Homework 4 Solutions

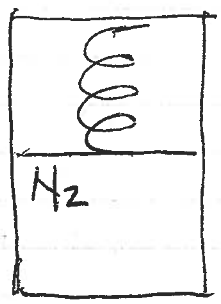
1. Relation $\Delta U = mC_v(\Delta T)$ applies to <u>any</u> process
   with ideal gas

2. Relation $\Delta H = mC_p(\Delta T)$ applies to <u>any</u> process
   with ideal gas

3. Both require the same energy: $\Delta U = mC_v(T_2 - T_1)$
   (volume not a factor)

4. Both require the same energy $\Delta H = mC_p(T_2 - T_1)$
   (pressure not a factor)

5.



$m = 10\,g \times 10^{-3}\,\frac{kg}{g} = 0.01\,kg$

$P_1 = 120\,kPa$

$T_1 = 27°C + 273 = 300\,K$

$R = 0.2968\,kJ/kgK$

$d_p = 10\,cm \times 10^{-2}\,\frac{m}{cm}$

$d_p = 0.1\,m$

$A_p = \pi\frac{(0.1m)^2}{4}$

$A_p = 0.00785\,m^2$

Spring constant $k = 1\,\frac{kN}{m}$

Hook's Law $F_s = kx$ where x = displacement (m)

Solution $\Delta U = mC_v(T_2 - T_1)$

$\Delta H = mC_p(T_2 - T_1)$

need T_2 from $P_2V_2 = mRT_2$ $\Rightarrow$ $T_2 = \frac{P_2(1.1V_1)}{mR}$

need P_2 and V_1 to solve

5. $P_1 = P_2 + \dfrac{F_s}{A} = P_1 + \dfrac{kx}{A}$

$$X = \frac{V_2 - V_1}{A} = \frac{0.1\,V_1}{A} = \frac{0.1V_1}{0.00785\,m^2}$$

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.01kg\,(0.2968\,kJ/kgk)\,(300k)}{120kPa} = 0.00742\,m^3$$

$$X = 0.1\frac{(0.00742)\,m^3}{0.00785\,M^2} = 0.0945\,m$$

$$P_2 = 120kPa + 1\frac{kN}{m}\frac{(0.0945\,m)}{0.00785\,m^2} = 120 + 12\,kPa$$

$$T_2 = \frac{132\,kPa\,(1.1)(0.00742\,m^3)}{0.01kg\,(0.2968\,kJ/kgk)} = 363.1\,K$$

$Cv@300k = 0.743\,kJ/kgk$ $\quad Cp = 1.04\,kJ/kgk$

$$\Delta U = mC_v(T_2 - T_1) = 0.01\,kg\left(0.743\frac{kJ}{kgk}\right)(363.1 - 300)k$$

$$\boxed{\Delta U = 0.47\,kJ}$$

$$\Delta H = mC_p(T_2 - T_1)$$

$$\Delta H = 0.01\,kg\left(1.04\frac{kJ}{kgk}\right)(363.1 - 300k)$$

$$\boxed{\Delta H = 0.66\,kJ}$$

6.

A steam	B sat mix
2 kg	3 kg
1 MPa	150°C
300°C	x = 0.5

$\rightarrow$

5 kg $\quad v = \dfrac{V_A + V_B}{5\text{ kg}}$
300 kPa
$V = V_A + V_B$

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A. steam is superheated ($T > T_s$ @ 1MPa)

$v_A = 0.25799 \text{ kJ/kg}$ (A-6)

$V_A = 2\text{ kg}(0.25799 \frac{m^3}{kg}) = 0.516\ m^3$

B. $v_B = 0.5(v_g - v_f) + v_f$ @ 150°C (A-4)

$= 0.5(0.39248 - 0.001091) + 0.001091 = 0.1968 \frac{m^3}{kg}$

$V_B = 3\text{ kg}(0.1968 \frac{m^3}{kg}) = 0.59\ m^3$

State 2 $\quad v = \dfrac{V_A + V_B}{5\text{kg}} = \dfrac{0.516 + 0.59\ m^3}{5}$

$v_2 = 0.2213\ m^3/kg$ @ 300kPa

$v_f < v_2 < v_g \Rightarrow$ sat. mixture

a) $T = T_{sat} = \boxed{133.52\ °C}$

b) $h_A = 3051.6 \text{ kJ/kg}$ (A-6)

$H_A = 2\text{kg}(3051.6 \text{ kJ/kg}) = 6{,}103.2\text{ kJ}$

$h_B = 0.5(2113.8) + 632.18 \frac{kJ}{kg} = 1689.1 \frac{kJ}{kg}$

$H_B = 3\text{kg}(1689.1 \frac{kJ}{kg}) = 5{,}067.2\text{ kJ}$

(4)

6. $H_A + H_B = 6103.2 + 5067.2 \text{ kJ} = 11{,}170.4 \text{ kJ}$

$h_2 = x(h_{fg}) + h_f$ @ 300kPa

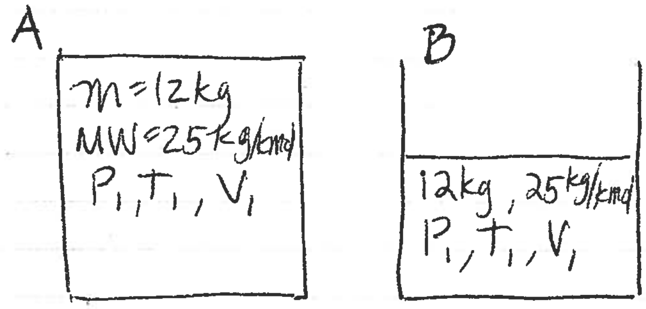
$$x = \frac{v_2 - v_f}{v_{fg}} = \frac{0.2213 - 0.001093}{0.60582 - 0.001073} = 0.364$$

$h_2 = 0.364(2163.5) + 561.43 = 1349.3 \text{ kJ/kg}$

$H_2 = 5\text{kg}(1349.3)\frac{\text{kJ}}{\text{kg}} = 6{,}746.5 \text{ kJ}$

$\Delta H = 6746.5 - 11170.4$ $\boxed{= -4424 \text{ kJ (LOST)}}$

7.



$\Delta T = 15°C$

$Q_A = mC_v(\Delta T)$ $\quad Q_B = mC_p\Delta T$

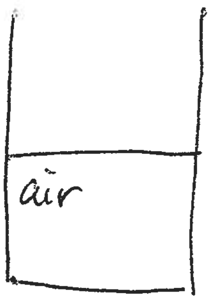
extra heat $= Q_B - Q_A = mC_p\Delta T - mC_v\Delta T$

$Q_B - Q_A = m\Delta T(C_p - C_v)$

$C_p - C_v = R \qquad R = R_u/25 = 8.314/25 = 0.333 \text{ kJ/kgK}$

$Q_A - Q_B = 12\text{kg}(15\text{K})0.333 \text{ kJ/kgK} = \boxed{60 \text{ kJ more}}$

8.



air compressed

$m = 2.4\,kg$

$P_1 = 150\,kPa \qquad P_2 = 600\,kPa$

$T_1 = 12°C = 285K = T_2$

isothermal work, ideal gas $\quad P_1V_1 = P_2V_2$

$$\frac{V_2}{V_1} = P_1/P_2$$

$$W_b = mRT\ln\left(\frac{P_1}{P_2}\right)$$

$$W_b = 2.4kg\left(0.287\frac{kJ}{kgK}\right)\left(\ln\left(\frac{150}{600}\right)\right)(285K)$$

$$\boxed{W_b = -272.1\,kJ}$$

4a

9. N_2 expanded as $PV^n = C$ (5)

$V_1 = 2\,m^3$ $\quad\quad$ $V_2 = 6\,m^3$

$P_1 = 500\,kPa$ $\quad\quad$ $P_2 = 250\,kPa$

$T_1 = 300°C = 573\,K$

a) find n

$$P_1 V_1^n = P_2 V_2^n$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$$

$$\ln\left(\frac{P_1}{P_2}\right) = n \ln\left(\frac{V_2}{V_1}\right) = n \ln(3)$$

$$\ln(2) = n \ln(3)$$

$$n = \frac{\ln(2)}{\ln(3)} = \boxed{0.631}$$

b) find T_2

ideal gas $\quad \dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}$

$$T_2 = \left(\frac{P_2}{P_1}\right)\left(\frac{V_2}{V_1}\right) T_1$$

$$T_2 = (0.5)(3)(573\,K) = \boxed{860\,K}$$

OR $\boxed{T_2 = 587°C}$

9 c) find $W_b = \frac{P_2V_2 - P_1V_1}{(1-n)} = \frac{250(6) - 500(2)}{1-0.631}$ (6

$W_b = 1{,}355 \text{ kJ}$

10. a) $1 \to 2$ $\quad P_1V_1 = P_2V_2$

$P_2 = \frac{P_1V_1}{V_2} = 100\text{ kPa}(3) \boxed{= 300\text{kPa}}$

b) $P_3 = 100/2 = 50\text{kPa} = P_4$

$W_{b34} = P(V_4 - V_3) = 50\text{kPa}(3-1)m^3$

$W_{b34} = \boxed{100 \text{ kJ}}$

c) $W_{b12} = P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 300\text{kPa}(1m^3)\ln\left(\frac{1}{3}\right)$

$W_{b1,2} = -330 \text{ kJ}$

d) $W_{net} = W_{b34} + W_{b12}$ (no W_b for $2\to3$ or $4\to1$)

$= 100\text{kJ} - 330\text{ kJ} = \boxed{-230\text{kJ}}$

Graph next page

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9. Nitrogen, an ideal gas, is expanded in a polytropic process according to the relation:

$$PV^n = \text{constant}$$

where P = pressure (kPa), V = volume (m^3) and n = a constant. The initial volume of the nitrogen is 2 m^3; the initial pressure is 500 kPa and the initial temperature is 300 °C. During the expansion, the volume triples and the pressure decreases to half its initial value.

a. (1 point) Find n.
b. (1 point) Calculate the equilibrium temperature after expansion.
c. (1 point) Calculate the boundary work by the gas during the process in kJ.

10. A device containing air is operated in a cycle consisting of four processes with no work exchanges other than boundary work.

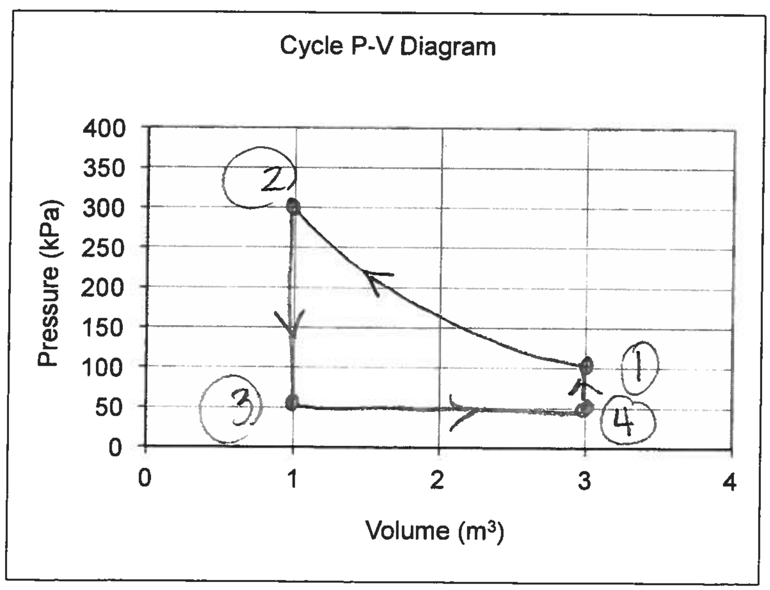
1 → 2: Isothermal compression, $V_1 = 3\ m^3$, $V_2 = 1\ m^3$; $P_1 = 100$ kPa

2 → 3: Isochoric heat loss, $P_3 = \frac{P_1}{2}\ m^3$

3 → 4: Isobaric expansion, $V_4 = 3\ m^3$

4 → 1: Isochoric heat addition, return to state 1

a. (1 point) Find P_2
b. (1 point) Find W_b for process 3 → 4
c. (1 point) Find W_b for process 1 → 2
d. (1 point) Find the net work for the cycle
e. (1 point) Graph the process on the P-V diagram below



11. free body diagram on piston : $F_{H_2O} = F_{He}$

$$P_{H_2O} A_2 = P_{He} A_1$$

condensing water is saturated mixture @ $T = 200°C$

$P = P_{sat} = 1,555 \text{ kPa}$

$$P_{He} = P_{H_2O} \left(\frac{A_2}{A_1}\right) \quad \text{where } \frac{A_2}{A_1} = \frac{d_2^2}{d_1^2} = \frac{4^2}{10^2} = 0.16$$

$$P_{He} = 1555 \text{ kPa} (0.16) = 249 \text{ kPa}$$

$$V_{He} = \frac{mRT}{P_{He}} = \frac{1 \text{ kg} (2.0769 \text{ kJ/kgK})(200+273) \text{K}}{249 \text{ kPa}}$$

$$\boxed{V_{He} = 3.95 \text{ m}^3}$$

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