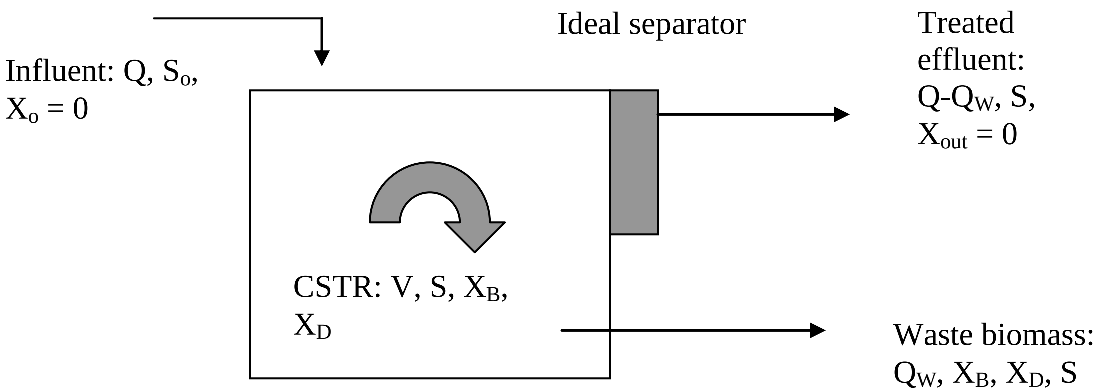
CSTR WITH SOLIDS RECYCLE

Components:

- Ideal CSTR, Volume = V
- Ideal solids separator, volume = 0

Flows

- Inflow (Q) and treated outflow ($Q-Q_W$)
- Waste biomass (Q_W) (wasting from CSTR mixed contents) to maintain steady-state biomass concentration
- $Q_W << Q$



Characteristics:

Hydraulic residence time: τ or HRT

$$\tau = \frac{V}{Q}$$

Solids residence time" Θ or SRT = mass solids in CSTR/solids mass wasting rate

$$\Theta = \frac{VX}{Q_W X} = \frac{V}{Q_W}$$

Note that since $Q_W << Q$, $\Theta >> \tau$

General Mass Balance Expression for one reactive component, S:

$$QS_O - (Q-Q_W)S - Q_WS + Vr_S = V\frac{dS}{dt}$$

At steady state condition for S:

$$QS_O - (Q-Q_W)S - Q_WS + Vr_S = 0$$

Rearranging:

$$Q(S_O-S) + Vr_S = 0$$

$$r_S = \frac{(S_O - S)}{\tau}$$

For heterotrophic growth and decay in a CSTR, four components and two reactions are of interest: SUBSTRATE COD (S_S), OXYGEN (S_O), CELL-COD (X_{BH}), and DEBRIS-COD (X_D)

	Components				**Rates**
Process	Soluble COD S_S (mg/L COD)	Dissolved O_2, S_O (mg/L O_2)	Heterotrophic biomass, X_{BH} (mg/L COD)	Debris, X_D (mg/L COD)	ρ_j
Aerobic Heterotrophic Growth	$-1/Y_H$	$-\frac{(1-Y_H)}{Y_H}$ (1)	1	na	$\mu_H X_{BH}$ (2)
Decay and Lysis of Heterotrophs		$1-f_D$ (3)	-1	f_D (6)	$b_H X_{BH}$ (4)

(1) heterotrophic cell yield = Y_H = g-heterotroph cell-COD/g-COD consumed

(2) heterotrophic growth rate (ρ_1) = $X_{BH}\mu_H = \hat{\mu}_H\left(\frac{S_S}{K_S + S_S}\right)X_{BH}$ where μ_H is specific growth rate (d^{-1})

(3) f_D = g debris-COD produced/g biomass-COD decayed. Assumes direct exertion of decayed biomass COD

(4) heterotrophic decay rate (ρ_2) $= X_{BH} b_H$ where b_H = specific decay rate (d^{-1})

net heterotrophic biomass growth rate $= r_{XB} = (\mu_H - b_H) X_{BH}$

debris generation rate $= r_D = f_D b_H X_{BH}$

COD consumption rate $= r_S = -\left(\frac{\mu_H}{Y_H}\right) X_{BH}$ (growth only)

O_2 consumption rate $= r_O = -\left(\frac{(1-Y_H)}{Y_H}\mu_H + (1-f_D) b_H\right) X_{BH}$

Apply rate expressions to steady-state mass balances on CSTR with solids retention:

Viable heterotrophic cells:

$$0 - 0 - Q_W X_{BH} + V X_{BH}(\mu_H - b_H) = 0$$

simplify and note that $\Theta = V/Q_W$

$$\frac{1}{\Theta} = \mu_H - b_H \qquad (1)$$

for CSTR with biomass retention (recycling), the growth rate is a function of Θ, not τ.

Assuming Monod kinetics with COD the limiting substrate:

$$\mu_H = \frac{\hat{\mu}_H S_S}{(K_S + S_S)} \qquad (2)$$

Substituting (2) → (1):

$$S_S = \frac{K_S\left(\frac{1}{\Theta} + b_H\right)}{\hat{\mu}_H - \left(\frac{1}{\Theta} + b_H\right)} \qquad \textbf{(I)}$$

Steady-state mass balance on COD substrate, S:

$$QS_{SO} - Q_W S_S - (Q-Q_W)S_S - (\mu_H/Y_H)X_{BH}V = 0$$

$$(\mu_H/Y_H)X_{BH} = (S_{SO} - S_S)/\tau$$

$$X_{BH} = \left(\frac{1}{\tau}\right)\frac{Y_H(S_{SO} - S_S)}{\mu_H} \qquad (3)$$

Substituting (1) into (3) for μ_H:

$$X_{BH} = \left(\frac{\Theta}{\tau}\right)\frac{Y_H(S_{SO} - S_S)}{(1 + b_H\Theta)} \qquad \textbf{(II)}$$

Heterotrophic cell concentration is proportional to Θ and inversely proportional to τ

Can substitute into (II) for S_S from (I)

$$X_{BH} = \left(\frac{\Theta}{\tau}\right)\frac{Y_H\left(S_{SO} - \left(\frac{K_S(1 + b_H\Theta)}{\hat{\mu}_H\Theta - (1 + b_H\Theta)}\right)\right)}{(1 + b_H\Theta)} \qquad \textbf{(IIa)}$$

Steady-state mass balance of debris:

$$-Q_W X_D + V f_D b_H X_{BH} V = 0$$

$$X_D = f_D b_H X_{BH} \Theta$$

Substituting for X_{BH} from (II):

$$X_D = f_D b_H \Theta \left(\frac{\Theta}{\tau}\right) \frac{Y_H (S_{SO} - S_S)}{(1 + b_H \Theta)} \qquad \textbf{(III)}$$

Total biomass, $X_T = X_{BH} + X_D$

$$X_T = (1 + f_D b_H \Theta) \left(\frac{\Theta}{\tau}\right) \frac{Y_H (S_{SO} - S_S)}{(1 + b_H \Theta)} \qquad \textbf{(IV)}$$

Define active fraction of biomass, $f_A = \frac{X_{BH}}{X_T}$

$$f_A = \frac{1}{(1 + f_D b_H \Theta)}$$

Define observed yield as the net yield, considering biomass decay (different from growth yield) = Y_{Hobs}

To maintain system in steady-state with respect to biomass components, net growth and accumulated debris must be wasted:

$$\text{Net growth} = Y_{Hobs} Q(S_{SO} - S_S)$$

$$\text{Wasted biomass} = Q_W X_T$$

Steady-state condition:

$$Q_W X_T = Y_{Hobs} Q(S_{SO} - S_S)$$

$$Y_{Hobs} = \frac{Q_W X_T}{Q(S_{SO} - S_S)}$$

Note that $Q_W/Q = (V/\Theta)/(V/\tau) = \tau/\Theta$

And substituting for X_T from (IV):

$$Y_{Hobs} = \frac{Y_H(1 + f_D b_H \Theta)}{(1 + b_H \Theta)} \qquad \textbf{(V)}$$

Since $f_D < 1$, $\frac{(1 + f_D b_H \Theta)}{(1 + b_H \Theta)} < 1$ and Y_{Hobs} always $< Y_H$

Waste biomass production:

$W_T = Q_W X_T$ (rate of total biomass wasting)

OXYGEN CONSUMPTION

$r_{o,1}$ = - $-\frac{(1-Y_H)}{Y_H} r_{XBH}$ for heterotrophic aerobic growth

incorporate recycling of decayed biomass assuming direct conversion of decayed biodegradable fraction of biomass to COD and exertion of oxygen demand:

	Components				**Rates**
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For decay stoichiometry that converts recycled COD directly to oxygen consumption:

$$-X_{BH} - (1-f_D)S_O + f_D X_D = 0$$

$$r_{O,2} = (1-f_D) r_{XBH}$$

combining

$$r_O = r_{O,1} + r_{O,2}$$

$$r_O = -\left(\frac{(1-Y_H)}{Y_H} \mu_H + (1-f_D) b_H \right) X_{BH}$$

substitute $\mu_H = 1/\Theta + b_H$ and for X_{BH}

$$r_O = \frac{(S_{SO} - S_S)}{\tau}\left[1 - \left(\frac{Y_H(1 + f_D b_H \Theta)}{(1 + b_H \Theta)}\right)\right] \qquad \textbf{(VIa)}$$

but
$$\left(\frac{Y_H(1 + f_D b_H \Theta)}{(1 + b_H \Theta)}\right) = Y_{Hobs}$$

so
$$r_O = \frac{(S_{SO} - S_S)}{\tau}\left[1 - Y_{Hobs}\right] \quad \text{(mg/L-d)} \qquad \textbf{(VIb)}$$

multiply by V to get overall system oxygen uptake rate **RO = $\mathbf{r_O}$V** = **$\mathbf{r_O}$Q$\boldsymbol{\tau}$** for COD oxidation and heterotrophic biomass decay producing additional oxygen demand

$$\mathbf{RO = Q(S_{SO} - S_S)(1 - Y_{obs})\ (mg/day)}$$

NITROGEN: Two processes for removing ammonia:

1. Heterotrophic net growth requirement for nitrogen = X_{Ncells} is also a function of Θ:

$$\mathbf{X_{Ncells}} = i_{NXB} Y_{Hobs}\ (S_{SO} - S_S) \text{ mg-N/l}$$

$$\mathbf{X_{Ncells}} = i_{NXB}\left(\frac{Y_H(1 + f_D b_H \Theta)}{(1 + b_H \Theta)}\right) * (S_{SO} - S_S) \text{ mg-N/l}$$

Ammonia nitrogen uptake rate in cell synthesis, $\mathbf{r_{NH}}$:

$$\mathbf{r_{NH}} = \frac{X_{Ncells}}{\tau} = \frac{i_{NXB} Y_{Hobs}(S_{SO} - S_S)}{\tau} \quad \text{(mgN/l/d)}$$

2. **Nitrification (ammonia substrate is rate-determining component)**

$$\mu_A = \frac{\hat{\mu}_A S_{NH}}{(K_{NH} + S_{NH})}$$

where μ_A is the autotrophic growth rate, S_{NH} is ammonia nitrogen concentration, and K_{NH} is the half-saturation constant for nitrifying bacteria for ammonia nitrogen.

And

$$S_{NH} = \frac{K_{NH}\left(\frac{1}{\Theta} + b_A\right)}{\hat{\mu}_A - \left(\frac{1}{\Theta} + b_A\right)}$$

where b_A is the decay coefficient for nitrifying bacteria (autotrophs).

And

$$\mathbf{X_{BA}} = \left(\frac{\Theta}{\tau}\right)\frac{Y_A\left(S^*_{NHO} - \left(\frac{K_{NH}(1 + b_A\Theta)}{\hat{\mu}_A\Theta - (1 + b_A\Theta)}\right)\right)}{(1 + b_A\Theta)}$$

Where $\mathbf{S^*_{NHO} = S_{NHO} - NR}$ and S_{NHO} = influent ammonia nitrogen.

The rate of oxygen consumption for nitrification, r_{ON}:

$$\mathbf{r_{ON}} = \left(\frac{4.57}{\tau}\right)\left[(S_{NHO} - S_{NH}) - i_{NXB}Y_{Hobs}(S_{SO} - S_S)\right] \qquad \textbf{(VII)}$$

Now calculate the total rate of oxygen consumption for both COD oxidation (growth plus decay) and nitrification, r_{OT} **by combining VIb and VII:**

$$\mathbf{r_{OT}} = \left(\frac{1}{\tau}\right)\left[(S_{SO} - S_S)(1 - Y_{Hobs}) + 4.57\,\left((S_{NHO} - S_{NH}) - i_{NXB}Y_{Hobs}(S_{SO} - S_S)\right)\right]$$