Kinetics of growth

It is often convenient to state stoichiometric expressions with cells as a reference, since many rates are a function of growth processes.

Notation:

In general, S designates soluble components, X, particulate.

S_S = soluble COD
S_O = dissolved oxygen
S_{NH} = ammonia nitrogen (NH_4-N)
S_{NO} = nitrate nitrogen (NO_3-N)
S_{NS} = soluble organic nitrogen
etc.

X_{BH} = heterotrophic viable cells
X_{BA} = autotrophic viable cells
X_S = biodegradable particulate organic matter
X_D = debris organic matter (nonbiodegradable)
X_{NS} = particulate organic nitrogen (biodegradable)
etc.

Reaction: Heterotrophic growth, reference is viable bacteria cells (as COD)

$$-\frac{1}{Y_H}S_S - \frac{(1-Y_H)}{Y_H}S_O + X_{BH} = 0$$

since nitrogen uptake in synthesis is growth-determined, it can "come along for the ride:"

$$-\frac{1}{Y_H}S_S - \frac{(1-Y_H)}{Y_H}S_O - i_{NXB}S_{NH} + X_{BH} = 0$$

associated rate relationship:

$$\frac{r_S}{(\frac{-1}{Y_H})} = \frac{r_O}{(\frac{-(1-Y_H)}{Y_H})} = r_{BH} = \text{overall growth reaction rate, } r_j \text{ for } j = 1 \rightarrow n$$

where r_S = rate of COD substrate consumption, r_O = rate of oxygen consumption and r_{BH} = heterotrophic growth rate.

If r_{BH} is defined along with values for Y_H and i_{NXB}, then all other rates are defined as above.

Bacteria grow by fission, so the population growth rate is a function of the number of viable cells. Define the growth rate for any bacteria, r_B

$$r_B = \mu X_B$$

where

μ = specific growth rate (hr^{-1}, d^{-1}, etc.)
X_B = viable cell concentration (mg-COD/L = g-COD/m^3)

Note that units of rate expressions for wastewater process analysis are always concentration/time. If m is constant, this is first order kinetics:

$$r_B = \mu X_B^{(1)}$$

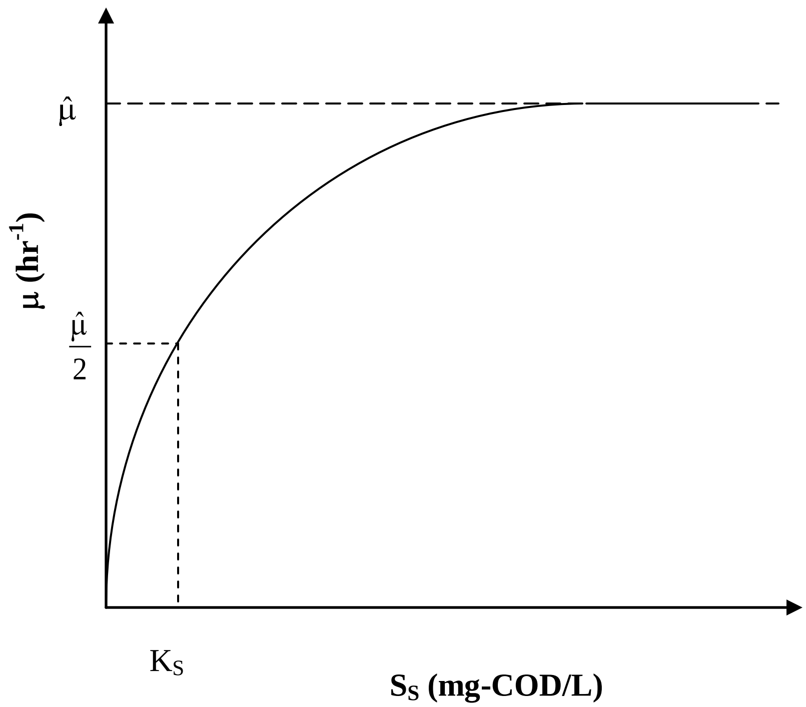
Note that yield links substrate removal with growth:

$$r_S = \frac{-\mu X_B}{Y}$$

In ~1948 French microbiologist Jacques Monod found that in continuous cultures of bacteria m was not constant. In fact it varied in non-linear proportion to a rate limiting substrate (often the electron donor).

$$\mu = \hat{\mu}\frac{S_S}{(K_S + S_S)}$$

where $\hat{\mu}$ = the maximum specific growth rate and K_S = the half-saturation (or half-velocity) constant.



Note that Monod kinetic expression developed for single culture growing on single limiting substrate. We will apply to mixed populations (e.g., heterotrophs) growing on compound mixture (COD)

Regions of Monod growth

First order:

If $S_S << K_S$, denominator → K_S and

$$\mu \cong \frac{\hat{\mu} S_S}{(K_S)}$$

growth rate is linearly proportional to rate limiting substrate concentration

Zero-order:

If $S_S >> K_S$, denominator → S_S and

$$\mu \cong \hat{\mu}$$

Complimentary Nutrients:

COD, N, P, O_2, etc.

Allowing simultaneous substrate limitation of growth by two or more components.

Interactive model uses a series of “switching functions” where 0 < SF < 1

$$\mu = \hat{\mu}\frac{S_S}{(K_S + S_S)}\frac{S_O}{(K_O + S_O)}\frac{S_{NH}}{(K_{NH} + S_{NH})}$$

where

$$0 < \frac{S_i}{(K_i + S_i)} < 1$$

Relation between a nutrient’s concentration and the half-saturation constant is key to determining value of each switching function.

Interactive model provides a smooth function for the specific growth rate which makes computational modeling easier.