CLASSIFICATION TREE ANALYSIS OF STOCHASTIC OPTIMAL CONTROL SEQUENCES FOR MIXED-MODE BUILDINGS.

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1. Abstract

In an attempt to increase energy efficiency in high-performance mixed-mode buildings, a control optimization is conducted, yielding a sequence of best-case energy saving and comfort preserving window opening schedules. The optimal sequence and a set of corresponding predictors are used in a classification tree analysis to glean simple control logic from the optimal solution that can be embedded in a building control system. Due to a comfort-heavy weighted objective function, the optimal control sequence is shown to perform worse than a default control sequence for the example building, however the extracted control rule performs as well as, and in some cases better than the default, improving comfort with no added energy cost.

2. Introduction

As energy consumption rates increase globally, it is of paramount importance to seek innovative and effective methods for decreasing energy demand and increasing efficiency. In the United States, building energy use accounts for over 40% of all energy use nationwide, and is split roughly in half between the commercial and residential sectors [14]. Clearly, any reductions in building energy use or
building energy efficiency can have a drastic impact on national energy demand and associated security risks.

With the advent of compression-based cooling in the past century, buildings have moved from being (largely) passive systems to very active dynamic environments, and in the last several decades energy costs have pushed building science towards higher and more stringent standards for energy performance. High performance buildings represent any buildings that seek to provide security and comfort at a lower cost to the environment than standard buildings currently do. One subset of high performance buildings, mixed-mode (MM) buildings, are an (often failed) attempt to bridge the gap between older building paradigms and new, providing superior occupant comfort and satisfaction while conserving energy and taking advantage of passive conditioning opportunities.

"MM refers to a hybrid approach to space conditioning that uses a combination of natural ventilation from operable windows (either manually or automatically controlled), and mechanical systems that include air distribution equipment and refrigeration equipment for cooling." [2] Control of MM building is typically broken up in to three types: concurrent, change-over, and zoned. In zoned control, a building is broken up according to space type, and certain zones are conditioned with natural ventilation while others employ conventional compression based air conditioning (AC). In change-over control, a given zone will have the means to use both conventional AC and natural ventilation, and switches between them such that when one system is on, the other is off. In concurrent controlled systems, both natural ventilation and conventional ventilation are allowed to occur simultaneously. This third type is potentially the most risky design because without proper control sequencing, it easily (and often) leads to buildings that attempt to condition the outdoors. It is also important to note that MM buildings typically incorporate a range of high-performance features such as radiant heating and cooling, active or passive shading devices, and geothermal heat exchangers to name a few. It is with the knowledge that MM buildings are complex, dynamic, and difficult to properly control that this paper seeks to evaluate more robust control strategies.

Major selling points of MM buildings are: elevated rates of occupant satisfaction, indoor air quality (IAQ), and occupant comfort. [2] This increased satisfaction is due to occupants’ access to operable features such as windows, shading devices, and lighting systems in and around their working environment. In the US, buildings have traditionally been constructed as well-sealed, highly conditioned and tightly controlled spaces. Recent studies have shown that occupants with access to operable windows typically experience greater comfort across a wider band of indoor conditions. [10, 4] The introduction of operable features in a building leads to considerable uncertainty in the performance of the heating, ventilating, and cooling (HVAC) system. Occupant - controlled shading devices like venetian blinds can block some or all solar radiation from directly reaching the internal surfaces of a
space, changing the operative temperature of the space and leading to increased heating, or decreased cooling requirements. When shades are drawn, occupants often need to turn on lights to achieve visual comfort with the indoor environment, and increased lighting leads to increased cooling, or decreased heating requirements. Clearly it is difficult to say just when or how occupants will modify these elements in their working environment, but several studies [13, 12, 15, 7] have led to detailed behavioral models that accurately reproduce occupant behavior in buildings. While lighting and shading devices can directly affect the heating, cooling, and electric lighting requirements of a space, occupant controlled windows can alter heating, cooling, and ventilation requirements. To summarize: manually operable lights, windows, and shades can lead to significant changes in the conditioning and electric loads of a building. Not only is controlling these systems a challenge, but the reliability of the controls is uncertain, due to the stochastic occupant-driven processes present in the building’s energy balance.

With the end goals of reducing energy consumption, greenhouse gas emissions, and carbon emissions of MM buildings while preserving occupant comfort, we seek to find simple, realistic control strategies through the combined processes of stochastic model predictive control (SMPC), and classification and regression tree (CART) analysis. In MPC, an optimizer discovers a time-series of control signals that minimize an objective function for a given model. In CART analysis, a set of predictor variables is divided discretely into similar groups which serve as representatives for predicting a response variable. In this paper, we conduct a CART analysis on the results of the MPC process after it is applied to a building in an attempt to extract logic from the optimal control sequence.

3. Methodology

Here we present the offline building control and data mining techniques used to determine energy-efficient control strategies for MM buildings.

3.1. Stochastic Model Predictive Control.

3.1.1. MPC. Before considering stochastic model predictive control (SMPC), it is important to understand deterministic model predictive control (MPC). Model Predictive Control is a method of process control, normally implemented in discrete time steps by implementing the following three steps. First a process model is used to predict over a finite horizon the response and future states of the process given a control input or other perturbation. Second, optimal control sequences are computed to minimize some cost associated with the process. Third, the first portion of the optimal control strategy is implemented and the controller begins predicting and optimizing again, iterating through each subsequent time step.

A model predictive controller consists of two major components: the controlled system model, and the optimization technique, both of which can present significant challenges. Often, simple system models are adequate to achieve satisfactory
control, as poor long-term predictions are disregarded as the prediction horizon approaches the time of action. When system models are linear or convex, optimization is straightforward and follows well established mathematical guidelines for convex optimization; more often MPC is applied to more complex systems, and more sophisticated optimization techniques are required. Often, the need for advanced optimization is sidestepped by simplifying the system model or transforming it to a form which lends itself to simple convex optimization. In the MPC software used in this paper, a highly nonlinear complex system is modeled in EnergyPlus and serves as a black-box objective function for the optimizer in the MPC algorithm.

3.1.2. Benefits of MPC. MPC has proven to be robust by nature, even overly conservative in some cases because it guarantees (in theory) that the implemented control sequence will never cause the system to stray outside of its defined operating limits. MPC is also very effective at controlling highly nonlinear systems.

3.1.3. History. MPC has its roots in the process industry, and excels where common PID controllers cannot achieve good control due to complex system dynamics and large time constants. Over the last three decades, computing and science have grown, making MPC applicable to nonlinear processes and systems with faster dynamics. MPC has found its way into the automotive, aerospace, and robotics industries, and in the last decade, MPC has begun working for building control as well.

3.1.4. SMPC. Stochastic model predictive control extends deterministic MPC to more unpredictable and unstable processes, in particular those processes that are subject to stochastic disturbances and stochastic variations in inputs or outputs. Several approaches to dealing with stochastic elements in system models and system dynamics are discussed in [3, 8, 5, 11, 1]. The defining feature of SMPC is that the controller assumes a range of possible system responses, and delivers a control strategy that is highly likely to bring the system to a given state in spite of stochastic influences, while (deterministic) MPC assumes perfect knowledge of the system, and delivers a single control strategy (with a single predicted response) that may not be robust to every possible system dynamic.

The most desirable and common treatment of SMPC is with mathematically closed-form solutions. Often unknown disturbances, errors, or other criteria are assumed to be independent and identically distributed (IID) and gaussian. This provides some insight into what might be the range of possible errors or outputs from a model. In reality these inputs often conform to other distributions, and when the true distributions are unknown, these assumptions are no longer accurate, as in the systems used in this study.
3.2. **Classification and Regression Trees.** Rooted in the fields of data mining and decision theory, classification and regression tree analysis provides a nonparametric means to relate a set of predictor variables to a response variable through a sequence of conditional statements. CART models can be visually represented and understood through dendrograms (see figure 1), and thereby lend themselves to implementation in building automation systems as simple if/then/else control rules. Regression trees are created by recursively splitting the set of predictor variables according to their skill at classifying the response variable; this process is referred to as ‘learning’ or ‘growing’ the decision tree. At each split (node) in the tree, the subset of predictor variables is examined, and a single predictor is selected which that divides the subset into groups (branches) that similarly classify the response variable. When classification or misclassification rate is the sole metric used to determine where to split the predictor set, this process continues until every branch leads to a terminal node where there is one and only one data point, called a leaf. Normally a fully grown tree (one in which every branch ends in a single value) is too complex and over-fitted to the data and must be simplified (pruned) to be useful. In order to prune the tree appropriately, a complexity criterion $C_\alpha$ is introduced that combines the rate of misclassification $R(T)$ with the number of terminal nodes ($N_m$) in the model. The misclassification error is given by:

$$ R(T) = 1/N_m \sum_{i \in R_m} I(y_i \neq k(m)) $$

as in [6], where $I$ is an indicator function that returns a 1 when the expression in parenthesis is true, $y_i$ is the the true class of a terminal node, and $k(m)$ is the predicted class. The complexity criterion is given by:

$$ C_\alpha(T) = \sum_{m=1}^{T} N_m Q_m(T) + \alpha |T| $$

where $T \in T_o$ is a subtree that is obtained by pruning $T_o$, $m$ is the index of a terminal node, $\alpha$ is a tuning parameter, and $Q_m$ is an average of the squared difference between the total number of nodes and the number of nodes in the full tree $T_o$. The reader is referred to [6] for more information on the derivation of CART classification and pruning, suffice it to say that parsimonious models are readily found by established and effective pruning techniques.

4. **Results**

The results presented here are the outcome of a 92-day optimization of building operation from May 30 to August 30, and a subsequent rule extraction. Due to computational and time constraints, the optimization could not be completed as one single-threaded process (i.e. optimize the entire period in one continuous fluid process, which requires approximately 5 months of computing time), so it was
discretized into 92 separate 1-day optimizations and run in parallel. In the preferable single-threaded optimization, the thermal history of the building is preserved as the optimization moves from one day to the next, but in the divided case, the building model is essentially at the same starting point for each 1-day optimization, and because of the adaptive warm-up routines in EnergyPlus, the building model may not have reached steady-state at the beginning of each day-long simulation. Additionally, the objective function used in this investigation placed extremely high weights on comfort violations (six orders of magnitude higher than the units of energy considered), and the optimizer was coerced by the objective function into choosing control sequences that preserve comfort conditions at all costs. A future investigation with a more appropriately tuned objective function should yield optimized decisions that lead to lower (or at least equal) energy consumption compared to the default case, while still preserving comfort.

The following analysis was conducted with the knowledge that the solution produced by the optimizer is in fact worse than a solution generated by white noise, or the default control sequence. Once a successful set of results is generated, this analysis can be reproduced for the new results quickly and easily.

From the 92-day data set, 2208 hours of data were available for use in constructing the CART model, a list of the predictor variables used is given in table 1 below. First a full CART model was constructed using the entire dataset, the associated dendrogram is given in figure 1. The full model was pruned from 11 splits to 4 according to the convention that the optimal pruned tree has \( n \) splits, where \( n \) is the number of splits with a complexity criterion value within one standard deviation of complexity for the full tree. A diagram showing complexity and splits for the full model is given in figure 2, the point corresponding to 4 splits falls just shy of the (blue, dashed) standard deviation line; this point is selected as the optimally pruned number of splits. The final pruned model is given in figure 3; note that over half of the observed points fall to the left of the first split, and that there are more false-positives in this first branch than there are true positives in the complete tree. This is the first indicator that this model likely doesn’t have high predictive ability. Observing the skill scores and misclassification metrics in table 2, we see that the full model follows a similar trend. Based on the RPSS values, both models perform worse than a white noise process seeded with the proportions of open/close states found by the optimization. Final evidence of the inferiority of the optimal control sequence is in the building’s total energy consumption calculated using each control sequence, however two interesting phenomena result, as seen in the cumulative energy consumption plot in figure 6. The first interesting conclusion is that both the full and partial CART models follow heuristic control logic; opening windows when indoor temperatures are high and outdoor temperatures are low. The second conclusion is that this heuristic doesn’t end up costing more energy than the default (windows closed) control scheme, and in a few cases may lead to some energy savings. These outcomes are very surprising, considering
that the optimal control sequence is considerably more energy-expensive; but in a way this makes sense: the optimizer was hunting for solutions that yielded greatest comfort, and the derived rule is in line with this objective, allowing natural ventilation when indoor and outdoor conditions are close to established comfort conditions.

Figures 4 and 5 show the performance of the full and pruned classification trees, respectively; the blue dashed lines represent the probability of a window opening, and the red lines indicate the result of comparing this value to a threshold value of 0.5; when the probability is greater than 0.5, we assume that a window opening is predicted. The period shown is the first week in June, and we can see that the full model outperforms the partial model, capturing two periods of early morning cooling in the second and third nights (hours 30 and 54), while the pruned tree misses them.

To arrive at the data shown in figure 6, the three control schemes (optimal, default, and CART-derived) were implemented in an EnergyPlus building model. Also implemented in the building model was a stochastic model of occupant behavior, which simulates the manual operation of windows. As a result of the behavioral model, each simulation yields a slightly different result; opened windows at different times lead to increased airflow and change the loads of the building. Figure 6 shows this range of energy consumption as transparent bands, where the uncertainty in energy consumption grows as the simulation time is increased. This confidence interval or consumption range is useful for building engineers as it provides a robustness to the results, showing how much variation in the building’s energy consumption can be expected from occupant behavior.

5. Outlook and Future Work

From the results presented here, there is a clear need for revised optimization results in order to prove the technology and show potential energy savings, instead of increased costs. Next steps in the rule extraction process include adding predictor variables, such as lagged or future predicted states, as well as changing the metric by which models are selected. It is shown in [9] that closed-loop performance of extracted control rules is often considerably worse than open-loop performance, so a closed-loop RPSS value may lead to selection of a model that works better in the simulation environment. Another potential addition to this work is the extension of CART models into random forests. Random Forests will give more robust classification trees and may demonstrate that some variables are more or less worth including in the CART models.

6. Conclusion

A methodology for extracting supervisory control rules for Mixed-Mode Buildings from stochastic control optimization is described and demonstrated on a sample dataset. The given results are synthetic and do not constitute an effective
strategy for reducing building energy consumption, however the methodology and analysis presented are sound and applicable to more meaningful datasets. The extracted control heuristic is shown to use no more energy than default operation, while maintaining occupant comfort.

REFERENCES

### Table 1. Variables used in CART model generation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Quantity</th>
<th>Range</th>
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</thead>
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<tr>
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</tr>
<tr>
<td>Window Status</td>
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<td>[0,1]</td>
</tr>
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<td>Day</td>
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<tr>
<td>Hour</td>
<td>1 Value</td>
<td>1:24</td>
</tr>
<tr>
<td>Dry bulb temperature</td>
<td>1 Value</td>
<td>-100:∞</td>
</tr>
<tr>
<td>Dew-point temperature</td>
<td>1 Value</td>
<td>0:∞</td>
</tr>
<tr>
<td>Relative Humidity</td>
<td>1 Value</td>
<td>0:100</td>
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<tr>
<td>Solar Radiation</td>
<td>1 Value</td>
<td>0:∞</td>
</tr>
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<td>Wind direction</td>
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<td>0:360</td>
</tr>
<tr>
<td>Wind speed</td>
<td>1 Value</td>
<td>0:∞</td>
</tr>
</tbody>
</table>

### Table 2. Goodness of fit in open-loop predictions.

| Model    | RPSS  | $R(T)$ | $R(0|1)$ | $R(1|0)$ |
|----------|-------|--------|----------|----------|
| Full     | -2.20 | .160   | .019     | .140     |
| Pruned   | -1.83 | .194   | .025     | .169     |
Figure 1. Full CART model dendrogram.
Figure 2. Full CART model complexity factors as a function of number of splits; the blue dashed line is one standard deviation from the final split.
Figure 3. Pruned CART model dendrogram.
Figure 4. Window predictions from the full CART model for the first week in June.

Figure 5. Window predictions from the pruned CART model for the first week in June.
Figure 6. Cumulative energy consumption for the entire 92-day period. Note that the CART model outperforms the 'optimal' solution, and in some cases outperforms the default (windows closed) solution.