AREN 2110
Spring 2010

Homework #3: Due Friday, Feb. 5, 6 PM **Solutions**

1. Oxygen is contained in a 0.05-m^3 steel tank at a pressure of 200 bar at 27 ^oC.
   a. What is the mass of oxygen in the tank?
   b. If the pressure in the tank gets too high, a fusible plug melts to release the pressure. At what temperature should the plug melt to keep the pressure below 250 bar?

Solution

a. $$M = \frac{PV}{RT} = \frac{200bar(100kPa/bar)(0.05m^3)}{(0.2598kJ/kg-K)(273+27)K}$$ **= 12.8 kg**

b. $$T_{melt} = \frac{P_{melt}V}{MR} = \frac{250bar(100kPa/bar)(0.05m^3)}{12.8kg(0.2598kJ/kg-K)}$$ **=376K OR 103 ^oC**

2. An ideal gas is compressed from a volume of 0.085 m^3 to a volume of 0.034 m^3 while the pressure is increased from 100 kPa to 390 kPa. The temperature increases by 146 K during this process. The specific gas constant, R, is 0.296 kJ/kg-K.
   a. Find the temperature of the gas at the end of the process.
   b. What is the mass of gas in the tank?
   c. What is the molecular weight of the gas?

Solution

a. T_1 = 261K =-12.3 ^oC and T_2 = 133.7 ^oC

b. $$M = \frac{P_1V_1}{RT_1} = \frac{100kPa(0.085m^3)}{(0.296kJ/kg-K)(261K)}$$ **= 0.11 kg**

Solution

c. $$MW = \frac{\overline{R}}{R} = \frac{8.314kJ/kmol-K}{0.296kJ/kg-K} = 28\ kg/kmol$$ **(gas is nitrogen, N_2, MW = 28 kg/kmol)**

3. The equation of state for a gas is

$$\mathbf{\bar{v}(P + \frac{10}{\bar{v}^2}) = \bar{R}T}$$

where $\bar{\mathbf{v}}$ = specific molar volume (m^3/kmol)
P = pressure (kPa), $\bar{\mathbf{R}}$ = universal gas constant (kJ/kmol-K) and T = temperature (K)

One kmol gas is expanded from 2 to 4 m^3 at a constant temperature of 300 K.

a. What is the boundary work done? Boundary work, W_b is calculated by:

$$W_b = \int_{V_1}^{V_2} P dV$$

And

$$\bar{v} = \frac{V}{N}$$

Where N = # moles of the gas.

<u>Solution</u>

$W_b = \int_{V_1}^{V_2} PdV$ **need P(V), by rearranging** $\bar{v}\left(P + \frac{10}{\bar{v}^2}\right) = \bar{R}T \quad \left(\frac{kJ}{kmol}\right)$ **and substituting**

$\bar{v} = \frac{V}{N}\left(\frac{m^3}{kmol}\right)$ **to obtain** $P = \frac{\bar{R}T}{\bar{v}} - \frac{10}{\bar{v}^2} = \frac{N\bar{R}T}{V} - \frac{10N^2}{V^2} \quad kPa$

$$W_b = \int_{V_1}^{V_2}\left(\frac{N\bar{R}T}{V} - \frac{10N^2}{V^2}\right)dV = \int_2^4\left(\frac{N\bar{R}T}{V} - \frac{10N^2}{V^2}\right)dV = N\bar{R}T\ln\left(\frac{V_2}{V_1}\right) + 10N^2\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

W_b = (1 mole)(8.314kJ/kmol-K)(300K)ln(2) + (10 N-m^4/$kmol^2$)(1 mole)(0.25 – 0.5)m^{-3}

W_b = + 1,726 kJ (+ sign indicates work done <u>by</u> gas)

4. Nitrogen, an ideal gas, is expanded in a polytropic process according to the relation:

$$\mathbf{PV^n = constant}$$

where P = pressure (kPa), V = volume (m^3) and n = a constant. The initial volume of the nitrogen is 2 m^3; the initial pressure is 500 kPa and the initial temperature is 300 ^oC. During the expansion, the volume triples and the pressure decreases to half its initial value.

a. Find n.

Solution

For polytropic process $P_1(V_1)^n = P_2(V_2)^n$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$$

$$ln\left(\frac{P_1}{P_2}\right) = n\,ln\left(\frac{V_2}{V_1}\right)$$

where

$$\left(\frac{V_2}{V_1}\right) = 3 \quad and \quad \left(\frac{P_1}{P_2}\right) = 2$$

$$n = \frac{ln(2)}{ln(3)} = \underline{0.631}$$

n = 0.631

b. Calculate the equilibrium temperature after expansion

Solution

$$T_2 = \frac{T_1 P_2 V_2}{P_1 V_1} = \frac{((273+300)K)(3)}{2}$$

T_2 = 859.5K OR 586.5 ^oC

c. Calculate the boundary work by the gas during the process in kJ.

Solution

$$W_b = \frac{P_2V_2 - P_1V_1}{(1-n)} = \frac{250(6)-500(2)}{(1-0.631)}(kJ)$$ **= 1,355 kJ (work by system)**

5. A device containing air is operated in a cycle consisting of four processes with no work exchanges other than boundary work.

   1 → 2: Isothermal compression, $V_1 = 3\ m^3$, $V_2 = 1\ m^3$; $P_1 = 100$ kPa

   2 → 3: Isochoric heat loss, $P_3 = \frac{P_1}{2}\ m^3$

   3 → 4: Isobaric expansion, $V_4 = 3\ m^3$

   4 → 1: Isochoric heat addition, return to state 1

   a. Find P_2

Solution

For isothermal process: $P_2 = P_1V_1/V_2$ = 100 kPa*(3/1)= 300 kPa

   b. Find W_b for process 3 → 4

Solution

For isobaric process: $W_{b,34} = P_3(V_4 - V_3)$ =50 kPa*(3-1)m^3 = 100 kJ (work by system)

   c. Find W_b for process 1 → 2

Solution

$$W_{b,12} = \int_{V_1}^{V_2} P * dV$$

For isothermal process: P = (mRT)/V, where mRT = constant = P_1V_1

$$W_{b,12} = P_1V_1 * \int_{V_1}^{V_2} \frac{dV}{V} = P_1V_1 * ln\left(\frac{V_2}{V_1}\right) = 300\ kJ * \ln(1/3)$$

= -330 kJ (work done on system)

   d. Find the net work for the cycle

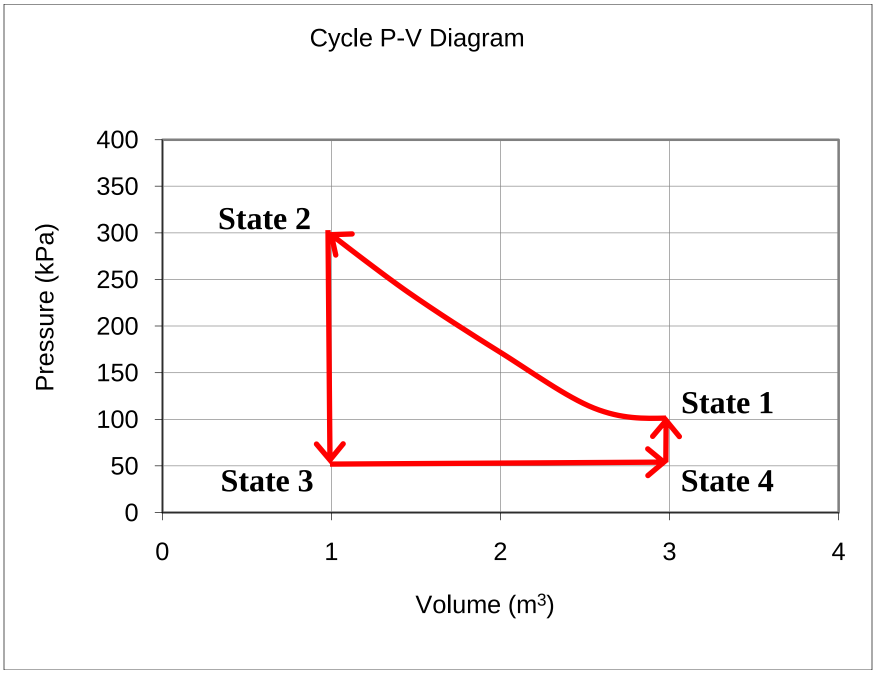
Solution

No work for 2 → 3 and 4 → 1 since both processes are isochoric (dV = 0)

$W_{b,cycle} = W_{b,12} + W_{b,34} = -330 + 100 = \underline{-230\ kJ}$ (work done <u>on</u> system during cycle)

e. Graph the process on the P-V diagram below

<u>Solution</u>



6. A rigid tank contains an ideal gas at 300 kPa and 600K. Now half the gas is withdrawn from the tank and the final pressure is 100kPa.

    a. Find the final temperature of the gas after half the mass is removed and the pressure is reduced.

<u>Solution</u>

$V_2 = V_1$ and $m_2 = 0.5\ m_1$

Ideal gas relation for isochoric process

$$\frac{P_1}{m_1 T_1} = \frac{P_2}{m_2 T_2}$$

$$T_2 = \frac{P_2 m_1 T_1}{P_1 0.5 m_1} = \frac{100kPa * 600K}{300kPa * (0.5)}$$

$T_2 = 400\ K$

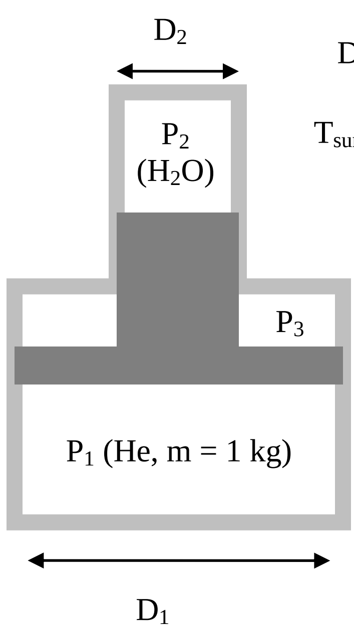
b. Compare the system to one where the final temperature is the same as in part a, but no mass was removed: What is the final pressure?

Solution

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

P_2 = (300 kPa*400K)/600K = 200kPa

7. A cylindrical piston cylinder device has three chambers, as shown below. Chamber 1 contains 1 kg helium; chamber 2 contains condensing water vapor; chamber 3 is evacuated. The device is placed in surroundings whose temperature is 200 ^oC and allowed to come to equilibrium. The inside diameter of chamber 1 is 10 cm and the inside diameter of chamber 2 is 4 cm. Find the volume of the helium in chamber 1 when equilibrium is reached. (Answer: 3.95 m^3)



DRAWING NOT TO SCALE

$T_{surroundings}$ = 200 ^oC

Solution

System is in mechanical and thermal equilibrium.

Thermal equilibrium: for chambers 1 and 2, respectively, $T_1 = T_2$ = 200 ^oC

Condensing water at T_2 = 200 ^oC has pressure = P_{sat} = P_2 = 1,554.9 kPa (Table A-4)

Mechanical equilibrium: assume weight of piston is negligible compared with pressure force. Justification: in chamber 2, pressure force = 1554.9 kPa*$(0.04\text{ m})^2$*3.14/4*10^3N/kN= 1,954 N and for comparable weight force, mass of piston would be: 1,954 N/9.81m/s^2 = 200 kg. Very unlikely that piston with dimensions of 4 cm and 10 cm would have mass more than 1% of 200 kg. So neglecting weight of piston is reasonable.

Force balance neglecting piston weight:

$$P_2A_2 = P_1A_1$$

$$P_1 = P_2(A_2/A_1)$$

$$\text{Area ratio } (A_2/A_1) = (D_2/D_1)^2 = (0.4)^2 = 0.16$$

$$\mathbf{P_1 = 1{,}554.9\ kPa*(0.16) = 248.78\ kPa}$$

Ideal gas law for He, R = 2.0769 kJ/kg-K (Table A-2)

$$\mathbf{V_1 = mRT_1/P_1 = (1\ kg*2.0769\ kJ/kg\text{-}K*473K)/248.78\ kPa = 3.95\ m^3}$$

(recall that 1 kPa = 1 kJ/m^3)