Reliability based nonlinear fracture mechanics analysis of a concrete dam; a simplified approach

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Reliability Based Nonlinear Fracture Mechanics
Analysis of a Concrete Dam; a Simplified Approach

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Abstract

At a time of decaying infrastructures, and limited resources, there is a need to prioritize rehabilitation by combining probabilistic methods with advanced structural analysis. This paper seeks to demonstrate such an investigation where the safety of an older dam is assessed through its reliability index and a nonlinear fracture mechanics based investigation. It is shown that such a complex evaluation is not only possible, but it provides the Engineer with much insight as to what are the key variables affecting the structure's safety, and which in turn may warrant additional resources for quantification.

Keywords: Dam Safety; Reliability Analysis; Nonlinear Analysis; Finite Element; Fracture Mechanics.

1 Introduction

Many dams in the United States are in serious need for remediation, (National Research Council 1985), (deSena 1985), yet because of limited resources, only some of them could indeed be rehabilitated. Accordingly, there is an increased need to prioritize the dams targeted for remediation through a formal and rigorous mechanism which accounts not only for the (multi) physics nature of the problem, but also for the uncertainties associated with the quantification of key parameters. Hence, there is an urgent need to marry probabilistic methods of investigation with modern computational tools for dam analysis. This is even more apparent when we consider that most of the dams in the United States were built in the thirties at a time when only simple methods could be used to properly dimension them, yet seventy years later (and despite progresses made in our capability to conduct sophisticated finite element studies) most current analysis procedures for safety investigation remain essentially the same as those used earlier on for design, (Federal Regulatory Commission 1991).

To the best of our knowledge Kreuzer and Bury (1983) were the first to focus on this emerging problem. Other key published work include (Baylosis 1988), (Lafitte 1993) and

1 Interestingly enough, the first reported finite element analysis of a civil engineering structure was the analysis of Norfolk Dam for the Wala-Wala Corps district by Clough (Sims, Rhodes and Clough 1964).
(Kreuzer 1994). Most recently, Ellingwood and Tekie (2001) introduced the concept of Fragility Analysis for concrete gravity dams where the fragility curve is the probability of failure in terms of pool elevation.

Furthermore, a number of private organizations (such as BC Hydro), and governmental agencies are actively pursuing guidelines for the probabilistic safety analysis of dams. Of particular interest is a US Army Corps of Engineers Technical Letter, (US Army Corps of Engineers 1992) which states that “Reliability analyses should be used in preparing Rehabilitation Evaluation Reports to distinguish between projects requiring restoration and those requiring modernization”. This letter is complemented by another one, (US Army Corps of Engineers 1993) which describes specific techniques for reliability investigation.

Concurrently, there is increased interest in applying fracture mechanics concepts to dam safety assessment, (Saouma, Dungar and Morris 1991) and (Bourdarot, Mazars and Saouma 1994). As such, another US Army Corps of Engineers Technical Letter stipulates that “A fracture mechanics analysis should be performed if current stability and strength criteria indicate that major structural rehabilitation is necessary because overturning instability and cracking is predicted”, (US Army Corps of Engineers 1991).

Whereas reliability analysis, most often, implies the determination of a structure’s Reliability index $\beta$, this is most simply determined through Monte-Carlo simulations. However, if a reliability based investigation is to be married with a sophisticated analysis (most likely through the finite element method), then Monte-Carlo simulation becomes prohibitively expensive.

Fortunately, alternatives (albeit with some degree of simplification) exist to Monte-Carlo simulations, (computationally prohibitive for non-linear analyses) namely the determination of the Reliability index through the Point Estimate Method, and the Taylor’s Series Finite Difference Estimation (US Army Corps of Engineers 1992, US Army Corps of Engineers 1993). It is the intent of this paper to demonstrate how these techniques can be coupled with fracture mechanics based non-linear finite element analysis to determine the reliability index of a dam prior and after remediation. Other more recent approaches are based on Latin Hypercube approach (Tekie and Ellingwood 2003). As a vehicle for our analyses we consider a dam which has been targeted for major rehabilitation. This dam was built in the thirties, and based on recent hydrological data can no longer sustain its probable maximum flood (PMF). Hence, based on an extensive study (primarily based on rigid body equilibrium), a multi-million dollar rehabilitation program has been considered. A detailed deterministic analysis of this dam has already been published by the author (Saouma and Morris 1998), and this very same dam was also investigated by Ellingwood and Tekie (2001).

2 Description of the Dam

The dam, Fig. 1, built in the thirties, is 53.64 m (176 ft) high; it has 53 monoliths with a crest elevation of 468 m (1,535 ft.) Based on recent hydrologic studies, it was determined (using classical analysis) that the Probable Maximum Flood (PMF) elevation is at 474.2 m (1,555.8 ft), yet its computed Imminent Failure Flood (IFF), which is the pool elevation which would trigger failure (through failure of the rock concrete interface and
ensuing sliding), is at 467.3 m (1,533 ft) or 6.7 m (22 ft) below the PMF. Hence, this dam has been targeted for major rehabilitation. Whereas numerous remediation schemes have been investigated, the one retained for this study entails prestressing the dam from crest to foundation using high-strength steel anchors and mass concrete thrust blocks to be added at the toe of the dam, Fig. 2. Our two-dimensional analysis (justified by the U shape of the valley) focuses on monolith 12 which was recognized as the most critical one.

![Figure 1: Dam Dimensions and Finite Element Mesh](image1.png)

![Figure 2: Proposed Retrofit and Applied Loads](image2.png)

3 Models
3.1 Nonlinear Fracture Mechanics

3.1.1 Analysis Procedure

The analysis was performed with the Merlin code developed by the author (Saouma, Červenka, Reich and Saouma 1997–2004). In this model, the only source of nonlinearity is the rock concrete interface and the uplift pressure, (Dewey, Reich and Saouma 1994).

First the entire rock-concrete contact zone was modeled by interface elements. Then the analysis proceeded incrementally, in terms of pool elevation (the first increment corresponding to the dam self weight). At each pool elevation, a full uplift based on that particular elevation was specified if the interface was to open up. Hence, should the code detect a crack opening, then and only then would the full uplift pressure be applied along the cracked ligament, and then the uplift pressure would reduce to the tail water pressure along the uncracked ligament in accordance with Federal Regulatory Commission (1991), Fig. 3.

![Fig. 3 Uplift Modelling](image)

3.1.2 Interface Crack Constitutive Model

The interface crack model (ICM) used in this investigation, originally developed by Červenka, J. (1994), is an extension of Hillerborg, A. and Modéer, M. and Petersson, P.E. (1976) to account for mixed mode loading.
In the present model, the rock-concrete contact is idealized as an interface between two dissimilar materials with zero thickness. Thus, the objective is to define relationships between normal and tangential stresses with opening and sliding displacements. The major premises upon which the model was developed are:

1. Shear strength depends on the normal stress.
2. Softening is present both in shear and tension.
3. There is residual shear strength due to the friction along the interface, which depends on the compressive normal stress.
4. Reduction in strength, i.e., softening, is caused by crack formation.
5. There is zero normal and shear stiffness when the interface is totally destroyed.
6. Under compressive normal stresses neither the shear nor the normal stiffness decrease to zero. In addition, should a compressive stress be introduced in the normal direction following a full crack opening, two faces of the interface come into contact, and both tangential and normal stiffness become nonzero.
7. Irreversible relative displacements are caused by broken segments of the interface material and by friction between the two crack surfaces.
8. Roughness of the interface causes opening displacements (i.e., dilatancy) when subjected to sliding displacements.
9. The dilatancy vanishes with increasing sliding or opening displacements.

In this model the strength of an interface is described by a failure function:

\[ \tau_1^2 + \tau_2^2 = \tan^2(\phi_f)\sigma^2 \]  

(1)

![Figure 4: Failure function.](image)

where:
• $c$ is the cohesion.
• $\Phi_f$ is the angle of friction.
• $\sigma_i$ is the tensile strength of the interface.
• $\tau_1$ and $\tau_2$ are the two tangential components of the interface traction vector.
• $\sigma$ is the normal traction component.

The shape of the failure function in the two-dimensional case is shown in Figure 4, and it corresponds to the failure criterion first proposed by Carol, I., Bažant, Z.P. and Prat, P.C., (1992). The general three dimensional failure function is obtained by mere rotation around the $\sigma$ axis.

In this model, the tensile strength $f_i$ can be obtained from
\[
\sigma_i = \frac{2c}{\cos \phi_f + \tan \phi_f (1 + \sin \phi_f)}
\]  
(2)

The evolution of the failure function is based on a softening parameter $u^{\text{ieff}}$ which is the norm of the inelastic displacement vector $u^i$. The inelastic displacement vector is obtained by decomposition of the displacement vector $u$ into an elastic part $u^e$ and an inelastic part $u^i$. The inelastic part can subsequently be decomposed into plastic (i.e. irreversible) displacements $u^p$ and fracturing displacements $u^f$. The plastic displacements are assumed to be caused by friction between crack surfaces and the fracturing displacements by the formation of micro cracks.

\[
F = F(c, \sigma_f, \Phi_f); \quad c = c(u^{\text{ieff}}); \quad \sigma_i = \sigma_i(u^{\text{ieff}})
\]

\[
u = u^e + u^i; \quad u^i = u^p + u^f
\]

\[
u^{\text{ieff}} = \|u^i\| = \left(\left(u_x^p + u_y^p + u_z^p\right)^2\right)^{1/2}
\]

(3)

Softening models are illustrated by Figure 5 where $G_{I}^{F}$ and $G_{IIa}^{F}$ are mode I and II fracture energies, $w_c$, $w_c$, are the maximum inelastic effective crack opening beyond which there is no transfer of cohesive stress or cohesion, $w_{Ie}$, $w_{IIe}$, $s_{Ie}$ and $s_{IIe}$ are the corresponding break points defined by Wittmann, Rokugo, Brühwiler, E., Mihashi and Simonin (1988), $\sigma_0$ and $c_0$ are the original tensile strength and cohesion respectively. It should be noted that $G_{IIa}^{F}$ is not the pure mode II fracture energy (i.e. the area under a $\tau-u$ curve), but rather is the energy dissipated during a shear test with high confining normal stress. The residual shear strength is obtained from the failure function by setting both $c$ and $\sigma$ equal to 0, which corresponds to the final shape of the failure function in Figure 4 and is given by:

\[
\tau_1^2 + \tau_2^2 = \tan^2(\Phi_f)\sigma^2
\]

(4)
Stiffness degradation is modeled through a damage parameter, \( D \in <0,1> \), which is a relative measure of the fractured surface. Thus, \( D \) is related to the secant of the normal stiffness \( K_{ns} \) in the uniaxial case:

\[
D = \frac{A_f}{A_o} = 1 - \frac{K_{ns}}{K_{no}} \quad (5)
\]

where \( K_{no} \) is the initial normal stiffness of the interface; \( A_o \) and \( A_f \) are the total interface area and the fractured area respectively. It is assumed, that the damage parameter \( D \) can be determined by converting the mixed mode problem into an equivalent uniaxial one, (Figure 6). In the equivalent uniaxial problem the normal inelastic displacement is set equal to \( u_{\text{eff}} \). Then, the secant normal stiffness can be determined from:

\[
K_{ns} = \frac{\sigma}{u-u^p} = \frac{\sigma_i(u_{\text{eff}})}{u^e+u^p+u^f-u^p} = \frac{\sigma_i(u_{\text{eff}})}{\sigma_i(u_{\text{eff}})/K_{no}+(1-\gamma)u_{\text{eff}}} \quad (6)
\]

where \( \gamma \) is the ratio of irreversible inelastic normal displacement to the total value of inelastic displacement.
Additional details on the joint formulation can be found in (Červenka, Chandra and Saouma 1998) and (Puntel, Bolzon and Saouma 2005). Table 1 summarizes the selected joint properties in the reported analyses.

<table>
<thead>
<tr>
<th>Concrete</th>
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<td>Poisson’s Ratio</td>
<td>Psi</td>
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<tr>
<td>Elastic Modulus</td>
<td>Psi</td>
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<tr>
<td>Poisson’s Ratio</td>
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</tr>
<tr>
<td>Angle of Friction</td>
<td>51 Degrees</td>
</tr>
<tr>
<td>$G_i^F$</td>
<td>0.37 and 0.7</td>
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<tr>
<td>$G_{IIa}^F$</td>
<td>3 $G_i^F$</td>
</tr>
<tr>
<td>$\Gamma_t$</td>
<td>Psi</td>
</tr>
<tr>
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<td>$E_{eff}/0.1$</td>
</tr>
<tr>
<td>$k_s$</td>
<td>$E_{eff}/0.1$</td>
</tr>
</tbody>
</table>

Table 1: Material Properties for NLFM Analysis of the Dam

### 3.2 Reliability Index

In a reliability based analysis, we define the performance function $F$ as the capacity to demand ratio $C/D$, and in general it is a function of one or more variables $x_i$ which describe the geometry, material, loads, and boundary conditions

$$F = F(x_i)$$

and thus $F$ is in turn a random variable with its own probability distribution function PDF, and mean $\mu$, Fig. 7 A performance function evaluation typically require a structural analysis, this may range from a simple calculation to a detailed finite element study.
Reliability index, in turn is used as a relative measure of the reliability or confidence in the ability of a structure to perform its function in a satisfactory manner. In other words it is a measure of the performance function and is defined as the distance between mean performance value and the limit state normalized with respect to the standard deviation, (Ang and Tang 1975).

$$
\beta = \frac{\ln C/D - \frac{\sigma_{\ln(C/D)}^2}{2}}{\sigma_{\ln(C/D)} \sqrt{\ln \left[ 1 + \left( \frac{\sigma_{C/D}}{\mu_{C/D}} \right)^2 \right]}}
$$

(8)

where $\mu_{\ln(C/D)}$ and $\sigma_{\ln(C/D)}$ are the mean and standard deviation of the natural logarithm of capacity divided by demand.

For standard distributions and for $\beta = 3.5$ (close to the target values set by the ACI and AISC codes), it can be shown that the probability of failure is $P_f = 1/9,091$ or $1.1 \times 10^{-4}$. That is 1 in every 10,000 structural members designed with $\beta = 3.5$ is more likely to fail because of either excessive load or under strength sometime in its lifetime. Target values for $\beta$ are shown in Fig. 8.
In practice, some of the analysis parameters can be considered as constants, while others must be variables. The variables, in turn, will have a distribution function characterized (for normal distributions) by a mean and a standard deviation.

In order to determine the reliability index, we must first solve for the mean and standard deviation of the performance function defined in terms of C/D. Unfortunately, many practical engineering problems are so complex that neither an analytical direct integration is possible, nor a Monte-Carlo simulation economically feasible.

Hence, a compromise must be thought, and two such alternatives will be separately considered. Those alternatives are further described in (US Army Corps of Engineers 1993). Whereas in the Monte Carlo simulations numerous random analyses had to be performed, in these methods a finite number of analyses must be undertaken.
3.2.1 Taylor’s Series-Finite Difference Estimation

Whereas in the preceding method we have cut down the number of deterministic analyses to $2^n$, in this one we reduce it even further to $2n + 1$, (Bryant, Brokaw and Mlakar 1993).

This simplified approach starts with the first order Taylor series expansion of Eq. 7 about the mean and limited to linear terms, (Benjamin and Cornell 1970).

$$\mu_F = F(\mu_i)$$  \hspace{1cm} (9)

where $\mu_i$ is the mean for all random variables. For independent random variables, the variance is given by

$$Var(F) = \sigma_F^2 = \sum \left( \frac{\partial F}{\partial \chi_i} \sigma_i \right)^2$$

$$\frac{\partial F}{\partial \chi_i} \approx \frac{F_i^+ - F_i^-}{2\sigma_i}$$  \hspace{1cm} (10)

$$F_i^+ = F(\mu_1, ..., \mu_i + \sigma_i, ..., \mu_n)$$

$$F_i^- = F(\mu_1, ..., \mu_i - \sigma_i, ..., \mu_n)$$

where $\sigma_i$ are the standard deviations of the variables. Hence,

$$\sigma_F = \sum \left( \frac{F_i^+ - F_i^-}{2} \right)$$  \hspace{1cm} (11)

Finally, the reliability index is given by Eq. 8.

The procedure can be summarized as follows:

1. Perform an initial analysis in which all variables are set equal to their mean value. This analysis provides the mean $\mu$.

2. Perform 2n analysis, in which all variables are set equal to their mean values, except variable $i$, which assumes a value equal to $\mu_i + \sigma_i$, and then $\mu_i - \sigma_i$.

3. For each pair of analysis in which variable $x_i$ is modified, determine the standard deviation component associated with the specific variable $i$, which will provide an indication of the sensitivity of the results to variation of this particular variable.

$$\left( \frac{F_i^+ - F_i^-}{2} \right)$$

4. The standard deviation of the entire structure is then determined by simply adding all the $\left( \frac{F_i^+ - F_i^-}{2} \right)$ terms.

Having determined the mean and standard deviation of C/D, the corresponding values for the logarithm of C/D, $\mu_{ln(C/D)}$ and $\sigma_{ln(C/D)}$, and are determined from Eq. 8.
3.2.2 Point Estimate Method

In the Point Estimate Method, we select only few points (in the design space) such that they match the mean, standard deviation, and coefficient of skewness of the distribution\(^2\), (Harr 1987, Wolff 1985). This is accomplished by limiting ourselves to all possible combinations of \(\mu_i \pm \sigma_i\). For each analysis we determine the ratio of capacity over demand, as well as its logarithm. Mean and standard deviation of the logarithmic values are determined from the \(2n\) analyses, and then is the ratio of the mean to the standard deviation of these \(2n\) analyses, Eq. 8.

4 Reliability Based Analysis

In the following analyses, we use a linear elastic fracture mechanics (LEFM) approach for the original dam, and a nonlinear fracture mechanics (NLFM) one for the retrofitted case. A more comprehensive study should consider the other two possible combinations. Hence, in the LEFM analysis, crack propagation stops when the stress intensity factor is equal to the fracture toughness \((K_I = K_{Ic})\). In this mode, there is no need to use interface elements. In the NLFM analysis, interface elements (as previously described) are used along the crack which growth is governed by a strength criterion \((\sigma^+ = f'_d)\) in the context of a nonlinear fracture mechanics model with cohesive cracks.

In this study, the assumed constants are density of concrete, and elastic moduli, whereas the assumed variables are the pool elevation \(\mu = 467.1\) m (1,532.5 ft), \(\sigma = 1.5\) m (5 ft); fracture toughness \(K_{Ic}: \mu = 54.95\) Mpa\(\sqrt{\text{m}}\) (50 ksi\(\sqrt{\text{in}}\)), \(\sigma = 54.95\) Mpa\(\sqrt{\text{m}}\) (50 ksi\(\sqrt{\text{in}}\)) (note: zero fracture toughness allowed); cohesion \(c: \mu = 75.8\) kPa (11 psi), \(\sigma = 13.8\) kPa (2 psi); and angle of friction \(\Phi: \mu = 50^\circ\), \(\sigma = 20^\circ\). The tail water elevation is directly correlated to head water elevation since only overtopping of the dam would cause a tail water presence. All variables are assumed to have a normal distribution, and whereas realistic values are given to the means, the standard deviations are arbitrarily selected using engineering judgment.

Finally, it is assumed that there is no correlation between the angle of friction and the cohesion, and that the drains are ineffective.

The performance function is the sliding safety factor (commonly referred to as SFF in dam literature) as determined from

\[
\text{SFF} = \frac{\text{Capacity}}{\text{Demand}} = \frac{\sum F_c \tan \Phi + c L_{uncr}}{\sum F_{ht}} \tag{12}
\]

where \(\Phi\), \(c\) and \(L_{uncr}\) are the friction angle, the cohesion and the length of the uncracked ligament respectively.

It should be noted that the “Demand” is governed by the PMF with which a return period must be associated (the higher the PMF, the longer the return period), hence there is an

---

\(^2\) Such a simplification is behind the development of the Whitney stress block in the ACI code; a complex stress distribution is replaced by a simpler one (rectangular), such that both have the same resultant force and location.
element of probability associated with the PMF, which will ultimately affect the
evaluation of the reliability index. In this study, the PMF was considered as a constant.
Hence we seek $P[Z = C/D \leq 1]$ or $P[Z = SFF - 1 \leq 0]$.

4.1 Original Dam: LEFM Analysis

Whereas for the sake of consistency, a nonlinear analysis should be undertaken for the
original dam in the context of a reliability study, a simpler linear elastic fracture
mechanics (LEFM) one was performed for illustrative purposes. In an LEFM analysis,
there is no energy dissipation along the crack, but only at the crack tip characterized by
its stress intensity factor (SIF) which should be smaller than the fracture toughness (a
material characteristic) for the crack not to propagate.

4.1.1 Taylor’s Series-Finite Difference Estimation

Two sets of analyses are performed. In the first one, Table 2 the original set of variables
was selected.

<table>
<thead>
<tr>
<th>Case</th>
<th>FE Kic</th>
<th>POOL</th>
<th>$c$</th>
<th>$\Phi$</th>
<th>Crack</th>
<th>SFF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,532.5</td>
<td>1,400.5</td>
<td>50</td>
<td>11</td>
<td>30</td>
<td>41.0</td>
</tr>
<tr>
<td>2</td>
<td>1,537.5</td>
<td>1,400.5</td>
<td>50</td>
<td>11</td>
<td>30</td>
<td>52.0</td>
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<tr>
<td>3</td>
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<td>1,363.0</td>
<td>50</td>
<td>11</td>
<td>30</td>
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<tr>
<td>4</td>
<td>1,532.5</td>
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<td>11</td>
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<td>0.01</td>
<td>0.085</td>
<td>0.245</td>
<td></td>
<td></td>
</tr>
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</table>

Table 2: Tabulation of Taylor Series-Finite Difference Method for the LEFM Study; Original $\Phi$

However because of the very small SFF values obtained (due to the unsafe nature of the
original dam), we conducted another analysis in which the angle $\Phi$ was arbitrarily
increased from a mean of 30 to 45 degrees, Table 3. It should be noted that whereas this
analysis required considerations of 9 $(2n+1)$ cases, those were extracted from only 3
separate finite element analyses (labeled as A-B-C).

The numerical procedure is as follows:

1. Identify set of parameters for each case, Table 3.
2. Perform a finite element analysis, and determine the crack length for which the crack tip stress intensity factor is equal to the fracture toughness.

3. Determine the SFF from Eq. 15.

4. Transfer the determined SFF to Table 3, and determine both $\mu_{C/D}$ and $\sigma_{C/D}$.

Using this procedure, we have $\mu_{C/D} = 1.22$ and $\sigma_{C/D} = 0.74$. Substituting in Eq. 8, this results in equal to 0.075. Indeed, as expected a very small value and unsafe value. Furthermore, we note from Table 3 that results are not very sensitive to variations in the fracture toughness $K_{IC}$, however they are most sensitive to the pool elevation and angle of friction, that is $\sigma_{C/D}^\phi$.

<table>
<thead>
<tr>
<th>Run</th>
<th>Head</th>
<th>Tail</th>
<th>$c$</th>
<th>$\Phi$</th>
<th>$K_{IC}$</th>
<th>$\sigma_{C/D}$</th>
<th>$\beta$</th>
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Table 3: Tabulation of Taylor Series-Finite Difference Method for the LEFM Study; $\Phi = 45^\circ$.

4.1.2 Point Estimate Method

With 4 variables, this study required 16 ($2^4$) analyses, however only two (A and B) distinct finite element analyses are needed. From each analysis, we determine the crack length for which the stress intensity factor is equal to the fracture toughness. Results are summarized in Table 4.

We observe that this analysis yields $\beta = 0.77$ value much smaller than one which is again clearly unacceptable and an indication that the dam can not sustain the selected pool elevation. Furthermore, we note that the values predicted by this method are much larger.
than the one obtained from the Taylor Series method. This discrepancy may be attributed to large errors associated with low $\beta$ values.

4.2 NLFM Analysis

The reliability analysis of the retrofitted dam is determined on the basis of a nonlinear finite element analysis with interface elements.

Again, the assumed constants are: density of concrete and elastic moduli, whereas the assumed variables are the pool elevation $\mu = 467.1$ m (1,532.5 ft), $\sigma = 1.52$ m (5 ft), the fracture energy $G_F^I \mu =61.40$ N/m (0.35 lbs/in), $\sigma = 17.5$ N/m (0.1 lbs/in), cohesion $c \mu = 0.29$ MPa (42 psi), $\sigma = 0.034$ Mpa (5 psi), and angle of friction $\mu = 51^\circ$, $\sigma = 5^\circ$. Some of the means were intentionally varied from their respective values in the linear elastic analysis.

In this analysis, the tensile strength is determined from the cohesion $c$ and angle of friction - (Eq. 2), and the mode II fracture energy $G_F^{II}$ is set to three times the mode I fracture energy $G_F^I$. Since this investigation was completed, there have been new evidences that the ratio should be closer to ten than to three.

<table>
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<th>Variables</th>
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<th>$K_{ic}$</th>
<th>$c$</th>
<th>$\Phi$</th>
<th>Crack</th>
<th>SFF</th>
<th>Ln (SFF)</th>
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\[
\mu_{lnC/D} = 0.29 \\
\sigma_{lnC/D} = 0.38 \\
\beta = 0.77
\]
4.2.1 Point Estimate Method

Two cases were considered for this analysis. Results are tabulated in Table 5, where h is the pool elevation, and Lc the cracked ligament length.

In the first we determine the average SFF value based on global equilibrium, and in the other we determine our computation on the basis of weighted average SFF as determined directly from the finite element analysis (this can be readily extracted from the interface elements), Fig. 9. As anticipated, both β values are in the acceptable range, however the one associated with the more accurate pointwise evaluation of SFF (6.62) is higher than the other (3.04). This discrepancy results from the simplifying (and conservative) assumption that the shear stress is constant along the base.
4.2.2 Taylor’s Series-Finite Difference Estimation

Again, two analyses are undertaken, the first based on a global average SFF, and the other based on the weighted average value of SFF along the base, SFF’, Table 6.

![Table 5: Tabulation of Point Estimate Method; Nonlinear Analysis; Retrofitted Dam.](image)

For the first case, we have $\mu_{C/D} = 4.22$ and $\sigma_{C/D} = 0.835$. Substituting in Eq. 8, this results in $\beta = 7.2$ which is indicative of a very safe structure. In terms of sensitivity, results are

![Table 6: Tabulation of Taylor’s Series Finite Difference Estimate Method; Nonlinear Analysis; Retrofitted Dam.](image)
most sensitive to pool elevation, cohesion, angle of friction, and fracture energy (in this order).

For the second case, we have $\mu_{CD} = 4.78$ and $\sigma_{CD} = 1.215$. Substituting in Eq. 8, this results in $\beta = 6.1$. Not so surprisingly, this analysis shows that the safety is most sensitive to the angle of friction, cohesion, pool elevation and finally fracture energy. The change in relative importance is attributed to the fact that the SFF is determined point-wise along the base.

When compared with the Point estimate method, there is a large discrepancy in $\beta$ when the calculations are based on SFF (7.2 versus 3.04), but a smaller one for those based on SFF’ (6.1 versus 6.62). Clearly, this discrepancy is indicative of some limitations of the reliability methods. Indeed misleading or erroneous results can be obtained when dealing with probabilities of rare events or when the failure design space is not sufficiently represented (as in this case). Nevertheless in all cases we have evidence of a very safe structure.

5 Conclusions

From the above analyses, we conclude Reliability based analysis of complex structures, requiring nonlinear finite element analysis, is a challenging task. Monte-Carlo simulation is computationally prohibitive, and alternate methods exist. This paper presented two such alternate techniques to obtain the reliability index of a dam based on nonlinear finite element analyses. The discrepancy between some of the results point to the limitation of these methods, and more recent ones should be explored. Possible alternatives include the so-called Latin Hypercube, (Iman, Helton and Campbell 1981), or the determination of the dam fragility curves.

Nevertheless, and despite some quantitative discrepancies between the two methods presented, results are consistent with engineering common sense. The reliability index of the original dam was clearly too low, and the one of the (to be) retrofitted dam is too high (for reference, it is estimated the reliability index of structures designed in accordance with the ACI or AISC codes is around 3.5). Hence, it appears that financial saving could be achieved by reducing the extent of the rehabilitation work; however it must be kept in mind that as in most major public works optimum financial considerations are not necessarily the only ones.

An important by-product of these analyses is the identification of the most critical parameters that is those whose small variation will result in large changes in the final results. In the reported analyses, it is evident that the fracture energy $GF$ plays a relatively small role. This is particular important in light of the extensive effort worldwide to develop a unified standard for concrete fracture energy.

Furthermore, this paper highlights the difference in Sliding safety factor when determined on the basis of the average normal and shear stresses, or when determined point-wise along the uncracked ligament. The two analyses reported yielded contradictory results in terms of the merit of one versus the other method. However, both cases yielded consistent values. Finally, this paper is nothing but a first attempt to apply semi-advanced probabilistic methods to advanced dam analysis (as opposed to apply advanced probabilistic techniques to an approximate dam analysis).
6 Acknowledgments

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7 References


