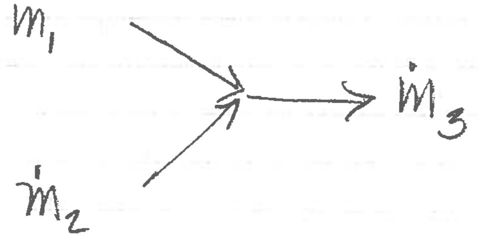
# Homework 6 SOLUTIONS

1. Adiabatic mixer, R-134a



① 1 MPa, 60°C, 2 kg/s

② 1 MPa, X=0, 0.0008695 $\frac{m^3}{s}$

③ 1 MPa, $T_3$, $1-X_3$

1st Law $\quad \underset{0}{\cancel{\dot Q}} - \underset{0}{\cancel{\dot W}} = \dot m_3 h_3 - \dot m_2 h_2 - \dot m_1 h_1$

$$\dot m_2 = \frac{\dot V_2}{v_f} = \frac{0.0008695\ m^3/s}{0.0008700\ m^3/kg} = 1\ \frac{kg}{s}$$

$$\dot m_3 = \dot m_2 + \dot m_1 = 3\ kg/s$$

$$h_1 = 293.38\ kJ/kg \quad (A-13)$$

$$h_2 = h_f = 107.32\ kJ/kg$$

Solve for $h_3$

$$0 = 3\frac{kg}{s} h_3 - 1\frac{kg}{s}\left(107.32\frac{kJ}{kg}\right) - 2\frac{kg}{s}\left(293.38\frac{kJ}{kg}\right)$$

$$h_3 = 231.36\ kJ/kg, \quad h_f < h_3 < h_g \text{ @ 1MPa}$$

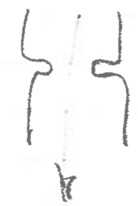
sat. mixture

a) $T = T_{sat} = \boxed{39.4°C}$

$$X_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{231.36 - 107.32}{163.67} = 0.76$$

b) $1 - X_3 = 1 - 0.76 = 0.24$ $\boxed{(24\%\ \text{liquid R-134a})}$

1. c)



adiabatic throttling valve

$$h_4 = h_3 = \boxed{231.36\ kJ/kg}$$

$P_4 = 200 kPa$   $h_f < h_4 < h_g$ @ 200kPa

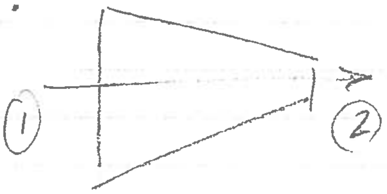
saturated mixture, $T = T_s = \boxed{-10.09°C}$

d) $$X_4 = \frac{231.36 - 38.43}{206.03} = 0.94$$

$1 - x = 0.06$   $\boxed{6\%\ \text{liquid}}$

$$v_3 = x_3(v_{fg}) + v_f = 0.76(0.020313 - 0.00087) + 0.00087$$
$$= 0.01565\ m^3/kg$$
$$v_4 = x_4(v_{fg}) + v_f = 0.94(0.099867 - 0.0007533) + 0.0007533$$
$$= 0.094\ m^3/kg$$

2.



adiabatic nozzle, air

① 300kPa, 200°C, 30 m/s, 80cm²

② 100kPa, 180 m/s

find $\dot{m}$, $T_2$, $A_2$

1st Law $$\underset{0}{\dot{Q}} - \underset{0}{\dot{W}} = \dot{m}\left[C_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2000}\right]$$

can find $T_2$, $C_p$ @ 475K ~ 1.025 kJ/kgK

$$T_2 = 200°C - \frac{1}{1.025\ \frac{kJ}{kgK}}\left(\frac{180^2 - 30^2}{2000}\right)\frac{kJ}{kg}$$

$$T_2 = 200 - 15.4 = \boxed{184.6°C}$$

2. $$\dot m = \frac{V_1 A_1}{v_1} = \frac{P_1}{RT_1} V_1 A_1$$

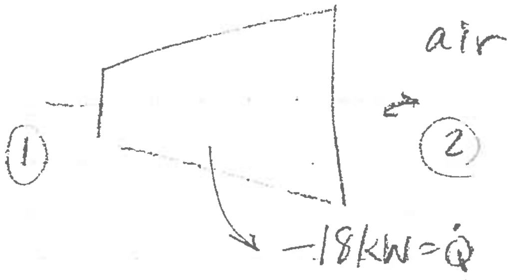
$$\dot m = \frac{300\,\text{kPa}}{0.287\frac{\text{kJ}}{\text{kgK}}(473\,\text{k})}\overset{\text{kJ}\cdot\text{m}^3}{} \frac{30\,\text{m}}{\text{s}}\left(80\,\text{cm}^2\times10^{-4}\frac{\text{m}^2}{\text{cm}^2}\right)$$

$$\boxed{\dot m = 0.53\ \text{kg/s}} = \frac{V_2 A_2}{v_2} = \frac{P_2}{RT_2} V_2 A_2$$

$$A_2 = 0.53\frac{\text{kg}}{\text{s}}\frac{\left(0.287\frac{\text{kJ}}{\text{kgK}}\right)(457.6\,\text{k})}{100\,\text{kPa}}\left(\frac{1}{180\ \text{m/s}}\right)$$

$$A_2 = 3.9\times10^{-3}\,\text{m}^2\times10^4\frac{\text{cm}^2}{\text{m}^2} = \boxed{39\ \text{cm}^2}$$

3.



① 27°C  220 m/s
2.5 kg/s

② $A_2 = 3A_1$, 42°C, 101 kPa

1st Law: $$\dot Q = \dot m\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2000}\right]$$

$$\dot Q = \dot m\left[C_p(T_2 - T_1) + \frac{(V_2^2 - V_1^2)}{2000}\right]$$

find $V_2$ and $P_1$

$$-18\ \text{kW} = 2.5\,\text{kg/s}\left[1.005\frac{\text{kJ}}{\text{kgk}}(42-27) + \frac{V_2^2 - 220^2}{2000}\right]$$

$$\boxed{V_2 = 62\ \text{m/s}}$$

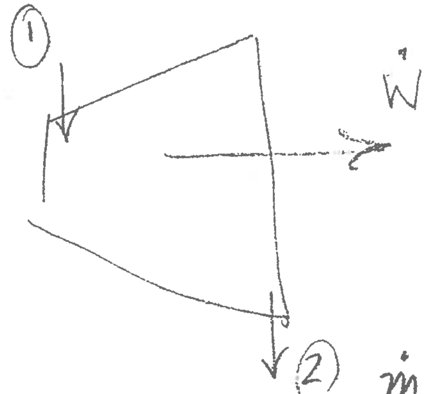
$$\frac{P_1}{RT_1} V_1 A_1 = \frac{P_2}{RT_2} V_2 A_2$$

3. $$P_1 = P_2\left(\frac{T_1}{T_2}\right)\frac{V_2}{V_1}\left(\frac{A_2}{A_1}\right)$$

$$P_1 = 101\,kPa\left(\frac{300k}{315k}\right)\left(\frac{62}{220}\right)(3)$$

$$\boxed{P_1 = 81.3\,kPa}$$

4.



adiabatic turbine

① 10 MPa, 450°C, 80 m/s

② 10 kPa, $x_2 = 0.92$, 50 m/s

$\dot m = 1.2\,kg/s$ find $\Delta ke$, $\dot W$, $A_1$

$$\Delta ke = \dot m\,\frac{(V_2^2 - V_1^2)}{2000} = 1.2\,\frac{(50^2 - 80^2)}{2000} = \boxed{-2.34\,kW}$$

$$-\dot W = \dot m\,(h_2 - h_1) - 2.34\,kW$$

A-6 $h_1 = 3{,}242.4\ kJ/kg$, $v_1 = 0.029782\ \frac{m^3}{kg}$

$h_2 = 0.92\,h_{fg} + h_f$ @ 10kPa

$= 0.92(2392.1) + 191.81 = 2392.5\ \frac{kJ}{kg}$

$$-\dot W = 1.2\,kg/s\,(2392.5 - 3242.4) - 2.34\,kW$$

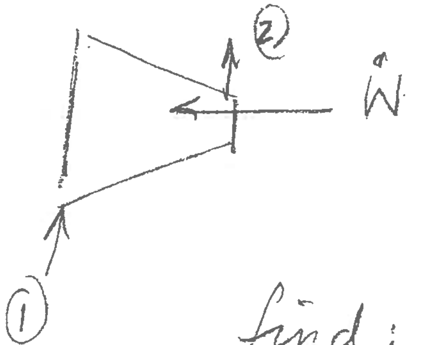
$$-\dot W = -1{,}022.2\,kW$$

note $\dot m\,(h_2 - h_1) = -1019.8\,kW \gg \Delta ke$

$\boxed{\dot W = 1{,}022.2\,kW}$ output

$$A_1 = \frac{\dot m\,v_1}{V_1} = \frac{1.2\,kg/s\,(0.029782\,m^3/kg)}{80\,m/s} = 4.5\times10^{-4}\,m^2 \times 10^4\,\frac{cm^2}{m^2}\ \boxed{= 4.5\,cm^2}$$

5. R-134a adiabatic Compressor



① $-20°C$, $x = 1$

② $70°C$, $0.7\,MPa$

$\dot{m} = 1.2\,kg/s$

find: $\dot{W}$, $\dot{V}_1$

1st Law: $\cancel{\dot{Q}}_{\,0} - \dot{W} = \dot{m}(h_2 - h_1)$

$h_1 = h_g @ -20°C$ (A-11) $= 238.41\,\frac{kJ}{kg}$

$h_2 = 308.33\,kJ/kg$ (A-13)

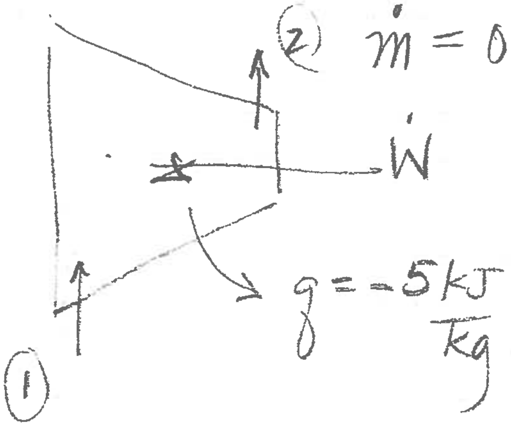
$-\dot{W} = 1.2\,\frac{kg}{s}(308.33 - 238.41)\frac{kJ}{kg}$

$$\boxed{\dot{W} = -83.9\,kW}$$

$\dot{V}_1 = \dot{m}v_1 = 1.2\,\frac{kg}{s}(v_g) = 1.2\,\frac{kg}{s}\left(0.14729\,\frac{m^3}{kg}\right)$

$$\boxed{\dot{V} = 0.18\,\frac{m^3}{s}}$$

6.



$\dot{m} = 0.1\,kg/s$ He in Compressor

① $100\,kPa$, $17°C$, $50\,cm^2$

② $500\,kPa$, $37°C$, $25\,cm^2$

6. a) $\dot{V}_1$ and $\dot{V}_2$

$$\dot{V} = \frac{\dot{m}}{\rho} = \dot{m} v$$

$$\dot{V}_1 = \frac{\dot{m}}{\rho_1} = \frac{0.1\,kg/s\,(2.0769\,kJ/kgK)(290\,K)}{100\,kPa}$$

$$\rho_1 = \frac{P_1}{RT_1} \qquad Pv = RT \qquad P = \rho RT$$

$$\boxed{\dot{V}_1 = 0.60\,m^3/s}$$

$$\dot{V}_2 = \frac{\dot{m}}{\rho_2} = \frac{0.1\,kg/s\,(2.0769\,kJ/kgK)(310\,K)}{500\,kPa}$$

$$\boxed{\dot{V}_2 = 0.13\,m^3/s}$$

b)

$$\mathcal{V}_1 = \frac{\dot{V}_1}{A} = \frac{0.6\,m^3/s}{50\times10^{-4}\,m^2} = 120\,\frac{m}{s}$$

$$\mathcal{V}_2 = \frac{\dot{V}_2}{A} = \frac{0.13\,m^3/s}{25\times10^{-4}\,m^2} = 52\,\frac{m}{s}$$

$$\Delta ke = 0.1\,\frac{kg}{s}\left(\frac{52^2 - 120^2}{2000}\right)\frac{kJ}{kg} = \boxed{-0.58\,kW}$$

c) 1st Law

$$\dot{m}\underset{0}{q} - \dot{W} = \dot{m}(C_p(T_2 - T_1)) + \Delta ke$$

$$-\dot{W} = 0.1\,\frac{kg}{s}\left(5.1926\,\frac{kJ}{kgK}(37-17)\right) = 0.58 + \underline{0.5}$$

$$\dot{m}\underset{0}{q} - \dot{W} = 10.39 - 0.08, \quad \boxed{\dot{W} = -10.3\,kW}$$

note $\Delta KE \ll \dot{m}(h_2 - h_1)$

7. adiabatic throttling

$h_2 = h_1$ but $T_2 \neq T_1$ so can't be ideal gas or liquid

only condition possible is if fluid is mixture and partially evaporates (see R-134a example)

8.

steam in adiabatic nozzle

① 200 kPa, 200°C

② 150 kPa, 150°C, $D_1/D_2 = 1.8$

1st Law $$0 = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2000}\right]$$

$$h_1 - h_2 = \frac{V_2^2 - V_1^2}{2000}$$

$$h_1 = 2870.7 \text{ kJ/kg} \quad (\text{A-6})$$

$$h_2 = \frac{2769.1 + 2776.2}{2} = 2772.7 \frac{\text{kJ}}{\text{kg}} \quad (\text{A-6 interpol})$$

$$V_2^2 - V_1^2 = 2000(2870.7 - 2772.7)$$

$$= 196{,}000 \, \text{m}^2/\text{s}^2$$

$$\dot{m}_1 = \dot{m}_2$$

$$\frac{V_1 A_1}{v_1} = \frac{V_2 A_2}{v_2}$$

$v_1 = 1.08049 \text{ m}^3/\text{kg}$

$$\frac{V_2}{V_1} = \left(\frac{A_1}{A_2}\right)\frac{v_2}{v_1}$$

$v_2 = \frac{0.95986 + 1.9367}{2}$

$v_2 = 1.448 \text{ m}^3/\text{kg}$

$\frac{A_1}{A_2} = \left(\frac{D_1}{D_2}\right)^2 = 1.8^2 = 3.24$

8.
$$\frac{V_2}{V_1} = 3.24\left(\frac{1.448}{1.08049}\right) = 4.34$$

$$(4.34V_1)^2 - V_1^2 = 196{,}000\ \text{m}^2/\text{s}^2$$

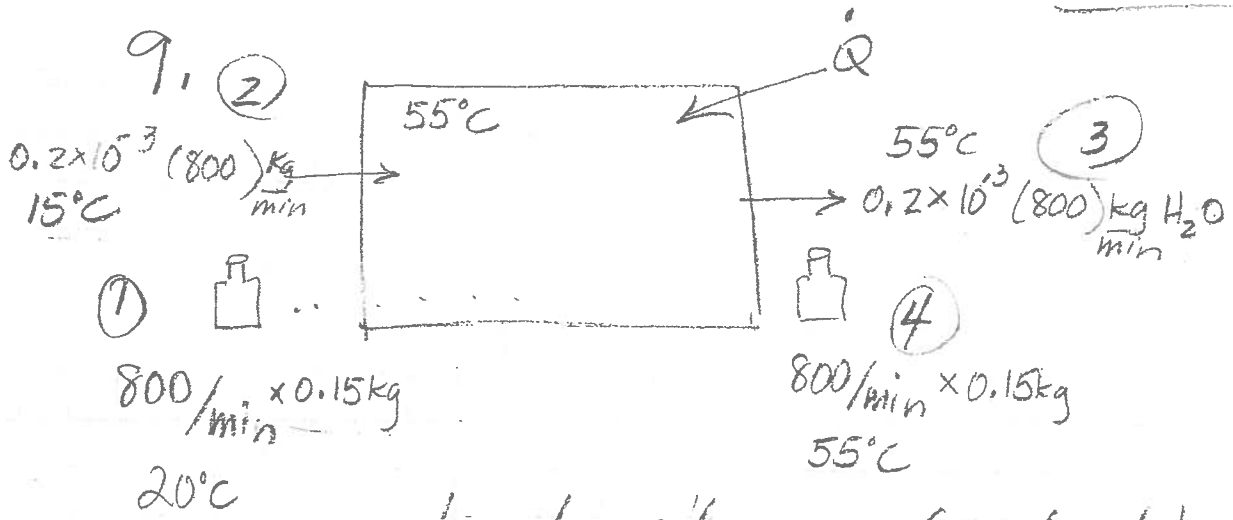
$$18.85V_1^2 - V_1^2 = 196{,}000$$

$$17.85V_1^2 = 196000\ \text{m}^2/\text{s}^2$$

$$V_1 = \boxed{105\ \text{m/s}}$$

$$V_2 = 4.34V_1 \boxed{= 456\ \text{m/s}}$$

9.



heat exchanger (no heat loss)

find $\dot{m}_w$, $\dot{Q}$ $(\dot{W} = 0)$

1st Law $\dot{Q} = \dot{m}_w(C_{pw}(T_3 - T_1)) + \dot{m}_g(C_{p,g}(T_4 - T_2))$

$C_{pw} = 4.184\ \text{kJ/kgK}$, $C_{pg} = 0.8\ \frac{\text{kJ}}{\text{kgK}}$

9. a) water loss (dragout)

$$\dot m_W = 800 \frac{\text{bottles}}{\text{min}}\ 0.2\times 10^{-3} \frac{\text{kg water}}{\text{bottle}} = \boxed{0.16 \frac{\text{kg water}}{\text{min}}}$$

b) 1st Law

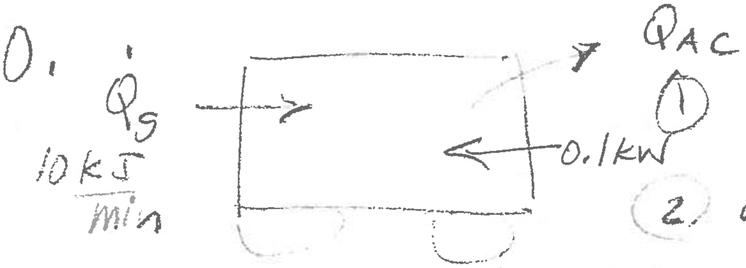
$$\dot Q = \dot m_W \left(C_{p_W}(T_3 - T_1)\right) + \dot m_g \left(C_{p_g}(T_4 - T_2)\right)$$

$$\dot m_g = 800 \frac{\text{bottles}}{\text{min}}\ 0.15 \frac{\text{kg}}{\text{bottle}} = 120 \frac{\text{kg}}{\text{min}}$$

$$\dot Q = 0.16 \frac{\text{kg}}{\text{min}} \left(4.184 \frac{\text{kJ}}{\text{kgK}} (55 - 15)\text{K} + 120 \frac{\text{kg}}{\text{min}} \left(0.8 \frac{\text{kJ}}{\text{kgK}} (55 - 20)\text{K}\right)\right.$$

$$\dot Q = 3{,}387 \frac{\text{kJ}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} = \overset{3360}{\boxed{56.4\text{ kW}}}$$

10.



① air, T = 60°C

② air, T = 22°C

1st Law, $\dot W = 0.1$ kW closed system

a) $\dot Q_S(t) + \dot Q_{AC}(t) - \dot W(t) = m C_V (T_2 - T_1)$

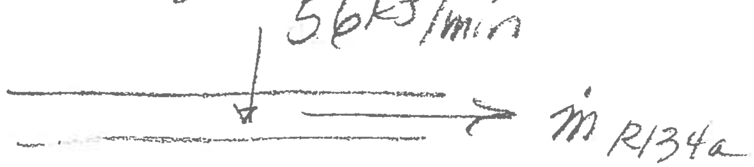
$t = 5$ min

$$m = \frac{P_1 V_1}{R T_1} = \frac{100\text{ kPa}\,(7\text{ m}^3)}{0.287 \frac{\text{kJ}}{\text{kgK}}\,(333\text{K})} = 7.3\text{ kg}$$

$$0.1(60)(5\text{min}) + 10(5\text{min}) + \dot Q_{AC}(5\text{ min}) = 7.3\text{ kg}\left(0.718 \frac{\text{kJ}}{\text{kgK}}\right)(22 - 60)$$

$$\dot Q_{AC} = \frac{1}{5\text{ min}}\left[-279.2\text{ kJ}\right] = \boxed{-56 \frac{\text{kJ}}{\text{min}}}$$

10. b) Heat exchanger



①

320 kPa
$x = 0.3$

320 kPa
$x = 1$

$\dot{W} = 0$

1st Law $\quad \dot{Q} = \dot{m}(h_2 - h_1)$

$h_2 = h_g @ 320\text{kPa} = 251.88 \frac{\text{kJ}}{\text{kg}}$

$h_1 = 0.3(h_{fg}) + h_f = 0.3(196.71) + 55.16$

$h_1 = 114.17 \text{ kJ/kg}$

$$\dot{m} = \frac{56 \text{ kJ/min}}{(251.88 - 114.17)\text{ kJ/kg}} = \boxed{0.41 \frac{\text{kg}}{\text{min}}}$$

c) adiabatic compressor, $T_3 = 50°\text{C}$, $P_3 = 1\text{MPa}$, $\dot{Q} = 0$

③ $h_3 = 282.74 \text{ kJ/kg}$ (A-13),

$-\dot{W} = \dot{m}(h_3 - h_2)$

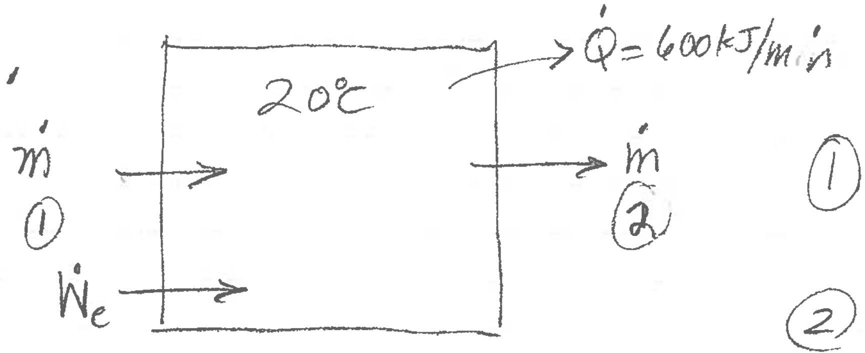
$-\dot{W} = 0.41 \frac{\text{kg}}{\text{min}} (282.74 - 251.88) \frac{\text{kJ}}{\text{kg}}$

$$\boxed{\dot{W} = -12.65 \text{ kJ/min}}$$

$v_1 = 0.3(0.063604 - 0.0007772) + 0.0007772 = 0.020 \text{ m}^3/\text{kg}$

$v_3 = 0.022 \text{ m}^3/\text{kg}$

11.



① 5°C, 100 kPa
0.005 m³/s

② 20°C, 100 kPa

$$\dot{Q}_L = \frac{600\,kJ}{60\,s} = 10\,kW \text{ (out)}$$

$$\dot{W}_e = 2\,kW \text{ (in)}$$

a) $\dot{m} = \rho_1 \dot{V}_1 = \frac{P_1}{RT_1}(\dot{V}_1) = \frac{100\,kPa\,(0.005\,m^3/s)}{0.287\,\frac{kJ}{kgK}\,(278K)} = 6.3\times10^{-3}\,\frac{kg}{s}$

$\boxed{\dot{m} = 6.3\times10^{-3}\,\frac{kg}{s}}$ or $63\,\frac{g}{s}$

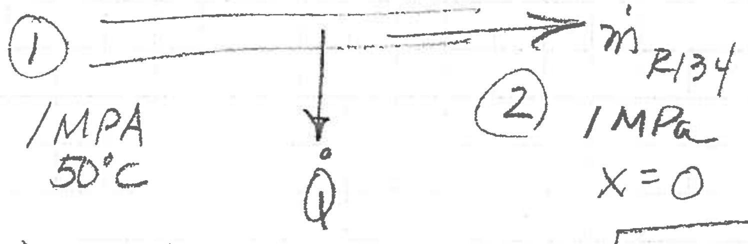
b) 1st Law

$$\dot{Q} + \dot{Q}_L - \dot{W} = \dot{m}\,c_p\,(T_2 - T_1)$$

$$\dot{Q} - 10\,kW - (-2\,kW) = 6.3\times10^{-3}(1.005)(20-5)$$

$$\dot{Q} = \boxed{8.09\,kW} \text{ input}$$

12.



a) $T_2 = T_s\,@\,1MPa = \boxed{39.37°C}$

b) $h_1 = 282.74\,kJ/kg$ (A-13), $h_2 = h_f\,@\,1MPa = 107.32\,kJ/kg$

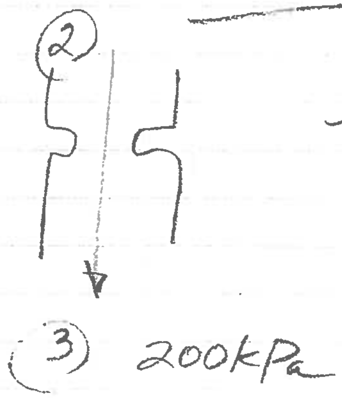
$$\Delta h = 107.32 - 282.74 = \boxed{-175.4\,kJ/kg}$$

12c) continues #11 house

$$\dot{Q} = -8.09\ kW = \dot{m}\left(-175.4\ \frac{kJ}{kg}\right)$$

$$\boxed{\dot{m}_{R134a} = 0.05\ \frac{kg}{s}}$$

d)



$$h_3 = h_2 = 107.32\ kJ/kg$$

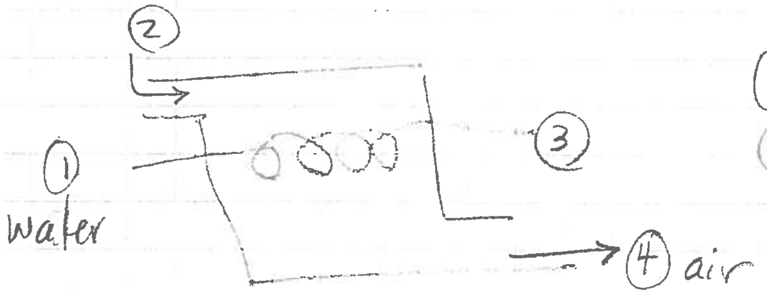
$$h_f < 107.32 < h_g \text{ @ } 200kPa$$

sat. mixture

$$T = T_s = \boxed{-10.09°C}$$

e. $$X_3 = \frac{107.32 - 38.43}{206.03} = \boxed{0.33 \text{ OR } 33\%}$$

13.



① water 80°C

③ water 30°C

② air 800 m³/min 27°C
100kPa

④ air, 60°C, 95kPa

adiabatic heat exchanger

a) $\dot{W} = 0$
$\dot{Q} = 0$

$$0 = \dot{m}_W\left(C_{P_W}(T_3 - T_1)\right) + \dot{m}_a C_{P_a}(T_4 - T_3)$$

$$\dot{m}_W = \frac{\dot{m}_a (1.005\ kJ/kgK)(60-27)}{4.184\ \frac{kJ}{kgK}(80-30)}$$

$$\dot{m}_a = \rho_1 \dot{V}_1 = \frac{P_1}{RT_1}\dot{V}_1 = \frac{100kPa\ (800\ m^3/min)}{0.287\frac{kJ}{kgK}(300k)} = 929\ \frac{kg}{min}$$

$$\dot{m}_W = \frac{929(1.005)(33)}{4.184(50)} = \boxed{147\ kg/min}$$

13 b) $\dot{V}_2 = \rho_2 \dot{m}_a = \frac{P_2}{RT_2}\left(929 \frac{kg}{min}\right)$

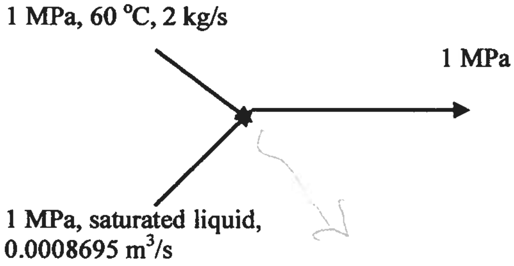
$$= \frac{95\,kPa}{0.287\frac{kJ}{kgK}(333K)}\left(929\frac{kg}{min}\right) = \boxed{923 \frac{m^3}{min}}$$

⑬

**AREN 2110: Thermodynamics**
**Spring 2011**

**HOMEWORK 6: Due Friday, March 4, 6 PM (13 problems, 45 points possible)**

1. (6 points: 1 per part) Refrigerant (R-134a) at a pressure of 1 MPa and 60 °C flows into a well-insulated mixing chamber at a rate of 2 kg/s. Saturated liquid R-134a at the same pressure enters the mixer at a rate of 0.0008695 m$^3$/s. Assume steady flow conditions.



a) What is the temperature of the refrigerant at the mixer outlet?

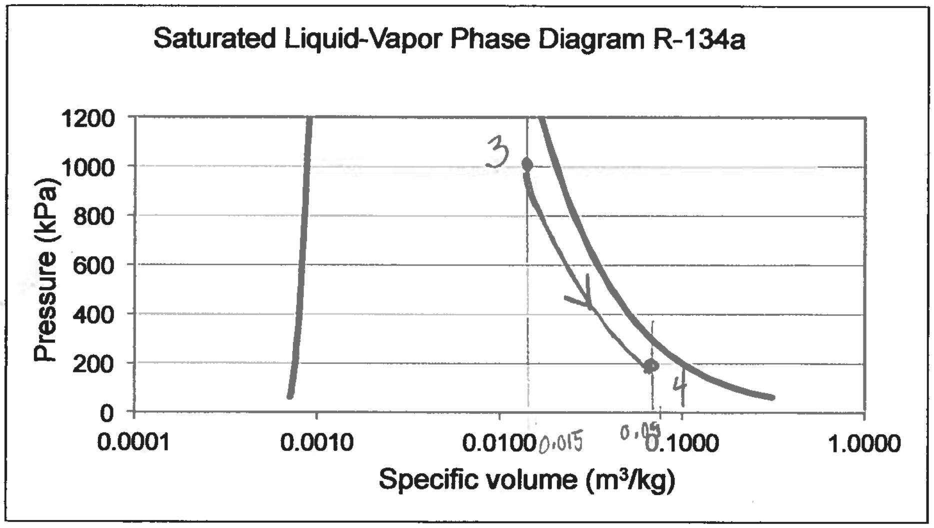
b) What is the percent liquid in the refrigerant at the mixer outlet?

c) After mixing, the refrigerant enters an adiabatic throttling valve that reduces the pressure to 200 kPa. What is the specific enthalpy of the refrigerant at the throttling valve outlet?

d) What is the temperature of the refrigerant at the throttling valve outlet?

e) What percent of the R-134a is liquid at the throttling valve outlet?

e) Draw the throttling valve process on the P-v diagram for refrigerant.



9. (2 points, 1 per part) A glass bottle washing facility uses a well-mixed hot water bath at 55 °C. The bottles enter the washer at a rate of 800 per minute at an ambient temperature of 20 °C and leave at the bath water temperature. Each bottle has a mass of 150 g and as it leaves the bath, takes 0.2 g water with it. Make up water with temperature of 15 °C is used to keep the mass of water in the bath constant. Assuming no heat loss from the outer surface of the bath tank, calculate:

a) The rate at which water must be supplied to maintain a constant mass of water
b) The rate at which heat must be supplied to maintain steady operation

10. (4 points. 1 per part) A car is left with its windows closed on a summer day and the interior air reaches a temperature of 60 °C.

a) At what rate must heat be removed by an air conditioner in the car to bring the temperature to 22 °C in 5 minutes? Assume the windows remain closed during cooling. The volume of air in the car is 7 $m^3$, and the air pressure = 100 kPa. Solar radiation heats the car at the rate of 10 kJ/min and the air conditioner has a 100-w fan.

The air conditioner uses R-134a refrigerant as a working fluid. The car air is cooled by blowing it across tubes in a heat exchanger. The R-134a enters the heat exchanger pipes as a saturated mixture at 320 kPa and quality = 0.3 and exits the exchanger as saturated vapor at the same pressure.

b) What mass flow rate of refrigerant is required to cool the car interior as for part a?

c) After evaporation in the heat exchanger, the saturated R-134a vapor is compressed to a pressure of 1 MPa and temperature = 50 °C in an adiabatic compressor. What is the power requirement for the compressor?

d) Graph the R-134a processes on a P-V diagram (below)