Kinetics of Decay

Stoichiometry for decay of viable cells producing debris and (eventually, soluble substrate:

Biomass-COD → f_DDebris-COD + $(1-f_D)S_S$

In usual notation:

$$(-1)X_B + (1-f_D)S_S + f_D X_D = 0 \qquad (1)$$

Where:
X_B = biomass concentration (mg-COD/L)
X_D = debris concentration (mg-COD/L)
S_S = soluble COD (mg/L)
f_D = fraction of decayed biomass that is nonbiodegradable debris (g-debris COD formed/g-biomass-COD decayed)

Assumption:
Simplification: all biodegradable particulate COD $(1-f_D)$ becomes soluble substrate COD eventually (hydrolysis is 100% effective)

Reaction stoichiometry for nitrogen in decay assuming nitrogen from decaying cells either becomes debris or soluble ammonia:

$$i_{NXB}\text{Biomass-COD} \rightarrow i_{NXD}f_D\text{debris-COD} + (i_{NXB} - i_{NXD}f_D)NH_4\text{-N}$$

$$(-1)i_{NXB}\, X_B + i_{NXD}f_D\, X_D + (i_{NXB} - i_{NXD}f_D)NH_4\text{-N} = 0 \qquad (2)$$

where:

i_{NXB} = 14 g-N/113 g-cells/1.42 g-COD/g-cells = 0.087 g-N/g-cell COD
i_{NXD} < (≈ 0.6) since debris are thought to contain less N than cytoplasm

Decay rate expressions:

Define biomass decay coefficient, b, for a first-order rate expression:

$$r_{XB} = -bX_B$$

where
r_{XB} = rate of decay of biomass (mg-COD/L-hr)
X_B = biomass concentration (mg-COD/L)
b = decay coefficient (hr^{-1})

$$\frac{r_{XB}}{-1} = \frac{r_{XD}}{f_D} = bX_B \qquad (3)$$

where r_{XD} = rate of formation of debris-COD (mg-COD/L/hr)

from (3):

$$r_{XD} = f_D b X_B$$

assuming all biodegradable particulate COD from decay becomes soluble COD:

$$\frac{r_{XB}}{-1} = \frac{r_{XD}}{f_D} = \frac{r_S}{(1-f_D)} = \frac{r_{SNH}}{(i_{NXB} - f_D i_{NXD})} = bX_B \qquad (3a)$$

Note that the general rate equation relationship is the same regardless of whether the organisms are heterotrophs or autotrophs.

A difference between the decay rate and growth rate expressions is that b is assumed to be constant for all process conditions (unlike the growth rate coefficient, μ, which is a function of substrate and other nutrient concentrations).

Hydrolysis

Modify (1):

$$(-1)X_B - (1-f_D)X_S + f_D X_D = 0 \qquad (1a)$$

where X_S = biodegradable particulate matter (mg-COD/L)

$$(-1)i_{NXB} X_B + i_{NXD} f_D X_D + (i_{NXB} - i_{NXD} f_D) X_{NS} = 0 \qquad (2a)$$

where X_{NS} = particulate organic nitrogen

Then:

$$\frac{r_{XB}}{-1} = \frac{r_{XD}}{f_D} = \frac{r_{XS}}{(1-f_D)} = \frac{r_{XNS}}{(i_{NXB} - f_D i_{NXD})} = bX_B \qquad (3b)$$

and

$$(-1)X_S + S_S = 0 \qquad (4)$$

For the hydrolysis rate, r_H:

$$r_H = k_H X_{BH} \frac{X_S / X_{BH}}{(K_X + (X_S / X_{BH}))} = \frac{r_{XS}}{-1} = \frac{r_S}{1} = \frac{r_{XNS}}{-(\frac{X_{NS}}{X_S})} = \frac{r_{SNH}}{(\frac{X_{NS}}{X_S})} \qquad (5)$$

where:
k_H is a pseudo-first-order rate coefficient (hr^{-1})
X_{BH} is heterotrophic biomass (mg-COD/L)
K_X = the half saturation coefficient for the switching function for the concentration of hydrolysis enzyme substrate (mg-particulate-COD/mg-heterotrophic biomass-COD)

note that k_H, like μ, is a function of the substrate for hydrolysis enzymes, which is particulate COD, X_S. Actually, the hydrolysis rate has been found to be a function of the ratio of the fraction of particulate COD normalized by the heterotrophic biomass (autotrophs do not make appreciable hydrolysis enzymes for particulate organics), as well as the biomass concentration.

Also note that with hydrolysis, another compartment, X_S, is added to the COD degradation process reactions.

DECAY CYCLE