Figure 1.10 as a function of input levels for the alternatives. These constitute a pair of objective functions to be “maximized” simultaneously:

$$\text{Maximize} \begin{Bmatrix} O_1 = f_1(\mathbf{x}) \\ \\ O_2 = f_2(\mathbf{x}) \end{Bmatrix} \qquad \text{...[1.10]}$$

subject to any applicable design functions and constraints:

$$g_i(\mathbf{x}) \leq 0 \qquad \text{...[1.11]}$$

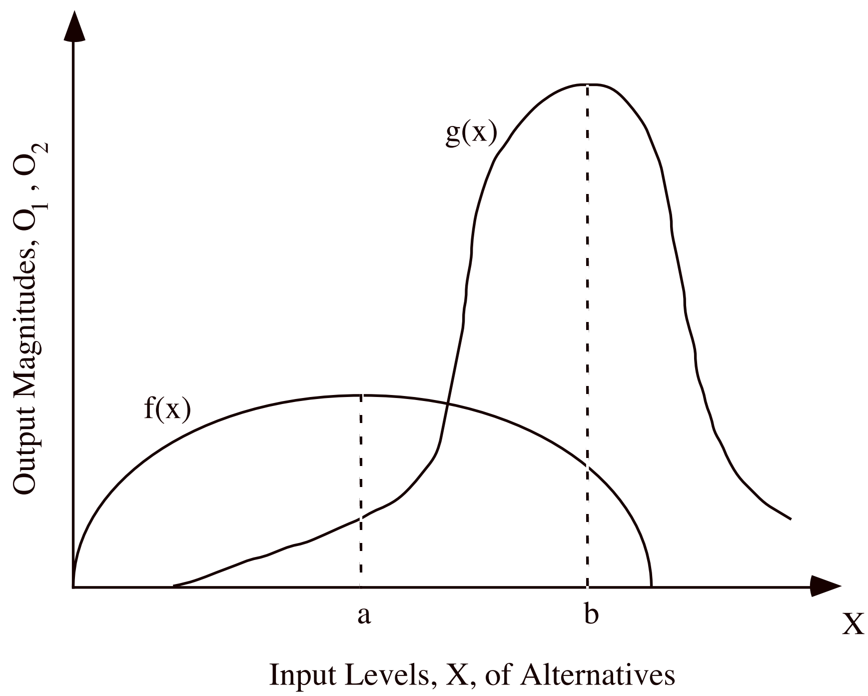


Figure 1.10: Joint Optimization of Multiple Outputs

Examination of Figure 1.10 indicates that some alternatives (input levels) can be immediately eliminated from further consideration because they are dominated by better combinations. This includes the area to the left of "a" and to the right of "b", since in these regions the functions O_1 and O_2 are both decreasing. This reduces the range of alternatives to between "a" and "b", called the "efficient region". However, to select the joint optimum point within the efficient region depends on the tradeoffs or relative weights for the outputs O_1 and O_2. The tradeoffs or weightings of the objectives cannot be deduced analytically, but are value judgments that must be supplied by the decision maker.

1.4.4 Summary

An engineering systems analysis is the basis for choosing among alternative project designs. The analysis may be performed at one or several levels depending on the nature of the project and the decision criteria. For most all engineered projects, a criterion that must be satisfied is that of maximizing the economic or monetary return. For this reason, optimization tools and methods usually attempt to maximize the monetary benefits of projects. In many instances, analysis of projects must be broadened to non-monetary environmental and social consequences to fully evaluate the worth or projects.

1.5 CLASSICAL OPTIMIZATION--A REVIEW

Before proceeding to modern operations research methods of optimization, it is useful to briefly review the concept of optimization (finding maximums or minimums) of functions from the classical approach of calculus. Engineers are usually familiar with calculus approaches to finding best facility capacities by taking a first derivative, setting it equal to zero and solving for stationary points. For example, consider the problem of a fixed flowrate to be pumped through a pipe. The pipe cost increases with diameter while the energy cost decreases. The optimal diameter is where df/dx = 0.

This is however an over simplified problem because usually several constraints exist (standard pipe diameters and pump sizes) and flow may very seasonally with demands so total cost is a function of several flow rates which cannot be calculated until diameter is known. We, therefore, need approaches to handling many constraints simultaneously. If the constraints are equalities, we can include them in the objective function using the Lagrangian concept. Consider the constrained problem:

$$\text{Minimize cost} = f(x) \qquad \text{...[1.12]}$$

$$\text{subject to } g(x) = b \qquad \text{...[1.13]}$$

Define the Lagrangian function as:

$$L = f(x) - \lambda\,[g(x) - b] \qquad \text{...[1.14]}$$

where λ is an artificial variable to be discussed later. Since we now have only a single function (L) we can take its derivative and set each partial derivative equal to zero as before for identifying the optimal value (the minimum possible value of f(x), which does not violate the constraint) as follows:

$$\frac{\partial L}{\partial x} = 0 \qquad \text{...[1.15]}$$

$$\frac{\partial L}{\partial \lambda} = 0 \qquad \text{...[1.16]}$$

A simultaneous solution of the above equations yields x = X*, where f(X*) will be either a minimum or maximum of f(x). If there are more than one x variable and/or multiple constraints, we have:

$$\text{Max or Min } f = f(x_1, x_2, ..., x_n) \qquad ...[1.17]$$

$$\text{subject to:} \quad g_i(x_1, x_2, ..., x_n) = b_i \qquad (i = 1, 2, ..., m) \qquad ...[1.18]$$

For example, consider:

$$\text{Maximize } f = 2\, x_1 + x_1\, x_2 + 5\, x_2 \qquad ...[1.19]$$

subject to the design constraint:

$$3\, x_1 + x_2 = 10 \qquad ...[1.20]$$

The Lagrangian for this problem can be formed as:

$$L = 2\, x_1 + x_1\, x_2 + 5\, x_2 - \lambda\, (3\, x_1 + x_2 - 10) \qquad ...[1.21]$$

Note, in forming the Lagrangian, all we have done is subtract a penalty from f(**x**) equal to λ for each unit by which g(x) ≠ b. Therefore, λ can be defined as the change in f(**x**) for each unit of change in the right hand side of g(**x**). This is what economists call an imputed value or shadow price at the optimal **x**:

$$\frac{\partial L}{\partial x_1} = 0 = 2 + x_2 - 3\,\lambda \qquad ...[1.22]$$

$$\frac{\partial L}{\partial x_2} = 0 = x_1 + 5 - \lambda \qquad ...[1.23]$$

$$\frac{\partial L}{\partial \lambda} = 0 = 3\, x_1 + x_2 - 10 \qquad \text{(the original constraint)} \qquad ...[1.24]$$

A simultaneous solution of this system of equations is:

$x_1 = -0.5$

$x_2 = 11.5$

$\lambda = 4.5$

$f(x_1,x_2) = 50.75$

Note that we could easily solve this problem because we had three linear equations, but even three nonlinear equations could have been very difficult to solve and most real-world water problems have many (perhaps hundreds) of variables. We clearly need a procedure for handling much larger problems. Also, the constraint was an equality, but most water problem constraints are inequalities (upper or lower bounds on resources or capacities on pipes, etc.). The above approach can be generalized to handle inequalities by using Kuhn-Tucker conditions (which are discussed in more detail in Section 11), but the solution then becomes even more difficult. The classical calculus approach to general optimization problems can be summarized as:

	Problem	Solution Technique
1.	Max or Min $f(\mathbf{x})$	Set: $\frac{df}{dx_i} = 0$ for each x_i, and solve the resulting system of equations.
2.	Max or Min $f(\mathbf{x})$ s.t.: $\mathbf{g}(\mathbf{x}) = \mathbf{b}$ (equality constraints)	Form the Lagrangian: $L = f(\mathbf{x}) - \sum [\lambda_i (g_i(\mathbf{x}) - b_i)]$ Set the partial derivatives of L with respect to each x_i and l_j equal to zero and solve the resulting simultaneous system.
3.	Max or Min $f(\mathbf{x})$ s.t.: $\mathbf{g}(\mathbf{x}) \leq,=,\geq \mathbf{b}$ (inequalities)	Form the Lagrangian: $L = f(\mathbf{x}) - \sum [\lambda_i (g_i(\mathbf{x}) - b_i)]$ and employ the Kuhn-Tucker conditions, including: $L'(\mathbf{x},\lambda) \geq 0$ $\lambda_i S_i = 0$ where S_i are slack or surplus variables.

In general we are not assured of a global optimum because of the nature of $f(\mathbf{x})$, as illustrated in Figure 1.11.

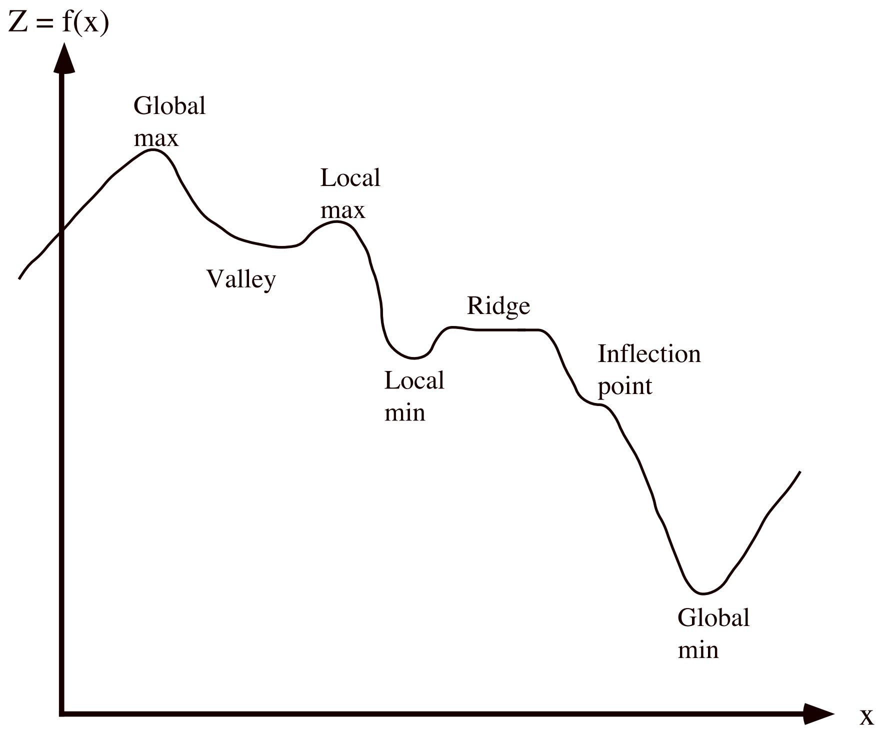


Figure 1.11: Possible Behaviors of a Nonlinear Function

1.6 MODEL DEVELOPMENT APPROACH

The proper way to approach development of the mathematical model of any system is:

1. Identify the essential parameters that must be modeled in order to capture quantitatively the decision variables of interest. For example, a thermal electric generating plant is a tremendously complex system with several sub-systems. If, however, our objectives is to model only cooling water requirements, these can be calculated by simple functions relating water supply to: capacity in KW, type of cooling tower, salinity at which effluent is removed.

2. Describe the system as accurately as possible as a series of equations and/or inequalities which capture the interaction of these essential parameters without regard to type of solution techniques which is to be used. If at this stage one sets out to build a linear programming (LP) model, then the solution tool is distorting (increasing errors in) the model. The solution tool rather than the real system is dictating the model structure. One should define the system as well as possible in mathematical terms using symbols for all parameters, which may vary, rather than assuming any constant values.

3. Now consider what solution approach may be best. The objective may be to find the "best" (the optimal) solution or more commonly it may be to analyze a range of "good"

solutions which are within the domain of the resources available. In either case a simulation or an optimization model may be appropriate. The essential differences is that an optimization model has one or more formal objective functions and all of the model's constraints must be solved simultaneously (unless dynamic programming or some or decomposition technique is used to solve a series of smaller systems of equation sequentially). All parameters over which some control is possible are considered to be variables and we seek the solution that maximizes or minimizes our objective.

4. Simulation models in contrast usually consist of a series of equations that are solved one at a time until all potential variables are fixed. This much simpler solution is obtained by fixing enough variables that equations can be solved, thereby giving a particular (but not optimal) solution. One can then change the fixed parameters, calculate a new solution and interactively develop the surface of the solution space (an objective function is not formally included in the model).

Simulation models are more flexible than optimization (due for example, to the capability of using "if-then-else" statements and "do" loops) and can produce more detailed solutions. However, for problems with many variables, manually varying multidimensional parameters may be very slow, very costly in terms of computations, and will never assure that one has really found the region of the solution space that is optimal. It is very common to use optimization models for preliminary design (identifying the domains of good solutions) and simulation models for more detailed design and operation analysis.

Non-linearities cause no difficulty in simulation models but may be very troublesome in most optimization models. If, however, an optimization model is considered appropriate and the system is non-liner there are several techniques available to either use piecewise linear segments of non-linear functions or to do truly non-linear optimization. The most common water resource planning models are those with linear constraints and non-linear objectives (due for example to economies of scale in cost functions). Another very common water resource model characteristic is discrete step functions due to standard pipe or treatment component sizes. Here we can use 0 or 1 variables for non-build or build decisions. This requires integer programming.

1.8 PROBLEMS

1. The storage volume for a proposed reservoir is given by the equation:

   $$S = 10\ A\ h$$

   where A is the area of the reservoir site in acres, and h is the height of the dam in feet.

   The cost of the land is \$1000 per acre, and the cost of the dam is \$5000 per foot of height. The total budget for the project is \$200,000.

   a. Determine the optimal size for the reservoir area and dam height within the given project budget.

   b. If the project budget were increased by \$1000, what corresponding increase in storage could be obtained?

2. Flood control for a region can be provided by some combination of reservoir storage and levees. It has been determined that:

   a. Cost of storage is inversely proportional to the flow released (or passed) from the reservoir.

   b. Cost of levees is directly proportional to the flow released.

   Your objective as project designer is to find the minimum cost project.

   a. Formulate equations and find an analytical solution.

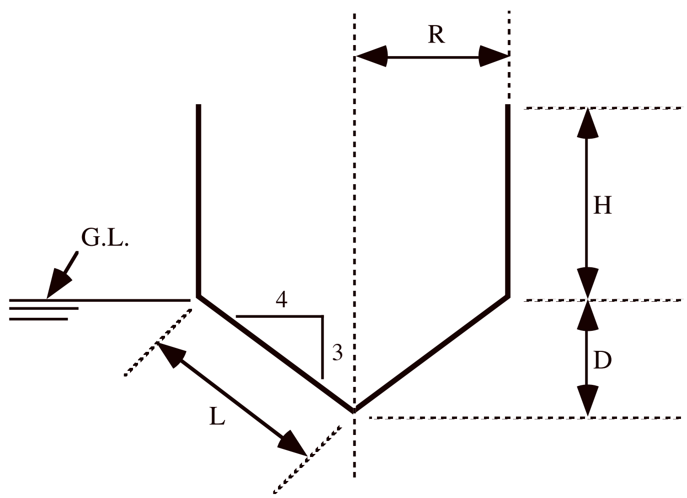
   b. Show the problem graphically and illustrate the solution.

3. A water company wants to pump water from its well field to the city, which is three miles away. The pipe in place will cost \$10D per foot, where D is the diameter of the pipe in feet. The annual pumping cost is \$150h per year, where h is the head loss in the pipe. The head loss is given by:

   $$h = \frac{0.02\ L\ V^2}{D\ 2g}$$

   where L is the pipe length in feet and V is the average flow velocity in feet/second. The pipe will last 15 years and the interest rate is 10 percent. The company wants to pump 12,000 gpm. What size pipe should they use to minimize cost?

4. A sedimentation tank is circular in plan with vertical sides above the ground and a conical hopper below the ground (refer to the figure, below). The slope of the conical part is 3 vertical to 4 horizontal. Determine the proportions to hold a volume of 4070 cubic meters for a minimum cost (which, in this case, will be a tank with a minimum total area of bottom and sides).



5. Consider a trapezoidal canal whose cross-sectional area, A, is to be 100 ft^2 (see the figure, below). It is desired to maximize discharge for the given cross-sectional area. The mean flow velocity increases with hydraulic radius, A/P, where P is the wetted perimeter. The discharge is a maximum when the mean velocity is maximum. Therefore, the design optimization reduces to minimizing the wetted perimeter, P.

For the trapezoid,

$$P = b + 2d \csc \theta$$

and

$$A = db + d^2 \cot \theta$$

Find the canal design parameters d, b, and θ.

Canal of trapezoidal section.