AREN 2110: Thermodynamics
Final Exam
Fall 2004

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Name

Test is open book and notes. Answer all questions and sign honor code statement: ***I have neither given nor received unauthorized assistance during this exam.***

Signed SOLUTIONS

Remember to show your work – partial credit will be given for a correct approach!

Question	Points
1	/50
2	/50
Total	/100

HAVE A GREAT HOLIDAY!

All table values are from 2nd edition.

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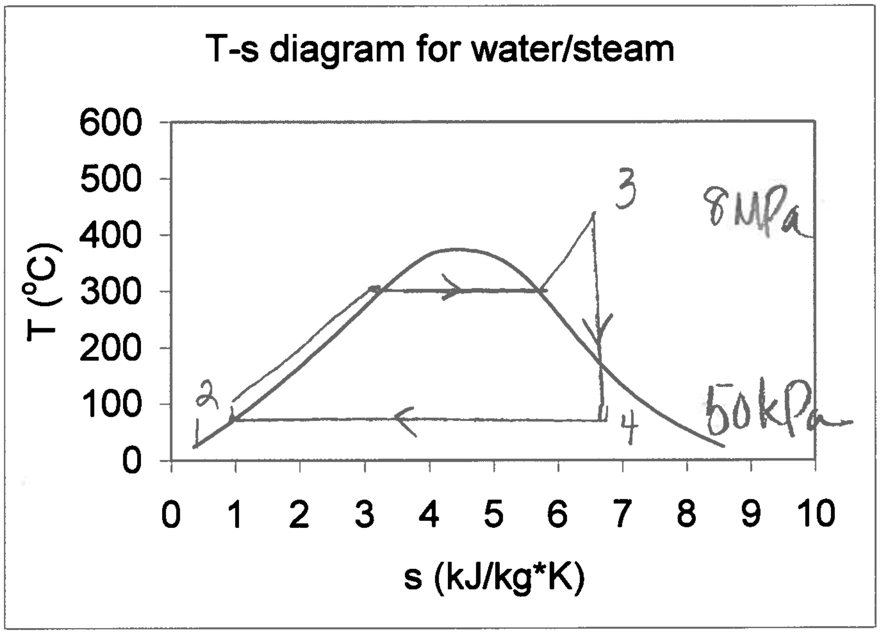
1. (50 points) A 25 MW ($\dot{W}_{net}$) steam power plant is a Rankine cycle with boiler pressure of 8 MPa and condenser pressure of 50 kPa. The moisture content of the steam at the turbine outlet = 16%.

a) Draw the cycle on the T-s diagram below, numbering the device inlet/outlet points. Name the device that corresponds to each process in the table below. Give the property or properties that is/are constant for each process. Give the heat or work interaction for each process with the appropriate sign (+ or -) (10)



Process	Device	Property/properties that are constant	Work (W)/Heat (Q) interaction	Sign (+ or -)
1 → 2	pump	entropy	W	−
2 → 3	boiler	pressure	Q	+
3 → 4	turbine	entropy	W	+
4 → 1	condenser	pressure & temp.	Q	−

b) Find the temperature of the steam at the <u>turbine inlet</u> (5) (don't need to interpolate)

$s_4 = 0.84\,(s_{fg}) + s_f\,)_{50kPa} = 0.84\,(6.5029) + 1.091 = 6.5534$ kJ/kgK

$= s_3$ Table A-6 (8 MPa), $\boxed{T_3 \approx 450°C}$

c) Calculate the efficiency of the power plant. (10)

$h_1 = h_f)_{50k} = 340.49\ \mathrm{kJ/kg}$

$h_2 = v(P_2 - P_1) + h_1$

$h_2 = 0.00103(8000-50) + 340.49$

$= 348.68\ \mathrm{KJ/kg}$

$h_3 = h_{450,8MPa} = 3272.0\ \mathrm{kJ/kg}$

$h_4 = 0.84(h_{fg}) + h_f)_{50kPa} = 0.84(2305.4) + 340.49$

$h_4 = 2277.03\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$$\eta = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

$$\eta = 1 - \frac{(2277.03 - 340.49)}{(3272 - 348.68)}$$

$$\eta = 0.338$$

$$\boxed{33.8\%}$$

d) Calculate the mass flow rate of steam required. (5)

$$\eta = \frac{\dot{W}_{net}}{\dot{m}_s(h_3 - h_2)} = \frac{25000\,\mathrm{kW}}{\dot{m}_s(3272-348.68)}$$

$$\dot{m}_s = \frac{25000}{0.338(3272-348.68)}$$

$$\boxed{\dot{m}_s = 25.3\ \mathrm{kg/s}}$$

The temperature at the turbine inlet is raised to 550 °C AND the condenser pressure is lowered to 20 kPa. Still sat liquid @ pump inlet, 8 MPa in boiler.

e) The amount of heat input to the boiler will (circle one AND justify your answer) (5)

$h_1 = 251.4 = h_f @ 20\mathrm{kPa}$

[✓ increase] decrease stay the same

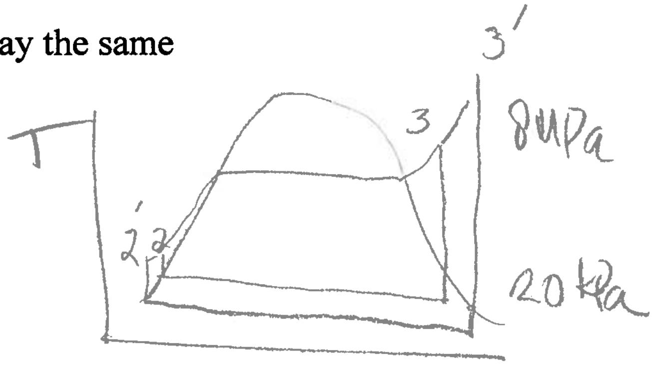
$h_2' = 251.4 + 0.001017(8000-20) = 259.5$

$h_3')_{8MPa,550} = 3521.0\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$q_{in}' = 3521.0 - 259.5$

$= 3261.5 > (3272 - 348.68)$

OR



area under 2'→3' > area under 2→3

f) The quality of the steam at the turbine outlet will (circle one <u>AND justify your answer</u>) (5)

[✓ increase] decrease stay the same

$s_3' = 6.8778 = s_4'$

$X_4' = \frac{6.8778 - 0.832}{7.0766}$

$X_4' = 0.854 > 0.84$

$h_4' = 0.854(h_{fg}) + h_f)_{20kPa} = 0.854(2358.3) + 251.4 = 2266.1$

g) The amount of heat rejected will (circle one <u>AND justify your answer</u>) (5)

[✓ increase] decrease stay the same

$q_{out}' = h_4' - h_1'$

$= 2266.1 - 251.4$

$= 2014.8 > (2277.03 - 340.49) = 1936.5$

h) The cycle efficiency will (circle one <u>AND justify your answer</u>) (5)

[✓ increase] decrease stay the same

$$\eta' = 1 - \left(\frac{2014.8}{3261.5}\right) = 0.382 > 0.338$$

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2. (50 points total) In winter, a heat pump transfers heat from outside air with an average temperature of 0 °C to a house maintained at 23 °C.

a) What is the <u>maximum</u> coefficient of performance possible for a heat pump under those conditions? (5)

$$\text{Carnot } COP_{HP\,air} = \frac{T_H}{T_H - T_L} = \frac{(273+23)}{(273+23)-273} = \boxed{12.9}$$

b) A homeowner is considering changing the heat pump so that it takes heat from groundwater with an average winter temperature of 13 °C. How much will the minimum power input decrease? (HINT: use ratio of COP values for air and groundwater heat sources) (5) (given same heating requirement ?

$$\text{Carnot } COP_{HP\,gw} = \frac{273+23}{(273+23)-(273+13)} = \boxed{29.6}$$

$$COP_{HP} = \frac{\dot{Q}_{out}}{\dot{W}} \qquad \frac{COP_{HP\,air}}{COP_{HP\,gw}} = \frac{\left(\frac{\dot{Q}_{out}}{\dot{W}_{air}}\right)}{\left(\frac{\dot{Q}_{out}}{\dot{W}_{gw}}\right)} = \frac{12.9}{29.6}$$

$\dot{Q}_{out}$ same for both

$$\frac{\dot{W}_{gw}}{\dot{W}_{air}} = \frac{12.9}{29.6} = 0.44$$

Could save $1 - 0.44 = 56\%$

The actual heat pump uses a vapor compression refrigeration (VCR) cycle and R-134a as the working fluid. The evaporator pressure = 0.24 MPa, and the condenser pressure = 0.9 MPa. The R-134a is saturated vapor at the compressor inlet and saturated liquid at the condenser outlet. (Groundwater temperature is 13 °C and the house air is 23 °C.)

c) Complete the table below and draw the process on the T-s diagram below. (10)

Property	Compressor inlet	Condenser inlet *	Throttling valve inlet	Evaporator inlet
T (°C)	−5.37	40	35.53	−5.37
s (kJ/kg-K)	0.9222	0.9222	0.3656	0.3824

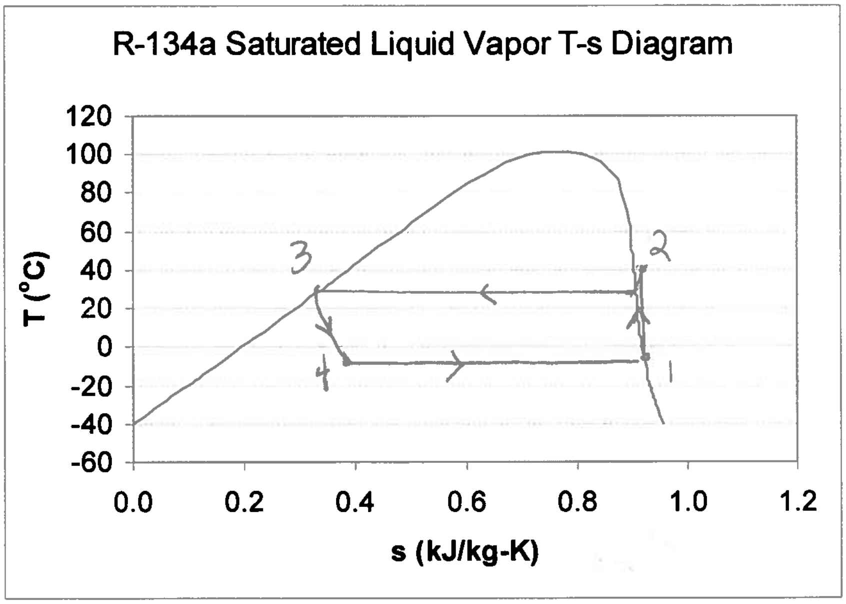
* don't need to interpolate T – estimate is close enough

$$h_3 = h_f\big)_{0.9MPa} = 99.56\,\frac{kJ}{kg} = h_4$$

$$x_4 = \frac{h_4 - h_f}{h_{fg}}\Big)_{0.24MPa} = \frac{99.56 - 42.95}{201.14} = 0.2815$$

$$S_4 = 0.2815(S_{fg}) + S_f\big)_{0.24MPa} = 0.8815(0.9222-0.171) + 0.1$$

$$= 0.3824\,kJ/kg\,K$$



d) Calculate the coefficient of performance of the VCR groundwater heat pump. (10)

$$COP_{Hp} = \frac{\dot{Q}_{out}}{\dot{W}} = \frac{q_{out}}{w} = \frac{h_2 - h_3}{h_2 - h_1}$$

$h_2 = h_{0.9,40°} = 271.25 \frac{kJ}{kg}$

$h_3 = h_f)_{0.9MPa} = 99.56 = h_4$

$h_1 = h_g)_{0.24MPa} = 244.09 \frac{kJ}{kg}$

$$COP_{Hp} = \frac{271.25 - 99.56}{271.25 - 244.09}$$

$$\boxed{COP_{Hp} = 6.32}$$

e) If the house loses 36,000 kJ/~~hr~~ on average during the winter, what is the compressor power requirement (in kW) to keep the inside temperature constant? (5)

$$\dot{Q}_{out} = \frac{36000}{3600} kW = 10 kW$$

$$\dot{W} = \frac{\dot{Q}_{out}}{COP_{HP}} = \frac{10kW}{6.32} = \boxed{1.6 kW}$$

f) What is the mass flow rate of refrigerant required? (5)

$$\dot{m} = \frac{\dot{Q}_{out}}{q_{out}} = \frac{-10kW}{(h_2 - h_3)} = \frac{10kW}{(271.25-99.56)} = \boxed{0.06 \frac{kg}{s}}$$

g) What is the total entropy generated in the surroundings by the heat pump? (5)

$$\dot{m}(\Delta s) = \sum_k \left(\frac{\dot{Q}}{T_k}\right) + \dot{S}_{gen}$$

$$\dot{S}_{gen} = -\sum_k \left(\frac{\dot{Q}}{T}\right)_k = -\dot{m}\left(\frac{(h_1-h_4)}{(273+13)} + \frac{(h_3-h_2)}{(273+23)}\right)$$

$$\dot{S}_{gen} = -0.06\left(\frac{(244.09-99.56)}{286} + \frac{(99.56-271.25)}{296}\right)$$

$$\dot{S}_{gen} = 0.0045 \frac{kW}{K}$$

h) In the summer the heat pump is "reversed." That is, the heat exchanger in the groundwater becomes the condenser (P = 0.9 MPa) and the heat exchanger in the room becomes the evaporator (P = 0.24 MPa), and it now cools the house air. If the average heat gain of the house in summer is 7 kw, does the compressor have sufficient power to keep the house cool? Justify your answer. No credit will be given for only a "yes" or "no." (5)

now HP is refrigerator: $COP_R = COP_{HP} - 1$

$$COP_R = 6.32 - 1 = 5.32$$

$COP_R = \frac{\dot{Q}_{in}}{\dot{W}}$ check $\dot{W} = 1.6kw \geq \frac{\dot{Q}_{in}}{COP_R}$

$1.6kw \geq \frac{7kw}{5.32} = 1.3\ kw$ yes

reversed heat pump will cool house