**AREN 2110: Thermodynamics**
**Midterm 2**
**Fall 2005**

SOLUTIONS
Name

*Test is open book and notes. Answer all questions and sign honor code statement:* ***I have neither given nor received unauthorized assistance during this exam.***

Signed________________________________

Remember to show your work – partial credit will be given for a correct approach!

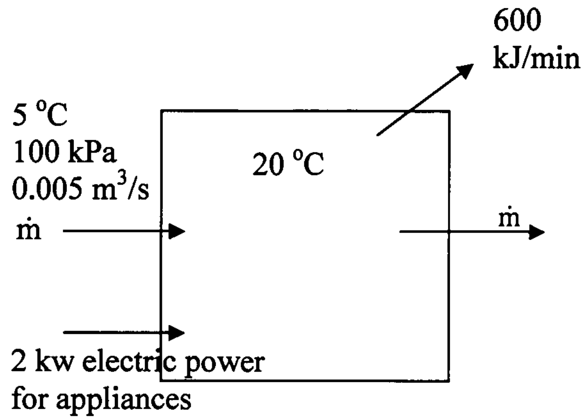
| Question | Points |
|---|---|
| 1 | /35 |
| 2 | /30 |
| 3 | /35 |
| Total | /100 |

**AREN 2110: Thermodynamics** ___________________________

**Midterm 2** **Name**

**Fall 2005**

1. (35 points) A house kept at 20 °C loses 600 kJ/min heat during January when the average outside temperature is 5 °C. In addition, cold air infiltrates the house at a rate of 0.005 $m^3/s$. Since the mass of air in the house does not change, the house loses warm air to the outdoors at the same mass flow rate. Finally, appliances use 2 kw electricity which adds energy to the house air.




a) What is the mass flow rate of infiltrating air? (5)

$$\dot{m} = \rho_1 \dot{V} = \left(\frac{P_1}{RT_1}\right)\dot{V}_1 = \frac{100\,kPa}{(0.287\frac{kJ}{kgK})(278K)} \times 0.005\,\frac{m^3}{s} = \boxed{0.0063\,\frac{kg}{s}}$$

b) At what rate must heat be added to keep the house at 20 °C? (5)

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1) = \dot{m}C_p(T_2 - T_1)$$

$$\dot{Q} - \frac{600}{60}\,kW - (-2\,kW) = 0.0063\,\frac{kg}{s}\,(1.004\,\frac{kJ}{kgK})\,(20-5)$$

$$\dot{Q} = 10\,kW - 2\,kW + 0.0063(1.004)(15)\,kW$$

$$\boxed{\dot{Q} = 8.1\,kW}$$

c) Heat for the house is provided from the condenser (heat exchanger) of a heat pump containing R-134a. The pressure of the condenser is constant at 1 MPa, and the temperature at the inlet is 50 °C. The refrigerant is saturated liquid at the outlet. What is the temperature of the R-134a at the outlet? (5)

$$T = T_s \;@\; 1\,MPa = \boxed{39.39\,°C}$$

**Name** ______________________________

1. (continued)

d) Calculate the mass flow rate of R-134a to required to keep the house at 20 °C. (5)

$\dot{Q} = -8.1\,kW = \dot{m}_{R\text{-}134a}(h_2 - h_1)$

$T_1 > T_s @ 1MPa \therefore h_1 = 280.19 \frac{kJ}{kg}$ (A-13)
superheated

$T_2 = T_s$ and $h_2 = h_f @ 1MPa = 105.29 \frac{kJ}{kg}$ (A-12)

$\dot{m}_{R134a} = \frac{-8.1\,kW}{(105.29 - 280.19)\frac{kJ}{kg}} = \boxed{0.046 \frac{kg}{s}}$

e) The refrigerant leaves the condenser and enters a throttling valve where the pressure is reduced from 1 MPa to 200 kPa. What is the temperature of the R-134a at the throttling valve outlet? Justify your answer. (5)

$h_3 = h_2 = h_f)_{1MPa} = 105.29\,kJ/kg$ @ 200kPa:

$h_f = 36.84\,kJ/kg < 105.29 < h_g = 241.3\,kJ/kg$, ∴ mixture

and $T_3 = T_s @ 200kPa \boxed{= -10\,°C}$

f) What is the vapor content of the refrigerant at the throttling valve outlet? (5)

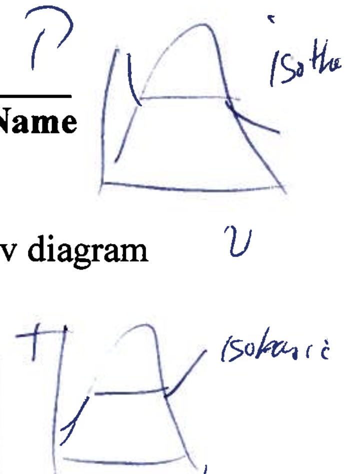
$X_3 = \frac{h_3 - h_f}{h_{fg}} @ 0.2MPa = \frac{105.29 - 36.84}{204.46}$

$X_3 = 0.335$

vapor content $= X_3 = \boxed{0.335 \text{ OR } 33.5\%}$

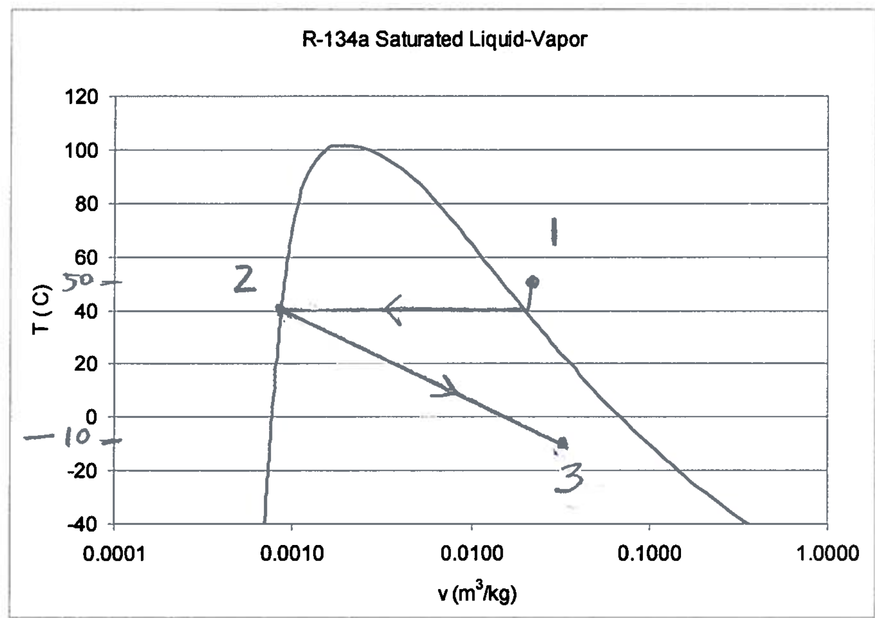
**AREN 2110: Thermodynamics**
**Midterm 2**
**Fall 2005**

Name ______________________




1. (continued)

g) Draw the throttling and condenser process sequence on the T–v diagram below. (5)

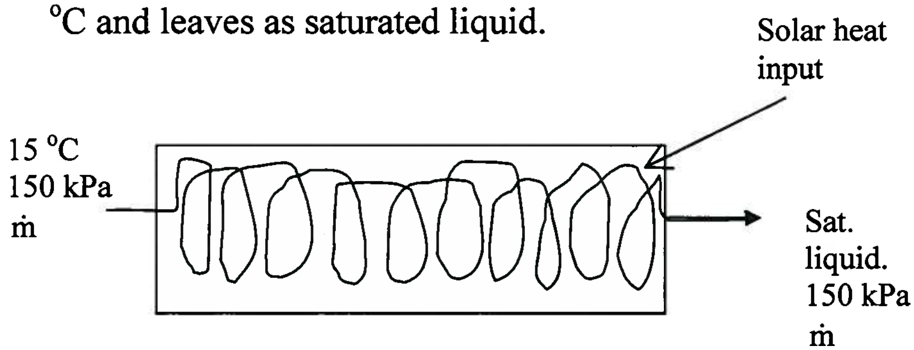



1 superheat
2 sat liquid
3. mix
1→2 isobaric

$v_2 = 0.02 \ m^3/kg$

$v_3 = 0.335(0.0993 - 0.000753) + 0.000753 = 0.034$

2. (30 points) A family uses 500 liters of hot water per day for showering, laundry, cooking, etc., that is to be supplied from a heat exchanger absorbing solar energy. Water enters the isobaric solar heat exchanger at 150 kPa and 15 °C and leaves as saturated liquid.




Solar Heat Exchanger for Hot Water

**AREN 2110: Thermodynamics** __________________________

**Midterm 2** **Name**

**Fall 2005**

2. (continued)

a) At what rate must solar heat be supplied to the heat exchanger to provide 500 L/day of hot water? (Assume density of water is ~1 kg/liter.) (5)

$500\,L/d \times 1\,kg/L = 500\,kg/d = \dot{m}$

$T_2 = T_s @ 150\,kPa = 111.4°C$

1st Law $\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1)$, $\dot{W} \to 0$

2 methods ok:

$h_1 \approx h_f @ 15°C = 62.99\,\frac{kJ}{kg}$

$h_2 = h_f @ 150\,kPa = 467.11\,\frac{kJ}{kg}$

$\dot{Q} = 500\frac{kg}{d} \cdot \frac{1\,d}{86400\,s}(467.11 - 62.99) = 2.34\,kW$ ✓ OR

$\dot{Q} = \dot{m} C_p (T_2 - T_1)$ (all liquid)

$= 500\frac{kg}{d}\left(4.18\frac{kJ}{kgK}\right)\frac{(111.4 - 15)K}{86400\,s/d}$ $C_p @ 400K = 4.18\,\frac{kJ}{kgK}$

$\dot{Q} = 2.33\,kW$ ✓

b) The solar hot water heat exchanger supplies 500 watts/m$^2$ of collector surface area. What is the area of the collector required to provide hot required flow of heated water?

$\frac{\dot{Q}}{A} = 0.5\,kW/m^2$ $A = \frac{\dot{Q}\,kW}{0.5\,kW/m^2} = \frac{2.34}{0.5} = 4.7\,m^2$

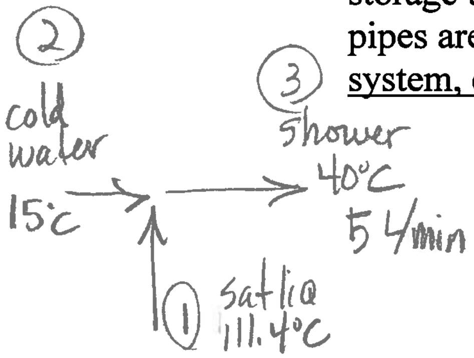
c) The house's hot water tank, also at 150 kPa, receives saturated liquid water from the solar heat exchanger outlet and is very well insulated. What is the temperature of the water in the tank? (5)

$T = T_s @ 150\,kPa = 111.4°C$

______________________________
**Name**

2 (continued)

d) For showering, cold water at 15 °C is mixed with hot water from the storage tank to obtain a shower flow of 5 L/min at 40 °C. The shower pipes are well insulated. Write the 1st Law equation for the shower system, consisting of pipes for cold and hot water and shower flows. (5)




$$0 = \dot{m}_3 h_3 - \dot{m}_2 h_2 - \dot{m}_1 h_1$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 \,, \quad \dot{m}_1 = \dot{m}_3 - \dot{m}_2$$

$$0 = \dot{m}_3 h_3 - \dot{m}_2 h_2 - (\dot{m}_3 - \dot{m}_2) h_1$$

e) Find the mass flow rate of cold water required for the 40 °C shower?

$$\boxed{0 = \dot{m}_3 (h_3 - h_1) + \dot{m}_2 (h_1 - h_2)}$$

from part a)

$h_1 = 467.11 \frac{kJ}{kg}$

$h_2 = 62.99 \frac{kJ}{kg}$

AND

$h_3 \cong h_f @ 40°C = 167.57 \frac{kJ}{kg}$

$$\dot{m}_2 = \frac{\dot{m}_3 (h_3 - h_1)}{(h_2 - h_1)} = 5 \frac{kg}{min} \frac{(167.57 - 467.11)}{(62.99 - 467.11)}$$

$$\boxed{\dot{m}_2 = 3.7 \frac{kg}{min}} \quad \left( \dot{V}_2 = 3.7 \frac{L}{min} \right)$$

OR

$$\dot{m}_2 = \frac{\dot{m}_3 (T_3 - T_1)}{(T_2 - T_1)} = 5 \frac{kg}{min} \frac{(40 - 111.4)}{(15 - 111.4)} \boxed{= 3.7 \text{ kg/min}} ✓$$

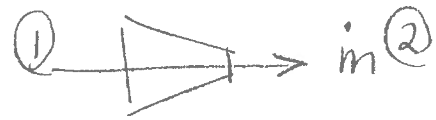
f) Find the mass flow rate of hot water for the 40 °C shower? (5)

$$\dot{m}_1 = (5 - 3.7) \frac{kg}{min} \boxed{= 1.3 \frac{kg}{min}}$$

**AREN 2110: Thermodynamics** ______________________________
**Midterm 2** **Name**
**Fall 2005**

3. (35 points) Air flows through a nozzle operating at steady state with the following conditions:

**Inlet**: 27 °C, 120 kPa, velocity = 200 m/s and inlet area = 0.04 $m^2$.
**Outlet**: 60 kPa, velocity = 500 m/s.




a) What is the mass flow rate of air? (5)

$$\dot{m} = \rho_1 V_1 A_1 = \left(\frac{P_1}{RT_1}\right) V_1 A_1 = \frac{120\,kPa}{(0.287\,\frac{kJ}{kgK})(300K)} \times 200\,\frac{m}{s} \times 0.04\,m^2$$

$$\boxed{\dot{m} = 11.15\,\frac{kg}{s}}$$

b) At what rate must heat be added (or removed) to (or from) the nozzle to keep the process <u>isothermal</u>? Use the appropriate sign convention for heat. (5)

$$\dot{Q} - \dot{W} = \dot{m}\left[C_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2000}\right] \quad (\dot{W} = 0,\ T_2 - T_1 = 0)$$

$$\dot{Q} = 11.15\,\frac{kg}{s}\left[\frac{500^2 - 200^2}{2000}\right]\frac{kJ}{kg} =$$

$$\boxed{\dot{Q} = 1{,}171\,kW}$$

c) Calculate the outlet area of the nozzle. (5)

$$A_2 = \frac{\dot{m}}{\rho_2 V_2} = 11.15\,\frac{kg}{s}\left(\frac{0.287\,\frac{kJ}{kgK}\,300K}{60\,kPa}\right) \times \frac{1\,s}{500\,m}$$

$$\boxed{A_2 = 0.032\,m^2}$$

AREN 2110: Thermodynamics
Midterm 2
Fall 2005

Name

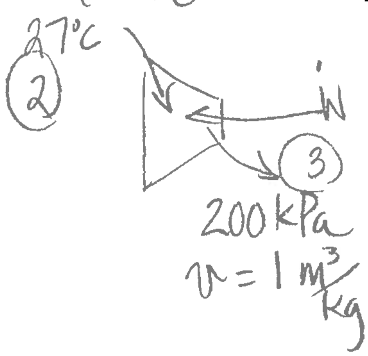
3. (continued)

d) Does the volumetric flow rate of air at the nozzle inlet = the volumetric flow rate at the outlet? Justify your answer. (5)

no $\dot{V} = VA$, $\dot{V}_1 = 200 \frac{m}{s}(0.04 m^2) = 8\ m^3/s$

$V_2 = 500 \frac{m}{s}(0.032 m^2) = 16 \frac{m^3}{s}$

e) Air leaving the nozzle enters an adiabatic compressor and exits at a pressure of 200 kPa and specific volume = 1 m$^3$/kg. What is the temperature of the air at the compressor outlet? (5)




$T_3 = \frac{P_3 v_3}{R} = \frac{200\ kPa\,(1\ m^3/kg)}{0.287\ kJ/kgK}$

$T_3 = 697 K$
$= 424°C$

f) Find the required power input for the compressor, assuming that kinetic energy changes can be neglected. (5)

$\dot{Q} - \dot{W} = \dot{m}(h_3 - h_2) = \dot{m} C_p (T_2 - T_1)$

$C_p @ 500K = 1.029 \frac{kJ}{kgK}$

$-\dot{W} = 11.15 \frac{kg}{s}\left(1.029 \frac{kJ}{kgK}\right)(424 - 27)K$

$\dot{W} = -4,555\ kW$

______________________________
**Name**

3. (continued)

g) What is the volumetric flow rate of air at the compressor outlet? (5 points)

$$\dot{V}_3 = v_3 \dot{m} = 1 \frac{m^3}{kg} \left(11.15 \frac{kg}{s}\right)$$

$$\boxed{\dot{V}_3 = 11.15 \frac{m^3}{s}}$$