# **CHAPTER**

9

# GAS MIXTURES AND PSYCHROMETRICS

p to this point, we have limited our consideration to thermodynamic systems that involve a single pure substance such as water, refrigerant-134a, or nitrogen. Many important thermodynamic applications, however, involve *mixtures* of several pure substances rather than a single pure substance. Therefore, it is important to develop an understanding of mixtures and learn how to handle them.

In this chapter, we deal with nonreacting gas mixtures. A nonreacting gas mixture can be treated as a pure substance since it is usually a homogeneous mixture of different gases. The properties of a gas mixture obviously will depend on the properties of the individual gases (called *components* or *constituents*) as well as on the amount of each gas in the mixture. Therefore, it is possible to prepare tables of properties for mixtures. This has been done for common mixtures such as air. It is not practical to prepare property tables for every conceivable mixture composition, however, since the number of possible compositions is endless. Therefore, we need to develop rules for determining mixture properties from a knowledge of mixture composition and the properties of the individual components. We do this first for ideal-gas mixtures and then for real-gas mixtures. The basic principles involved are also applicable to liquid or solid mixtures, called *solutions*.

At temperatures below the critical temperature, the gas phase of a substance is frequently referred to as a vapor. The term vapor implies a gaseous state that is close to the saturation region of the substance, raising the possibility of condensation during a process.

When we deal with a gas—vapor mixture, the vapor may condense out of the mixture during a process, forming a two-phase mixture. This may complicate the analysis considerably. Therefore, a gas—vapor mixture needs to be treated differently from an ordinary gas mixture.

Several gas-vapor mixtures are encountered in engineering. In this chapter, we consider the *air-water-vapor mixture*, which is the most commonly encountered gas-vapor mixture in practice. We also discuss *air-conditioning*, which is the primary application area of air-water-vapor mixtures.

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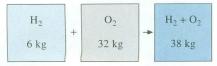
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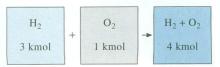
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#### FIGURE 9-1

The mass of a mixture is equal to the sum of the masses of its components.



#### FIGURE 9-2

The number of moles of a nonreacting mixture is equal to the sum of the number of moles of its components.

$$H_2 + O_2$$

$$y_{H_2} = 0.75$$

$$y_{O_2} = \frac{0.25}{1.00}$$

#### FIGURE 9-3

The sum of the mole fractions of a mixture is equal to 1.

 $3 \text{ kg O}_2$   $5 \text{ kg N}_2$  $12 \text{ kg CH}_4$ 

#### FIGURE 9-4

Schematic for Example 9–1.

# 9-1 • COMPOSITION OF A GAS MIXTURE: MASS AND MOLE FRACTIONS

To determine the properties of a mixture, we need to know the *composition* of the mixture as well as the properties of the individual components. There are two ways to describe the composition of a mixture: either by specifying the number of moles of each component, called **molar analysis**, or by specifying the mass of each component, called **gravimetric analysis**.

Consider a gas mixture composed of k components. The mass of the mixture  $m_m$  is the sum of the masses of the individual components, and the mole number of the mixture  $N_m$  is the sum of the mole numbers of the individual components\* (Figs. 9–1 and 9–2). That is,

$$m_m = \sum_{i=1}^k m_i$$
 and  $N_m = \sum_{i=1}^k N_i$  (9-1*a*, *b*)

The ratio of the mass of a component to the mass of the mixture is called the **mass fraction** mf, and the ratio of the mole number of a component to the mole number of the mixture is called the **mole fraction** y:

$$\mathrm{mf_i} = \frac{m_i}{m_m}$$
 and  $y_i = \frac{N_i}{N_m}$  (9–2a, b)

Dividing Eq. 9–1a by  $m_m$  or Eq. 9–1b by  $N_m$ , we can easily show that the sum of the mass fractions or mole fractions for a mixture is equal to 1 (Fig. 9–3):

$$\sum_{i=1}^{k} \mathbf{mf}_i = 1 \quad \text{and} \quad \sum_{i=1}^{k} y_i = 1$$

The mass of a substance can be expressed in terms of the mole number N and molar mass M of the substance as m = NM. Then the **apparent** (or **average**) **molar mass** and the **gas constant** of a mixture can be expressed as

$$M_m = \frac{m_m}{N_m} = \frac{\sum m_i}{N_m} = \frac{\sum N_i M_i}{N_m} = \sum_{i=1}^k y_i M_i$$
 and  $R_m = \frac{R_u}{M_m}$  (9-3a, b)

The molar mass of a mixture can also be expressed as

$$M_{m} = \frac{m_{m}}{N_{m}} = \frac{m_{m}}{\sum m_{i}/M_{i}} = \frac{1}{\sum m_{i}/(m_{m}M_{i})} = \frac{1}{\sum_{i=1}^{k} \frac{\text{mf}_{i}}{M_{i}}}$$
(9-4)

Mass and mole fractions of a mixture are related by

$$\text{mf}_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_{\cdots} M_{\cdots}} = y_i \frac{M_i}{M_{\cdots}}$$
 (9–5)

#### **EXAMPLE 9-1** Mass and Mole Fractions of a Gas Mixture

Consider a gas mixture that consists of 3 kg of  $O_2$ , 5 kg of  $N_2$ , and 12 kg of  $CH_4$ , as shown in Fig. 9–4. Determine (a) the mass fraction of each component, (b) the mole fraction of each component, and (c) the average molar mass and gas constant of the mixture.

<sup>\*</sup>Throughout this chapter, the subscript m will denote the gas mixture and the subscript i will denote any single component of the mixture.

**SOLUTION** The schematic of the gas mixture is given in Fig. 9–4. We note that this is a gas mixture that consists of three gases of known masses.

Analysis (a) The total mass of the mixture is

$$m_m + m_{O_2} + m_{N_2} + m_{CH_4} = 3 + 5 + 12 = 20 \text{ kg}$$

Then the mass fraction of each component becomes

$$mf_{O_2} = \frac{m_{O_2}}{m_m} = \frac{3 \text{ kg}}{20 \text{ kg}} = \mathbf{0.15}$$

$$mf_{N_2} = \frac{m_{N_2}}{m_m} = \frac{5 \text{ kg}}{20 \text{ kg}} = \mathbf{0.25}$$

$$mf_{CH_4} = \frac{m_{CH_4}}{m_m} = \frac{12 \text{ kg}}{20 \text{ kg}} = \mathbf{0.60}$$

(b) To find the mole fractions, we need to determine the mole numbers of each component first:

$$N_{\rm O_2} = \frac{m_{\rm O_2}}{M_{\rm O_2}} = \frac{3 \text{ kg}}{32 \text{ kg/kmol}} = 0.094 \text{ kmol}$$

$$N_{\rm N_2} = \frac{m_{\rm N_2}}{M_{\rm N_2}} = \frac{5 \text{ kg}}{28 \text{ kg/kmol}} = 0.179 \text{ kmol}$$

$$N_{\rm CH_4} = \frac{m_{\rm CH_4}}{M_{\rm CH_4}} = \frac{12 \text{ kg}}{16 \text{ kg/kmol}} = 0.750 \text{ kmol}$$

Thus.

$$N_m = N_{O_2} + N_{N_2} + N_{CH_4} = 0.094 + 0.179 + 0.750 = 1.023 \text{ kmol}$$

and

$$y_{O_2} = \frac{N_{O_2}}{N_m} = \frac{0.094 \text{ kmol}}{1.023 \text{ kmol}} = \mathbf{0.092}$$

$$y_{N_2} = \frac{N_{N_2}}{N_m} = \frac{0.179 \text{ kmol}}{1.023 \text{ kmol}} = \mathbf{0.175}$$

$$y_{CH_4} = \frac{N_{CH_4}}{N_m} = \frac{0.750 \text{ kmol}}{1.023 \text{ kmol}} = \mathbf{0.733}$$

(c) The average molar mass and gas constant of the mixture are determined from their definitions,

$$M_m = \frac{m_m}{N_m} = \frac{20 \text{ kg}}{1.023 \text{ kmol}} = 19.6 \text{ kg/kmol}$$

or

$$M_m = \sum y_i M_i = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} + y_{CH_4} M_{CH_4}$$
  
= (0.092)(32) + (0.175)(28) + (0.733)(16)  
= 19.6 kg/kmol

Also,

$$R_m = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/(kmol} \cdot \text{K})}{19.6 \text{ kg/kmol}} = 0.424 \text{ kJ/(kg} \cdot \text{K})$$

**Discussion** When mass fractions are available, the molar mass and mass fractions could also be determined directly from Eqs. 9–4 and 9–5.

# 9-2 • P-v-T BEHAVIOR OF GAS MIXTURES: **IDEAL AND REAL GASES**

An ideal gas is defined as a gas whose molecules are spaced far apart so that the behavior of a molecule is not influenced by the presence of other molecules—a situation encountered at low densities. We also mentioned that real gases approximate this behavior closely when they are at a low pressure or high temperature relative to their critical-point values. The P-v-T behavior of an ideal gas is expressed by the simple relation Pv = RT, which is called the ideal-gas equation of state. The P-v-T behavior of real gases is expressed by more complex equations of state or by Pv = ZRT, where Z is the compressibility factor.

When two or more ideal gases are mixed, the behavior of a molecule normally is not influenced by the presence of other similar or dissimilar molecules, and therefore a nonreacting mixture of ideal gases also behaves as an ideal gas. Air, for example, is conveniently treated as an ideal gas in the range where nitrogen and oxygen behave as ideal gases. When a gas mixture consists of real (nonideal) gases, however, the prediction of the P-v-T behavior of the mixture becomes rather involved.

The prediction of the P-v-T behavior of gas mixtures is usually based on two models: Dalton's law of additive pressures and Amagat's law of additive volumes. Both models are described and discussed below.

Dalton's law of additive pressures: The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and volume (Fig. 9-5).

Amagat's law of additive volumes: The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure (Fig. 9-6).

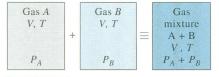
Dalton's and Amagat's laws hold exactly for ideal-gas mixtures, but only approximately for real-gas mixtures. This is due to intermolecular forces that may be significant for real gases at high densities. For ideal gases, these two laws are identical and give identical results.

Dalton's and Amagat's laws can be expressed as follows:

Dalton's law: 
$$P_{m} = \sum_{i=1}^{k} P_{i}(T_{m}, V_{m})$$
 exact for ideal gases, approximate for real gases (9-6)

Amagat's law: 
$$V_m = \sum_{i=1}^k V_i(T_m, P_m)$$
 for real gases (9-7)

In these relations,  $P_i$  is called the **component pressure** and  $V_i$  is called the **component volume** (Fig. 9–7). Note that  $V_i$  is the volume a component would occupy if it existed alone at  $T_m$  and  $P_m$ , not the actual volume occupied by the component in the mixture. (In a vessel that holds a gas mixture, each component fills the entire volume of the vessel. Therefore, the volume of each component is equal to the volume of the vessel.) Also, the ratio  $P_i/P_m$  is called the **pressure fraction** and the ratio  $V_i/V_m$  is called the **volume fraction** of component i.



#### FIGURE 9-5

Dalton's law of additive pressures for a mixture of two ideal gases.

$$\begin{bmatrix} \operatorname{Gas} A \\ P, T \\ V_A \end{bmatrix} + \begin{bmatrix} \operatorname{Gas} B \\ P, T \\ V_B \end{bmatrix} \equiv \begin{bmatrix} \operatorname{Gas} \operatorname{mixture} \\ A + B \\ P, T \\ V_A + V_B \end{bmatrix}$$

#### FIGURE 9-6

Amagat's law of additive volumes for a mixture of two ideal gases.

$$\begin{array}{c|c} O_2 + N_2 \\ 100 \text{ kPa} \\ 400 \text{ K} \\ 1 \text{ m}^3 \end{array} \longrightarrow \begin{array}{c|c} O_2 \\ 100 \text{ kPa} \\ 400 \text{ K} \\ 0.3 \text{ m}^3 \end{array} \begin{array}{c} N_2 \\ 100 \text{ kPa} \\ 400 \text{ K} \\ 0.7 \text{ m}^3 \end{array}$$

#### FIGURE 9-7

The volume a component would occupy if it existed alone at the mixture T and P is called the component volume (for ideal gases, it is equal to the partial volume  $y_i V_m$ ).

### **Ideal-Gas Mixtures**

For ideal gases,  $P_i$  and  $V_i$  can be related to  $y_i$  by using the ideal-gas relation for both the components and the gas mixture:

$$\begin{split} \frac{P_i(T_m, V_m)}{P_m} &= \frac{N_i R_u T_m / V_m}{N_m R_u T_m / V_m} = \frac{N_i}{N_m} = y_i \\ \frac{V_i(T_m, P_m)}{V_m} &= \frac{N_i R_u T_m / P_m}{N_m R_u T_m / P_m} = \frac{N_i}{N_m} = y_i \end{split}$$

Therefore,

$$\frac{P_i}{P_m} = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i \tag{9-8}$$

Equation 9–8 is strictly valid for ideal-gas mixtures since it is derived by assuming ideal-gas behavior for the gas mixture and each of its components. The quantity  $y_i P_m$  is called the **partial pressure** (identical to the *component pressure* for ideal gases), and the quantity  $y_i V_m$  is called the **partial volume** (identical to the *component volume* for ideal gases). Note that for an ideal-gas mixture, the mole fraction, the pressure fraction, and the volume fraction of a component are identical.

The composition of an ideal-gas mixture (such as the exhaust gases leaving a combustion chamber) is frequently determined by a volumetric analysis (called the Orsat Analysis) and Eq. 9–8. A sample gas at a known volume, pressure, and temperature is passed into a vessel containing reagents that absorb one of the gases. The volume of the remaining gas is then measured at the original pressure and temperature. The ratio of the reduction in volume to the original volume (volume fraction) represents the mole fraction of that particular gas.

# **Real-Gas Mixtures**

Dalton's law of additive pressures and Amagat's law of additive volumes can also be used for real gases, often with reasonable accuracy. This time, however, the component pressures or component volumes should be evaluated from relations that take into account the deviation of each component from ideal-gas behavior. One way of doing that is to use more exact equations of state (van der Waals, Beattie–Bridgeman, Benedict–Webb–Rubin, etc.) instead of the ideal-gas equation of state. Another way is to use the compressibility factor (Fig. 9–8) as

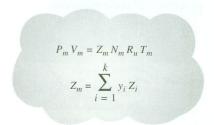
$$PV = ZNR_{u}T ag{9-9}$$

The compressibility factor of the mixture  $Z_m$  can be expressed in terms of the compressibility factors of the individual gases  $Z_i$  by applying Eq. 9–9 to both sides of Dalton's law or Amagat's law expression and simplifying. We obtain

$$Z_m = \sum_{i=1}^{k} y_i Z_i {(9-10)}$$

where  $Z_i$  is determined either at  $T_m$  and  $V_m$  (Dalton's law) or at  $T_m$  and  $P_m$  (Amagat's law) for each individual gas. It may seem that using either law will give the same result, but it does not.

The compressibility-factor approach, in general, gives more accurate results when the  $Z_i$ 's in Eq. 9–10 are evaluated by using Amagat's law instead of



#### FIGURE 9-8

One way of predicting the *P-v-T* behavior of a real-gas mixture is to use compressibility factors.

Dalton's law. This is because Amagat's law involves the use of mixture pressure  $P_m$ , which accounts for the influence of intermolecular forces between the molecules of different gases. Dalton's law disregards the influence of dissimilar molecules in a mixture on each other. As a result, it tends to underpredict the pressure of a gas mixture for a given  $V_m$  and  $T_m$ . Therefore, Dalton's law is more appropriate for gas mixtures at low pressures. Amagat's law is more appropriate at high pressures.

Note that there is a significant difference between using the compressibil-

Note that there is a significant difference between using the compressibility factor for a single gas and for a mixture of gases. The compressibility factor predicts the *P-v-T* behavior of single gases rather accurately, as discussed in Chap. 3, but not for mixtures of gases. When we use compressibility factors for the components of a gas mixture, we account for the influence of like molecules on each other; the influence of dissimilar molecules remains largely unaccounted for. Consequently, a property value predicted by this approach may be considerably different from the experimentally determined value.

# 9-3 - PROPERTIES OF GAS MIXTURES: IDEAL AND REAL GASES

Consider a gas mixture that consists of 2 kg of  $N_2$  and 3 kg of  $CO_2$ . The total mass (an *extensive property*) of this mixture is 5 kg. How did we do it? Well, we simply added the mass of each component. This example suggests a simple way of evaluating the **extensive properties** of a nonreacting ideal- or realgas mixture: *Just add the contributions of each component of the mixture* (Fig. 9–9). Then the total internal energy, enthalpy, and entropy of a gas mixture can be expressed, respectively, as

$$U_m = \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} m_i u_i = \sum_{i=1}^{k} N_i \bar{u}_i$$
 (kJ) (9–11)

$$H_m = \sum_{i=1}^{k} H_i = \sum_{i=1}^{k} m_i h_i = \sum_{i=1}^{k} N_i \bar{h}_i$$
 (kJ) (9–12)

$$S_m = \sum_{i=1}^k S_i = \sum_{i=1}^k m_i s_i = \sum_{i=1}^k N_i \bar{s}_i$$
 (kJ/K) (9–13)

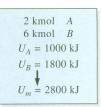
By following a similar logic, the changes in internal energy, enthalpy, and entropy of a gas mixture during a process can be expressed, respectively, as

$$\Delta U_m = \sum_{i=1}^k \Delta U_i = \sum_{i=1}^k m_i \Delta u_i = \sum_{i=1}^k N_i \Delta \bar{u}_i \qquad \text{(kJ)}$$

$$\Delta H_m = \sum_{i=1}^k \Delta H_i = \sum_{i=1}^k m_i \Delta h_i = \sum_{i=1}^k N_i \Delta \bar{h}_i \qquad \text{(kJ)}$$

$$\Delta S_m = \sum_{i=1}^k \Delta S_i = \sum_{i=1}^k m_i \Delta s_i = \sum_{i=1}^k N_i \Delta \bar{s}_i \qquad \text{(kJ/K)}$$

Now reconsider the same mixture, and assume that both  $N_2$  and  $CO_2$  are at 25°C. The temperature (an *intensive* property) of the mixture is, as you would expect, also 25°C. Notice that we did not add the component temperatures to



#### FIGURE 9-9

The extensive properties of a mixture are determined by simply adding the properties of the components.

determine the mixture temperature. Instead, we used some kind of averaging scheme, a characteristic approach for determining the **intensive properties** of a gas mixture. The internal energy, enthalpy, and entropy of a gas mixture *per unit mass* or *per unit mole* of the mixture can be determined by dividing the equations above by the mass or the mole number of the mixture  $(m_m \text{ or } N_m)$ . We obtain (Fig. 9–10)

$$u_m = \sum_{i=1}^k \mathrm{mf}_i u_i$$
 and  $\bar{u}_m = \sum_{i=1}^k y_i \bar{u}_i$  (kJ/kg or kJ/kmol) (9–17)

$$h_m = \sum_{i=1}^{k} \text{mf}_i h_i \text{ and } \bar{h}_m = \sum_{i=1}^{k} y_i \bar{h}_i$$
 (kJ/kg or kJ/kmol) (9–18)

$$s_m = \sum_{i=1}^k \mathrm{mf}_i s_i$$
 and  $\bar{s}_m = \sum_{i=1}^k y_i \bar{s}_i$  (kJ/kg·K or kJ/kmol·K) (9–19)

Similarly, the specific heats of a gas mixture can be expressed as

$$C_{v,m} = \sum_{i=1}^{k} \mathrm{mf}_{i} C_{v,i} \quad \text{and} \quad \overline{C}_{v,m} = \sum_{i=1}^{k} y_{i} \overline{C}_{v,i} \qquad (kJ/kg \cdot {}^{\circ}C \text{ or } kJ/kmol \cdot {}^{\circ}C)$$
(9-20)

$$C_{p,\,m} = \sum_{i=1}^k \mathrm{mf}_i C_{p,\,i} \quad \text{and} \quad \overline{C}_{p,\,m} = \sum_{i=1}^k y_i \, \overline{C}_{p,\,i} \qquad (\mathrm{kJ/kg} \cdot \mathrm{^\circ C} \text{ or kJ/kmol} \cdot \mathrm{^\circ C}) \quad \textbf{(9-21)}$$

Notice that *properties per unit mass involve mass fractions* ( $mf_i$ ) *and properties per unit mole involve mole fractions* ( $y_i$ ).

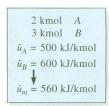
The relations just given are exact for ideal-gas mixtures, and approximate for real-gas mixtures. (In fact, they are also applicable to nonreacting liquid and solid solutions especially when they form an "ideal solution.") The only major difficulty associated with these relations is the determination of properties for each individual gas in the mixture. The analysis can be simplified greatly, however, by treating the individual gases as an ideal gas, if doing so does not introduce a significant error.

### **EXAMPLE 9-2** Mixing Two Ideal Gases in a Tank

An insulated rigid tank is divided into two compartments by a partition. One compartment contains 7 kg of oxygen gas at 40°C and 100 kPa, and the other compartment contains 4 kg of nitrogen gas at 20°C and 150 kPa. Now the partition is removed, and the two gases are allowed to mix. Determine (a) the mixture temperature and (b) the mixture pressure after equilibrium has been established.

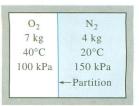
**SOLUTION** We take the entire contents of the tank (both compartments) as the system (Fig. 9–11). This is a *closed system* since no mass crosses the boundary during the process. We note that the volume of a rigid tank is constant and thus  $v_2 = v_1$  and there is no boundary work done.

**Assumptions** 1 We assume both gases to be ideal gases, and their mixture to be an ideal-gas mixture. This assumption is reasonable since both the oxygen and nitrogen are well above their critical temperatures and well below their critical pressures. 2 The tank is insulated and thus there is no heat transfer. 3 There are no other forms of work involved.



#### FIGURE 9-10

The intensive properties of a mixture are determined by weighted averaging.



**FIGURE 9–11** Schematic for Example 9–2.

**Analysis** (a) Noting that there is no energy transfer to or from the tank, the energy balance for the system can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

$$0 = \Delta U = \Delta U_{\text{N}_2} + \Delta U_{\text{O}_2}$$

$$[mC_v (T_m - T_1)]_{\text{N}_2} + [mC_v (T_m - T_1)]_{\text{O}_2} = 0$$

By using  $C_{\nu}$  values at room temperature (from Table A–2a), the final temperature of the mixture is determined to be

$$(4 \text{ kg})(0.743 \text{ kJ/kg} \cdot ^{\circ}\text{C})(T_m - 20^{\circ}\text{C}) + (7 \text{ kg})(0.658 \text{ kJ/kg} \cdot ^{\circ}\text{C})(T_m - 40^{\circ}\text{C}) = 0$$

$$T_m = 32.2^{\circ}\text{C}$$

(b) The final pressure of the mixture is determined from the ideal-gas relation

$$P_m V_m = N_m R_u T_m$$

where

$$N_{\rm O_2} = \frac{m_{\rm O_2}}{M_{\rm O_2}} = \frac{7 \text{ kg}}{32 \text{ kg/kmol}} = 0.219 \text{ kmol}$$

$$N_{\rm N_2} = \frac{m_{\rm N_2}}{M_{\rm N_2}} = \frac{4 \text{ kg}}{28 \text{ kg/kmol}} = 0.143 \text{ kmol}$$

$$N_m = N_{\rm O_2} + N_{\rm N_2} = 0.219 + 0.143 = 0.362 \text{ kmol}$$

and

$$V_{O_2} = \left(\frac{NR_u T_1}{P_1}\right)_{O_2} = \frac{(0.219 \text{ kmol})(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(313 \text{ K})}{100 \text{ kPa}} = 5.70 \text{ m}^3$$

$$V_{N_2} = \left(\frac{NR_u T_1}{P_1}\right)_{N_2} = \frac{(0.143 \text{ kmol})(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(293 \text{ K})}{150 \text{ kPa}} = 2.32 \text{ m}^3$$

$$V_m = V_{O_2} + V_{N_2} = 5.70 + 2.32 = 8.02 \text{ m}^3$$

Thus

$$P_m = \frac{N_m R_u T_m}{V_m} = \frac{(0.362 \text{ kmol})(8.314 \text{ kPa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(305.2 \text{ K})}{8.02 \text{ m}^3} = 114.5 \text{ kPa}$$

**Discussion** We could also determine the mixture pressure by using  $P_mV_m = m_mR_mT_m$ , where  $R_m$  is the apparent gas constant of the mixture. This would require a knowledge of mixture composition in terms of mass or mole fractions.

# 9-4 DRY AND ATMOSPHERIC AIR

Air is a mixture of nitrogen, oxygen, and small amounts of some other gases. Air in the atmosphere normally contains some water vapor (or *moisture*) and is referred to as **atmospheric air**. By contrast, air that contains no water vapor is called **dry air**. It is often convenient to treat air as a mixture of water vapor and dry air since the composition of dry air remains relatively constant, but the amount of water vapor changes as a result of condensation and evaporation from oceans, lakes, rivers, showers, and even the human body. Although the amount of water vapor in the air is small, it plays a major role in human comfort. Therefore, it is an important consideration in air-conditioning applications.

The temperature of air in air-conditioning applications ranges from about -10 to about  $50^{\circ}$ C. In this range, dry air can be treated as an ideal gas with a constant  $C_p$  value of 1.005 kJ/(kg · K) [0.240 Btu/(lbm · R)] with negligible error (under 0.2 percent), as illustrated in Fig. 9–12. Taking  $0^{\circ}$ C as the reference temperature, the enthalpy and enthalpy change of dry air can be determined from

$$h_{\rm dry \ air} = C_p T = (1.005 \ {\rm kJ/kg \cdot ^{\circ}C})T$$
 (kJ/kg) (9-22a)

and

$$\Delta h_{\text{dry air}} = C_p \Delta T = (1.005 \text{ kJ/kg} \cdot ^{\circ}\text{C}) \Delta T$$
 (kJ/kg) (9–22*b*)

where T is the air temperature in  ${}^{\circ}$ C and  $\Delta T$  is the change in temperature. In air-conditioning processes we are concerned with the *changes* in enthalpy  $\Delta h$ , which is independent of the reference point selected.

It certainly would be very convenient to also treat the water vapor in the air as an ideal gas and you would probably be willing to sacrifice some accuracy for such convenience. Well, it turns out that we can have the convenience without much sacrifice. At 50°C, the saturation pressure of water is 12.3 kPa. At pressures below this value, water vapor can be treated as an ideal gas with negligible error (under 0.2 percent), even when it is a saturated vapor. Therefore, water vapor in air behaves as if it existed alone and obeys the ideal-gas relation Pv = RT. Then the atmospheric air can be treated as an ideal-gas mixture whose pressure is the sum of the partial pressure of dry air\*  $P_a$  and that of water vapor  $P_v$ :

$$P = P_a + P_v$$
 (kPa) (9–23)

The partial pressure of water vapor is usually referred to as the **vapor pressure**. It is the pressure water vapor would exert if it existed alone at the temperature and volume of atmospheric air.

Since water vapor is an ideal gas, the enthalpy of water vapor is a function of temperature only, that is, h = h(T). This can also be observed from the T-s diagram of water given in Fig. A–9 and Fig. 9–13 where the constant-enthalpy lines coincide with constant-temperature lines at temperatures below 50°C. Therefore, the enthalpy of water vapor in air can be taken to be equal to the enthalpy of saturated vapor at the same temperature. That is,

$$h_{\nu}(T, \text{low } P) \cong h_{\varrho}(T)$$
 (9–24)

The enthalpy of water vapor at 0°C is 2501.3 kJ/kg. The average  $C_p$  value of water vapor in the temperature range -10 to 50°C can be taken to be  $1.82 \text{ kJ/kg} \cdot ^{\circ}\text{C}$ . Then the enthalpy of water vapor can be determined approximately from

$$h_{\varrho}(T) \cong 2501.3 + 1.82T$$
 (kJ/kg)  $T \text{ in } {}^{\circ}\text{C}$  (9–25)

or

$$h_g(T) \cong 1061.5 + 0.435T$$
 (Btu/lbm)  $T \text{ in } {}^{\circ}\text{F}$  (9–26)

DRY AIR					
T, °C	$C_p$ , kJ/kg · °C				
-10	1.0038				
0	1.0041				
10	1.0045				
20	1.0049				
30	1.0054				
40	1.0059				
50	1.0065				

#### FIGURE 9-12

The  $C_p$  of air can be assumed to be constant at 1.005 kJ/kg · °C in the temperature range -10 to 50°C with an error under 0.2 percent.

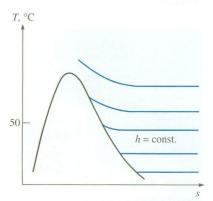


FIGURE 9-13

At temperatures below 50°C, the h = constant lines coincide with the T = constant lines in the superheated vapor region of water.

<sup>\*</sup>Throughout this chapter, the subscript a will denote dry air and the subscript  $\nu$  will denote water vapor.

	WATER	R VAPOR	
	$h_g$ , k	Difference,	
T, °C	Table A-4	Eq. 9-25	kJ/kg
-10	2482.9	2483.1	-0.2
0	2501.3	2501.3	0.0
10	2519.8	2519.5	0.3
20	2538.1	2537.7	0.4
30	2556.3	2555.9	0.4
40	2574.3	2574.1	0.2
50	2592.1	2592.3	-0.2

#### FIGURE 9-14

In the temperature range -10 to 50°C, the  $h_g$  of water can be determined from Eq. 9–25 with negligible error.

AIR  

$$25^{\circ}$$
C,  $100 \text{ kPa}$   
 $(P_{\text{sat, H}_2\text{O} \oplus 25^{\circ}\text{C}} = 3.169 \text{ kPa})$   
 $P_{\upsilon} = 0 \rightarrow \text{dry air}$   
 $P_{\upsilon} < 3.169 \text{ kPa} \rightarrow \text{unsaturated air}$   
 $P_{\upsilon} = 3.169 \text{ kPa} \rightarrow \text{saturated air}$ 

#### FIGURE 9-15

For saturated air, the vapor pressure is equal to the saturation pressure of water.

AIR 25°C, 1atm 
$$m_a = 1 \text{ kg}$$
 
$$m_v = 0.01 \text{ kg}$$
 
$$m_{v,\text{max}} = 0.02 \text{ kg}$$
 Specific humidity:  $\omega = 0.01$  
$$\frac{\text{kg H}_2\text{O}}{\text{kg dry air}}$$
 Relative humidity:  $\phi = 50\%$ 

#### FIGURE 9-16

Specific humidity is the actual amount of water vapor in 1 kg of dry air, whereas relative humidity is the ratio of the actual amount of moisture in the air to the maximum amount of moisture air can hold at that temperature.

in the temperature range -10 to  $50^{\circ}$ C (or 15 to  $120^{\circ}$ F), with negligible error, as shown in Fig. 9–14.

# 9-5 • SPECIFIC AND RELATIVE HUMIDITY OF AIR

The amount of water vapor in the air can be specified in various ways. Probably the most logical way is to specify directly the mass of water vapor present in a unit mass of dry air. This is called **absolute** or **specific humidity** (also called *humidity ratio*) and is denoted by  $\omega$ :

$$\omega = \frac{m_v}{m_a}$$
 (kg water vapor/kg dry air) (9–27)

The specific humidity can also be expressed as

$$\omega = \frac{m_v}{m_a} = \frac{P_v V / (R_v T)}{P_a V / (R_o T)} = \frac{P_v / R_v}{P_a / R_a} = 0.622 \frac{P_v}{P_a}$$
(9–28)

or

$$\omega = \frac{0.622 P_{v}}{P - P_{v}}$$
 (kg water vapor/kg dry air) (9–29)

where P is the total pressure.

Consider 1 kg of dry air. By definition, dry air contains no water vapor, and thus its specific humidity is zero. Now let us add some water vapor to this dry air. The specific humidity will increase. As more vapor or moisture is added, the specific humidity will keep increasing until the air can hold no more moisture. At this point, the air is said to be saturated with moisture, and it is called **saturated air**. Any moisture introduced into saturated air will condense. The amount of water vapor in saturated air at a specified temperature and pressure can be determined from Eq. 9–29 by replacing  $P_{\nu}$  by  $P_{g}$ , the saturation pressure of water at that temperature (Fig. 9–15).

The amount of moisture in the air has a definite effect on how comfortable we feel in an environment. However, the comfort level depends more on the amount of moisture the air holds  $(m_v)$  relative to the maximum amount of moisture the air can hold at the same temperature  $(m_g)$ . The ratio of these two quantities is called the **relative humidity**  $\phi$  (Fig. 9–16)

$$\phi = \frac{m_v}{m_g} = \frac{P_v V / (R_v T)}{P_o V / (R_v T)} = \frac{P_v}{P_o}$$
 (9-30)

where

$$P_o = P_{\text{sat @ }T} \tag{9-31}$$

Combining Eqs. 9–29 and 9–30, we can also express the relative humidity as

$$\phi = \frac{\omega P}{(0.622 + \omega)P_g}$$
 and  $\omega = \frac{0.622\phi P_g}{P - \phi P_g}$  (9-32a, b)

The relative humidity ranges from 0 for dry air to 1 for saturated air. Note that the amount of moisture air can hold depends on its temperature. Therefore, the relative humidity of air changes with temperature even when its specific humidity remains constant.

Atmospheric air is a mixture of dry air and water vapor, and thus the enthalpy of air is expressed in terms of the enthalpies of the dry air and the water vapor. In most practical applications, the amount of dry air in the air—water-vapor mixture remains constant, but the amount of water vapor changes. Therefore, the enthalpy of atmospheric air is expressed *per unit mass of dry air* instead of per unit mass of the air—water-vapor mixture.

The total enthalpy (an extensive property) of atmospheric air is the sum of the enthalpies of dry air and the water vapor:

$$H = H_a + H_v = m_a h_a + m_v h_v$$

Dividing by  $m_a$  gives

$$h = \frac{H}{m_a} = h_a + \frac{m_v}{m_a} h_v = h_a + \omega h_v$$

or

$$h = h_a + \omega h_g$$
 (kJ/kg dry air) (9–33)

since  $h_v \cong h_g$  (Fig. 9–17).

Also note that the ordinary temperature of atmospheric air is frequently referred to as the **dry-bulb temperature** to differentiate it from other forms of temperatures that shall be discussed.

#### **EXAMPLE 9-3** The Amount of Water Vapor in Room Air

A 5-m  $\times$  5-m  $\times$  3-m room shown in Fig. 9–18 contains air at 25°C and 100 kPa at a relative humidity of 75 percent. Determine (a) the partial pressure of dry air, (b) the specific humidity, (c) the enthalpy per unit mass of the dry air, and (d) the masses of the dry air and water vapor in the room.

**SOLUTION** A sketch of the room is given in Fig. 9–18. Both the air and the vapor fill the entire room, and thus the volume of each gas is equal to the volume of the room.

**Assumptions** The dry air and the water vapor in the room are ideal gases. **Analysis** (a) The partial pressure of dry air can be determined from

$$P_a = P - P_{v+}$$

where

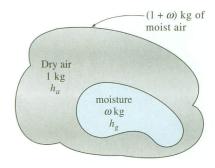
$$P_v = \phi P_g = \phi P_{\text{sat } \oplus 25^{\circ}\text{C}} = (0.75)(3.169 \text{ kPa}) = 2.38 \text{ kPa}$$

Thus,

$$P_a = (100 - 2.38) \text{ kPa} = 97.62 \text{ kPa}$$

(b) The specific humidity of air is

$$\omega = \frac{0.622 P_{_{0}}}{P-P_{_{0}}} = \frac{(0.622)(2.38 \text{ kPa})}{(100-2.38) \text{ kPa}} = \textbf{0.0152 kg H}_{2} \textbf{O/kg dry air}$$



 $h = h_a + \omega h_o$ , kJ/kg dry air

#### FIGURE 9-17

The enthalpy of moist (atmospheric) air is expressed per unit mass of dry air, not per unit mass of moist air.



FIGURE 9-18

Schematic for Example 9-3.

(c) The enthalpy of air per unit mass of dry air is determined from Eq. 9–33, where  $h_{\rm g}$  is taken from Table A–4:

$$\begin{split} h &= h_a + \omega h_v \cong C_p T + \omega h_g \\ &= (1.005 \text{ kJ/kg} \cdot ^{\circ}\text{C})(25^{\circ}\text{C}) + (0.0152)(2547.2 \text{ kJ/kg}) \\ &= \textbf{63.8 kJ/kg dry air} \end{split}$$

The enthalpy of water vapor (2547.2 kJ/kg) could also be determined from the approximation given by Eq. 9–25:

$$h_{g \otimes 25^{\circ}\text{C}} \cong 2501.3 + 1.82(25) = 2546.8 \text{ kJ/kg}$$

which is very close to the value obtained from Table A-4.

(d) Both the dry air and the water vapor fill the entire room completely. Therefore, the volume of each gas is equal to the volume of the room:

$$V_a = V_v = V_{\text{room}} = (5)(5)(3) = 75 \text{ m}^3$$

The masses of the dry air and the water vapor are determined from the ideal-gas relation applied to each gas separately:

$$m_a = \frac{P_a V_a}{R_a T} = \frac{(97.62 \text{ kPa})(75 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} = 85.61 \text{ kg}$$

$$m_v = \frac{P_v V_v}{R_v T} = \frac{(2.38 \text{ kPa})(75 \text{ m}^3)}{(0.4615 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} = 1.3 \text{ kg}$$

The mass of the water vapor in the air could also be determined from Eq. 9–27:

$$m_{v} = \omega m_{a} = (0.0152)(85.61 \text{ kg}) = 1.3 \text{ kg}$$

# 9-6 DEW-POINT TEMPERATURE

If you live in a humid area, you are probably used to waking up most summer mornings and finding the grass wet. You know it did not rain the night before. So what happened? Well, the excess moisture in the air simply condensed on the cool surfaces, forming what we call *dew*. In summer, a considerable amount of water vaporizes during the day. As the temperature falls during the night, so does the "moisture capacity" of air, which is the maximum amount of moisture air can hold. (What happens to the relative humidity during this process?) After a while, the moisture capacity of air equals its moisture content. At this point, air is saturated, and its relative humidity is 100 percent. Any further drop in temperature results in the condensation of some of the moisture, and this is the beginning of dew formation.

The **dew-point temperature**  $T_{\rm dp}$  is defined as the temperature at which condensation begins when the air is cooled at constant pressure. In other words,  $T_{\rm dp}$  is the saturation temperature of water corresponding to the vapor pressure:

$$T_{\rm dp} = T_{\rm sat @ Pv} \tag{9-34}$$

This is also illustrated in Fig. 9–19. As the air cools at constant pressure, the vapor pressure  $P_{\nu}$  remains constant. Therefore, the vapor in the air (state 1)

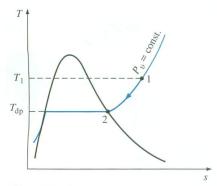


FIGURE 9-19

Constant-pressure cooling of moist air and the dew-point temperature on the *T-s* diagram of water.

undergoes a constant-pressure cooling process until it strikes the saturated vapor line (state 2). The temperature at this point is  $T_{\rm dp}$ , and if the temperature drops any further, some vapor condenses out. As a result, the amount of vapor in the air decreases, which results in a decrease in  $P_v$ . The air remains saturated during the condensation process and thus follows a path of 100 percent relative humidity (the saturated vapor line). The ordinary temperature and the dew-point temperature of saturated air are identical.

You have probably noticed that when you buy a cold canned drink from a vending machine on a hot and humid day, dew forms on the can. The formation of dew on the can indicates that the temperature of the drink is below the dew-point temperature of the surrounding air (Fig. 9–20).

The dew-point temperature of room air can be determined easily by cooling some water in a metal cup by adding small amounts of ice and stirring. The temperature of the outer surface of the cup when dew starts to form on the surface is the dew-point temperature of the air.

#### **EXAMPLE 9-4** Fogging of the Windows in a House

In cold weather, condensation frequently occurs on the inner surfaces of the windows due to the lower air temperatures near the window surface. Consider a house, shown in Fig. 9–21, that contains air at 20°C and 75 percent relative humidity. At what window temperature will the moisture in the air start condensing on the inner surfaces of the windows?

**SOLUTION** The interior of a house is maintained at a specified temperature and humidity. The window temperature at which fogging will start is to be determined. *Analysis* The temperature distribution in a house, in general, is not uniform. When the outdoor temperature drops in winter, so does the indoor temperature near the walls and the windows. Therefore, the air near the walls and the windows remains at a lower temperature than at the inner parts of a house even though the total pressure and the vapor pressure remain constant throughout the house. As a result, the air near the walls and the windows will undergo a  $P_v = \text{constant cooling process until the moisture in the air starts condensing.}$  This will happen when the air reaches its dew-point temperature  $T_{\rm dp}$ . The dew point is determined from Eq. 9–34 to be

$$T_{\rm dp} = T_{\rm sat @ P_{\rm u}}$$

where

$$P_v = \phi P_{g \otimes 20^{\circ}\text{C}} = (0.75)(2.339 \text{ kPa}) = 1.754 \text{ kPa}$$

Thus,

$$T_{\rm dp} = T_{\rm sat @ 1.754 \, kPa} = 15.4^{\circ}{\rm C}$$

Therefore, the inner surface of the window should be maintained above 15.4°C if condensation on the window surfaces is to be avoided.

# 9-7 - ADIABATIC SATURATION AND WET-BULB TEMPERATURES

Relative humidity and specific humidity are frequently used in engineering and atmospheric sciences, and it is desirable to relate them to easily measurable quantities such as temperature and pressure. One way of determining the



#### FIGURE 9-20

When the temperature of a cold drink is below the dew-point temperature of the surrounding air, it "sweats."

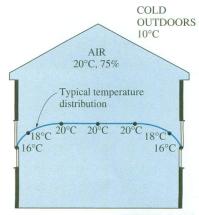
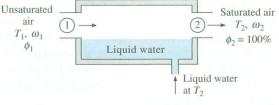


FIGURE 9–21

Schematic for Example 9-4.



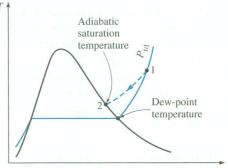


FIGURE 9-22

The adiabatic saturation process and its representation on a *T-s* diagram of water.

relative humidity is to determine the dew-point temperature of air, as discussed in the last section. Knowing the dew-point temperature, we can determine the vapor pressure  $P_{\upsilon}$  and thus the relative humidity. This approach is simple, but not quite practical.

Another way of determining the absolute or relative humidity is related to an *adiabatic saturation process*, shown schematically and on a *T-s* diagram in Fig. 9–22. The system consists of a long insulated channel that contains a pool of water. A steady stream of unsaturated air that has a specific humidity of  $\omega_1$  (unknown) and a temperature of  $T_1$  is passed through this channel. As the air flows over the water, some water will evaporate and mix with the airstream. The moisture content of air will increase during this process, and its temperature will decrease, since part of the latent heat of vaporization of the water that evaporates will come from the air. If the channel is long enough, the airstream will exit as saturated air ( $\phi = 100$  percent) at temperature  $T_2$ , which is called the **adiabatic saturation temperature**.

If makeup water is supplied to the channel at the rate of evaporation at temperature  $T_2$ , the adiabatic saturation process described above can be analyzed as a steady-flow process. The process involves no heat or work interactions, and the kinetic and potential energy changes can be neglected. Then the conservation of mass and conservation of energy relations for this two-inlet, one-exit steady-flow system reduces to the following:

Mass balance:

$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$
 (The mass flow rate of dry air remains constant)

 $\dot{m}_{w_1} + \dot{m}_f = \dot{m}_{w_2}$  (The mass flow rate of vapor in the air increases by an amount equal to the rate of evaporation  $\dot{m}_f$ )

 $\dot{m}_a \omega_1 + \dot{m}_f = \dot{m}_a \omega_2$ 

or

Thus,

$$\dot{m}_f = \dot{m}_a(\omega_2 - \omega_1)$$

Energy balance:

$$\dot{E}_{\rm in}=\dot{E}_{\rm out} \qquad ({\rm since}~\dot{Q}~=0~{\rm and}~\dot{W}=0)$$
 
$$\dot{m}_ah_1+\dot{m}_fh_f=\dot{m}_ah_2$$

or

or

$$\dot{m}_a h_1 + \dot{m}_a (\omega_2 - \omega_1) h_{f_2} = \dot{m}_a h_2$$

Dividing by  $\dot{m}_a$  gives

$$h_1 + (\omega_2 - \omega_1)h_{f_2} = h_2$$

$$(C_pT_1 + \omega_1h_{g_1}) + (\omega_2 - \omega_1)h_{f_2} = (C_pT_2 + \omega_2h_{g_2})$$

which yields

$$\omega_1 = \frac{C_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$
 (9–35)

where, from Eq. 9-32b,

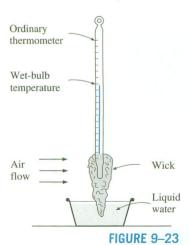
$$\omega_2 = \frac{0.622 P_{g_2}}{P_2 - P_{g_2}} \tag{9-36}$$

since  $\phi_2 = 100$  percent. Thus we conclude that the specific humidity (and relative humidity) of air can be determined from Eqs. 9–35 and 9–36 by measuring the pressure and temperature of air at the inlet and the exit of an adiabatic saturator.

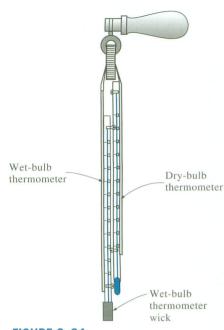
If the air entering the channel is already saturated, then the adiabatic saturation temperature  $T_2$  will be identical to the inlet temperature  $T_1$ , in which case Eq. 9–35 yields  $\omega_1 = \omega_2$ . In general, the adiabatic saturation temperature will be between the inlet and dew-point temperatures.

The adiabatic saturation process discussed above provides a means of determining the absolute or relative humidity of air, but it requires a long channel or a spray mechanism to achieve saturation conditions at the exit. A more practical approach is to use a thermometer whose bulb is covered with a cotton wick saturated with water and to blow air over the wick, as shown in Fig. 9–23. The temperature measured in this matter is called the **wet-bulb temperature**  $T_{\rm wb}$ , and it is commonly used in air-conditioning applications.

The basic principle involved is similar to that in adiabatic saturation. When insaturated air passes over the wet wick, some of the water in the wick evaporates. As a result, the temperature of the water drops, creating a temperature lifference (which is the driving force for heat transfer) between the air and the water. After a while, the heat loss from the water by evaporation equals the neat gain from the air, and the water temperature stabilizes. The thermometer reading at this point is the wet-bulb temperature. The wet-bulb temperature



A simple arrangement to measure the wet-bulb temperature.



**FIGURE 9–24** Sling psychrometer.

can also be measured by placing the wet-wicked thermometer in a holder attached to a handle and rotating the holder rapidly, that is, by moving the thermometer instead of the air. A device that works on this principle is called a *sling psychrometer* and is shown in Fig. 9–24. Usually a dry-bulb thermometer is also mounted on the frame of this device so that both the wet- and dry-bulb temperatures can be read simultaneously.

Advances in electronics made it possible to measure humidity directly in a fast and reliable way. It appears that sling psychrometers and wet-wicked thermometers are about to become things of the past. Today, hand-held electronic humidity measurement devices based on the capacitance change in a thin polymer film as it absorbs water vapor are capable of sensing and digitally displaying the relative humidity within 1 percent accuracy in a matter of seconds.

In general, the adiabatic saturation temperature and the wet-bulb temperature are not the same. However, for air—water-vapor mixtures at atmospheric pressure, the wet-bulb temperature happens to be approximately equal to the adiabatic saturation temperature. Therefore, the wet-bulb temperature  $T_{\rm wb}$  can be used in Eq. 9–14 in place of  $T_2$  to determine the specific humidity of air.

#### **EXAMPLE 9-5** The Specific and Relative Humidity of Air

The dry- and the wet-bulb temperatures of atmospheric air at 1 atm (101.325 kPa) pressure are measured with a sling psychrometer and determined to be 25 and  $15^{\circ}$ C, respectively. Determine (a) the specific humidity, (b) the relative humidity, and (c) the enthalpy of the air.

**SOLUTION** Dry- and wet-bulb temperatures are given. The specific humidity, relative humidity, and enthalpy are to be determined.

**Analysis** (a) The specific humidity  $\omega_1$  is determined from Eq. 9–35,

$$\omega_1 = \frac{C_p(T_2 - T_1) + \omega_2 h_{fg_2}}{h_{g_1} - h_{f_2}}$$

where  $\mathcal{T}_2$  is the wet-bulb temperature and  $\omega_2$  is

$$\omega_2 = \frac{0.622 P_{g_2}}{P_2 - P_{g_2}} = \frac{(0.622)(1.705 \text{ kPa})}{(101.325 - 1.705) \text{ kPa}}$$
$$= 0.01065 \text{ kg H}_2\text{O/kg dry air}$$

Thus,

$$\begin{split} \omega_1 &= \frac{(1.005 \text{ kJ/kg} \cdot ^{\circ}\text{C})[(15-25)^{\circ}\text{C}] + (0.01065)(2465.9 \text{ kJ/kg})}{(2547.2-62.99) \text{ kJ/kg}} \\ &= \textbf{0.00653 kg H}_2\text{O/kg dry air} \end{split}$$

(b) The relative humidity  $\phi_1$  is determined from Eq. 9–32a to be

$$\phi_1 = \frac{\omega_1 P_2}{(0.622 + \omega_1) P_{g_1}} = \frac{(0.00653)(101.325 \text{ kPa})}{(0.622 + 0.00653)(3.169 \text{ kPa})} = \textbf{0.332 or 33.2\%}$$

(c) The enthalpy of air per unit mass of dry air is determined from Eq. 9-33:

$$h_1 = h_{a_1} + \omega_1 h_{v_1} \cong C_p T_1 + \omega_1 h_{g_1}$$
  
 $h_1 = [1.005 \text{ kJ/(kg} \cdot ^{\circ}\text{C})](25^{\circ}\text{C}) + (0.00653)(2547.2 \text{ kJ/kg})$   
 $= 41.8 \text{ kJ/kg dry air}$ 

### —8 • THE PSYCHROMETRIC CHART

the state of the atmospheric air at a specified pressure is completely specified by two independent intensive properties. The rest of the properties can be calculated easily from the relations already given. The sizing of a typical ar-conditioning system involves numerous such calculations, which may wentually get on the nerves of even the most patient engineers. Therefore, here is clear motivation to computerize calculations or to do these calculations once and to present the data in the form of easily readable charts. Such that are called **psychrometric charts**, and they are used extensively in air-conditioning applications. A psychrometric chart for a pressure of 1 atm 101.325 kPa or 14.696 psia) is given in Fig. A–33 in SI units and in Fig. A–33E in English units. Psychrometric charts at other pressures (for use at considerably higher elevations than sea level) are also available.

The basic features of the psychrometric chart are illustrated in Fig. 9–25. he dry-bulb temperatures are shown on the horizontal axis, and the specific umidity is shown on the vertical axis. (Some charts also show the vapor presare on the vertical axis since at a fixed total pressure P there is a one-to-one orrespondence between the specific humidity  $\omega$  and the vapor pressure  $P_{\nu}$ , as an be seen from Eq. 9–29.) On the left end of the chart, there is a curve (called ne saturation line) instead of a straight line. All the saturated air states are loated on this curve. Therefore, it is also the curve of 100 percent relative hunidity. Other constant relative-humidity curves have the same general shape. Lines of constant wet-bulb temperature have a downhill appearance to the ght. Lines of constant specific volume (in m<sup>3</sup>/kg dry air) look similar, except ney are steeper. Lines of constant enthalpy (in kJ/kg dry air) lie very nearly arallel to the lines of constant wet-bulb temperature. Therefore, the constantet-bulb-temperature lines are used as constant-enthalpy lines in some charts. For saturated air, the dry-bulb, wet-bulb, and dew-point temperatures are lentical (Fig. 9-26). Therefore, the dew-point temperature of atmospheric air any point on the chart can be determined by drawing a horizontal line line of  $\omega = \text{constant}$  or  $P_{\nu} = \text{constant}$ ) from the point to the saturated curve. he temperature value at the intersection point is the dew-point temperature. The psychrometric chart also serves as a valuable aid in visualizing the airinditioning processes. An ordinary heating or cooling process, for example, Il appear as a horizontal line on this chart if no humidification or dehumidieation is involved (that is,  $\omega = \text{constant}$ ). Any deviation from a horizontal e indicates that moisture is added or removed from the air during the ocess.

# **EXAMPLE 9-6** The Use of the Psychrometric Chart

Consider a room that contains air at 1 atm,  $35^{\circ}$ C, and 40 percent relative hunidity. Using the psychrometric chart, determine (a) the specific humidity, b) the enthalpy, (c) the wet-bulb temperature, (d) the dew-point temperature, nd (e) the specific volume of the air.

**OLUTION** At a given total pressure, the state of atmospheric air is completely pecified by two independent properties such as the dry-bulb temperature and re relative humidity. Other properties are determined by directly reading their rules at the specified state.

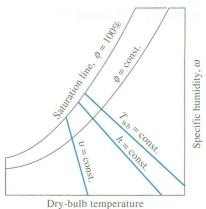


FIGURE 9–25

Schematic for a psychrometric chart.

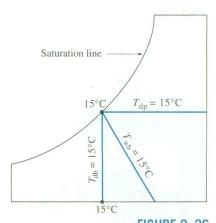


Figure 9–26

For saturated air, the dry-bulb, wet-bulb, and dewpoint temperatures are identical.

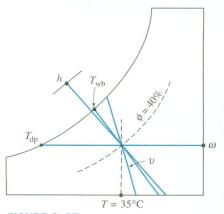


FIGURE 9–27 Schematic for Example 9–6.

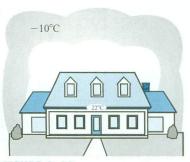


FIGURE 9-28

We cannot change the weather, but we can change the climate in a confined space by air-conditioning.

**Analysis** (a) The specific humidity is determined by drawing a horizontal lin from the specified state to the right until it intersects with the  $\omega$  axis, as show in Fig. 9–27. At the intersection point we read

$$\omega = 0.0142 \text{ kg H}_2\text{O/kg dry air}$$

(b) The enthalpy of air per unit mass of dry air is determined by drawing a lin parallel to the h= constant lines from the specific state until it intersects the enthalpy scale. At the intersection point we read

$$h = 71.5 \text{ kJ/kg dry air}$$

(c) The wet-bulb temperature is determined by drawing a line parallel to th  $T_{\rm wb}=$  constant lines from the specified state until it intersects the saturatio line. At the intersection point we read

$$T_{\rm wb} = 24^{\circ}{\rm C}$$

(d) The dew-point temperature is determined by drawing a horizontal line from the specified state to the left until it intersects the saturation line. At the intersection point we read

$$T_{\rm dp} = 19.4^{\circ}{\rm C}$$

(e) The specific volume per unit mass of dry air is determined by noting the distances between the specified state and the  $\nu =$  constant lines on both sides of the point. The specific volume is determined by visual interpolation to be

$$v = 0.893 \text{ m}^3/\text{kg dry air}$$

# 9-9 - HUMAN COMFORT AND AIR-CONDITIONING

Human beings have an inherent weakness—they want to feel comfortab They want to live in an environment that is neither hot nor cold, neither hum nor dry. However, comfort does not come easily since the desires of human body and the weather usually are not quite compatible. Achiev comfort requires a constant struggle against the factors that cause discomf such as high or low temperatures and high or low humidity. As engineers, i our duty to help people feel comfortable. (Besides, it keeps us employed.)

It did not take long for people to realize that they could not change weather in an area. All they can do is change it in a confined space such a house or a workplace (Fig. 9–28). In the past, this was partially complished by fire and simple indoor heating systems. Today, modern conditioning systems can heat, cool, humidify, dehumidify, clean, and e deodorize the air—in other words, *condition* the air to peoples' desires. conditioning systems are designed to *satisfy* the needs of the human be therefore, it is essential that we understand the thermodynamic aspects of body.

The human body can be viewed as a heat engine whose energy input food. As with any other heat engine, the human body generates waste heat must be rejected to the environment if the body is to continue operating, rate of heat generation depends on the level of the activity. For an ave