IDEAL PLUG FLOW REACTOR

Characteristics of ideal plug flow

- PERFECT MIXING IN THE RADIAL DIMENSION (UNIFORM CROSS SECTION CONCENTRATION)

- NO MIXING IN THE AXIAL DIRECTION, OR NO AXIAL DISPERSION (SEGREGATED FLOW)

TRACER PULSE INPUT AT $t = 0$ TRANSLATED TO EQUAL PULSE OUTPUT AT $t = \tau$, $\tau = L/v$ ($L =$ PFR length, $v =$ average velocity)

COMPARE WITH CSTR RESPONSE TO TRACER PULSE DISPERSION
In an ideal PFR, concentration is a function of both distance along the flow path, $x$, and time, $t$:

$$ C = C(x,t) $$

For a mass balance on a reacting compound, take mass balance on differential axial element with uniform reaction potential (concentration), where

- $dV$ = differential volume
- $A$ = cross sectional area
- $dx$ = differential distance

and

$$ dV = Adx $$

Mass balance over differential element on a reactant, $C$

$$ \text{In} = QCx $$
$$ \text{Out} = QCx + dx $$
$$ \text{Generation} = dVr_C = Adxr_C $$

Accumulation = $dV \frac{\delta Cx}{\delta t} = Adx \frac{\delta Cx}{\delta t}$

$$ QCx - QCx + dx + dVr_C = dV \frac{\delta Cx}{\delta t} $$

$Cx + dx = Cx + dCx$

$$ Q(Cx - Cx - dCx) + dVr_C = dV \frac{\delta Cx}{\delta t} $$

$$ -Q \frac{\delta Cx}{\delta V} + r_c \frac{\delta Cx}{\delta t} = -\frac{\delta Cx}{\delta \left(\sqrt{\frac{V}{Q}}\right)} + r_c \quad \text{since } Q \text{ is constant} $$
\[ \delta(V/Q) = \delta \tau \]

\[ \frac{\delta C_x}{\delta \tau} + r_c = \frac{\delta C_x}{\delta t} \]

is the non-steady state ideal PFR mass balance for a reactant.

At steady state, \( \frac{\delta C_x}{\delta t} = 0 \)

And the ordinary differential can be substituted for the partial differential

\[ \frac{dC_x}{d\tau} = r_c \]

Comments

1. At steady-state, the concentration of a reactant at any single point along the PFR is constant at \( C_x \). Overall a stable concentration profile is obtained at steady state, with the concentration varying in space as the reaction occurs along the flow path.

2. In an ideal PFR, \( \tau \) is the absolute residence time for mass flowing through the reactor, not the average residence time as in a CSTR.

3. Compare ideal batch and ideal PFR mass balances:

   **Ideal PFR**: \( \frac{dC}{d\tau} = r_c \)

   **Ideal batch**: \( \frac{dC}{dt} = r_c \)

   Position in a PFR is equivalent to time in a batch reactor
For a 1st order reaction, \( r = -kC \), in a PFR at steady state

\[
\frac{dC}{d\tau} = -kC
\]

\[
\int_{C_0}^{C} \frac{dC}{C} = \int_{0}^{\tau} -kd\tau
\]

\[
C_L = C_0 \exp(-k\tau)
\]

**Ideal PFR, steady-state 1st order reaction profile**

Example:

Chlorine contact basin for disinfection
Where
\[ Q = 0.25 \text{ m}^3/\text{s} \]
\[ A = \text{channel cross section between baffles} = 18 \text{ m}^2 \]
\[ r_d = \text{rate of microorganism kill in presence of chlorine} = -k_d X \]
\[ X = \text{concentration of microorganisms at any point in contact reactor} \]
\[ X_0 = \text{influent concentration of microorganisms} = 10^6 \text{ E. coli/100 ml} \]
\[ k_d = 5 \text{ hr}^{-1} \]
\[ r_c = \text{rate of chlorine decay (from microorganism Cl-demand)} = -k_c X \]
\[ k_c = 10^{-5} \text{ (mg-chlorine/L)(#/100mL)}^{-1} \text{hr}^{-1} \]

2 rate expressions, 2 constituents, 2 coupled mass balances

find:
1. reactor volume and flow path length, L, such that \( X_L < 10^3 \text{ cells/100 ml} \)
2. chlorine concentration which must be added to insure that there is detectable chlorine at PFR exit (detection level = \( C_L = 0.05 \text{ mg/L} \))

1. Steady-state mass balance on cells
\[
X_L = X_0 \exp(-k_d \tau) \\
\tau = (1/k_d) \ln(X_0/X_L) = (1/5)(\text{hr}) \ln(10^6/10^3) = 1.4 \text{ hr} \\
V = Q\tau = 0.25 \text{ m}^3/\text{s} \times 3600 \text{ s/hr} \times 1.4 \text{ hr} = 1260 \text{ m}^3 \\
L = V/A = 1260 \text{ m}^3/18 \text{ m}^2 = 70 \text{ m} \\
3. Steady state mass balance on chlorine
\[
\frac{dC_c}{d\tau} = -k_cX = -k_cX_o \exp(-k_d\tau)
\]

\[
\int_{C_{co}}^{C_c} dC_c = -k_cX_o \int_{0}^{\tau} \exp(-k_d\tau) d\tau
\]

\[
C_L = C_{co} - \frac{(k_cX_o)}{k_d} + \frac{k_cX_o \exp(-k_d\tau)}{k_d}
\]

\[
C_L = C_{co} - \frac{(k_cX_o)}{k_d} (1 - \exp(-k_d\tau))
\]

\[C_{co} = 0.05 + (10^{-5}(10^6)/5)(1-\exp(-5(1.4))) = 2.05 \text{ mg/L}\]