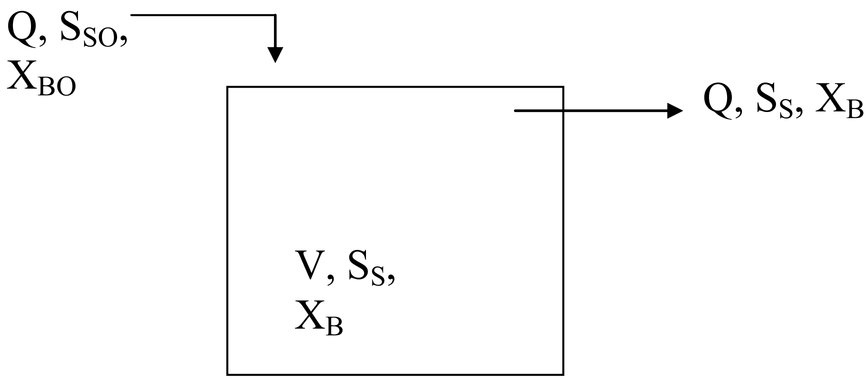
CSTR with COD consumption, cell growth and decay



$Q = 10{,}000\ m^3/d$
$S_{SO} = 400\ g/m^3$ COD
$X_{BO} = 0$
$Y = 0.6$ g-cell COD/g-COD
$\hat{\mu} = 1\ d^{-1}$
$b = 0.1\ d^{-1}$
$K_S = 50\ g/m^3$ COD

Analysis

Steady state mass balance on cells

$$Vr_{XB} = QX_B$$

$$r_{XB} = (\mu - b)X_B$$

$$(\mu - b)X_B = X_B/\tau$$

$$1/\tau = \mu - b \qquad 1$$

$$\mu = \hat{\mu}\frac{S_S}{K_S + S_S}$$

$$\frac{1}{\tau} = \hat{\mu}\frac{S_S}{K_S + S_S} - b \qquad 2$$

rearranging EQUATION 2:

$$S_S = \frac{K_S(1 + b\tau)}{\hat{\mu}\tau - (1 + b\tau)} \qquad 3$$

note that mass balance on cells produces relation for substrate COD in terms of kinetic and reactor parameters

also, S_S is not dependent on S_{SO}

other relations:

τ_{min} is detention time associated with cell washout, at washout, no cells and no COD consumption, $S_S \rightarrow S_{SO}$

$$S_{SO} = \frac{K_S(1 + b\tau_{\min})}{\hat{\mu}\tau_{\min} - (1 + b\tau_{\min})}$$

rearranging

$$\tau_{\min} = \frac{K_S + S_{SO}}{\hat{\mu}S_{SO} - b(K_S + S_{SO})} \qquad 4$$

S_{Smin} indicates the minimum substrate concentration which will sustain cell growth greater than loss from decay, corresponds to maximum substrate removal, the maximum detention time, and the minimum growth rate

$$\frac{1}{\tau_{\max}} = \mu_{\min} - b \rightarrow o$$

and

$$\mu_{\min} = b$$

and

$$\frac{\hat{\mu}(S_{S\min})}{(K_S + S_{S\min})} = b$$

rearranging

$$S_{S\min} = \frac{K_S b}{(\hat{\mu} - b)} \qquad 5$$

steady state mass balance on substrate

$$QS_{SO} - QS_S + Vr_S = 0$$

$$S_{SO} - S_S + \tau r_s = 0$$

$$-r_S = \frac{(S_{SO} - S_S)}{\tau}$$

substituting: $r_S = \frac{-\mu X_B}{Y}$ into mass balance

$$X_B = \frac{Y(S_{SO} - S_S)}{\mu\tau}$$

from equation 1: $1/\tau = \mu - b$ and $\mu = 1/\tau + b$

$$X_B = \frac{Y(S_{SO} - S_S)}{\tau(\frac{1}{\tau} + b)} = \frac{Y(S_{SO} - S_S)}{(1 + b\tau)} \qquad 6$$

for this system, mass balance on substrate produces relation for cell concentration

where

$$S_S = \frac{K_S(1 + b\tau)}{\hat{\mu}\tau - (1 + b\tau)}$$

Cell growth and substrate consumption are coupled.