Homework 3 solutions

1. CSTR, no decay, $r_x = \mu X_B$

mb cells, $X_0 = 0$

$$Q X_B = V r_g = \mu X_B V$$

$$\frac{1}{\tau} = \mu = \frac{\hat{\mu} S_S}{K_S + S_S}$$

$$K_S + S_S = \tau \hat{\mu} S_S$$

$$S_S(\hat{\mu}\tau - 1) = K_S$$

$$S_S = \frac{K_S}{\hat{\mu}\tau - 1}$$

$$S_S = \frac{K_S(1 + b\tau)}{\hat{\mu}\tau - (1 + b\tau)}$$

@ τ_{min}, $S_S \rightarrow S_{S0}$

$$S_{S0} = \frac{K_S}{\hat{\mu}\tau_{min} - 1}$$

$$\hat{\mu}\tau_{min} - 1 = \frac{K_S}{S_{S0}}$$

$$\tau_{min} = \frac{1}{\hat{\mu}}\left(\frac{K_S}{S_{S0}} + 1\right) = \frac{K_S + S_{S0}}{\hat{\mu} S_{S0}} = \frac{50 + 250}{2(250)}$$

a) $\tau_{min} = 0.6$ days @ washout

b) $S_S = 0.1 S_{S0} = 25$ mg/L

$$\tau = \frac{K_S + S_S}{\hat{\mu} S_S} = \frac{50 + 25}{2(25)} = 1.5 \text{ d}$$

$$V = Q\tau = 10^4 \text{ m}^3/\text{d} \, (1.5 \text{ d}) = 15{,}000 \text{ m}^3$$

1. c, mass balance S:

$$S_{S0} - S_S + \tau r_S = 0$$

$$-r_S = \frac{S_{S0} - S_S}{\tau} = +\frac{\mu X_B}{Y}$$

$$X_B = \frac{Y(S_{S0} - S_S)}{\mu \tau}$$

$$\mu = \frac{1}{\tau}$$

$$X_B = Y(S_{S0} - S_S) = (250 - 25)\,0.6 = 135 \frac{mg\,COD}{L}$$

AND $X_B' = 135 \frac{mg\,COD}{L} \frac{1\,g\,cells}{1.42\,g\,COD} = 95 \frac{mg\,cells}{L}$

d, PFR

$$\frac{dS}{d\tau} = r_S = -\frac{\hat{\mu} X}{Y}\left(\frac{S_S}{K_S + S_S}\right)$$

$$\int_{S_{S0}}^{S_S} \left(\frac{K_S}{S_S} + 1\right) dS = -\frac{\hat{\mu}}{Y} X \int_0^{\tau} d\tau$$

$$K_S \ln S_S + S_S \Big|_{S_{S0}}^{S_S} = -\frac{\hat{\mu} X \tau}{Y}$$

$$K_S \ln \frac{S_S}{S_{S0}} + S_S - S_{S0} = -\frac{\hat{\mu} X \tau}{Y}$$

$$\tau = \frac{Y}{\hat{\mu} X}\left[K_S \ln\left(\frac{S_{S0}}{S_S}\right) + (S_{S0} - S_S)\right]$$

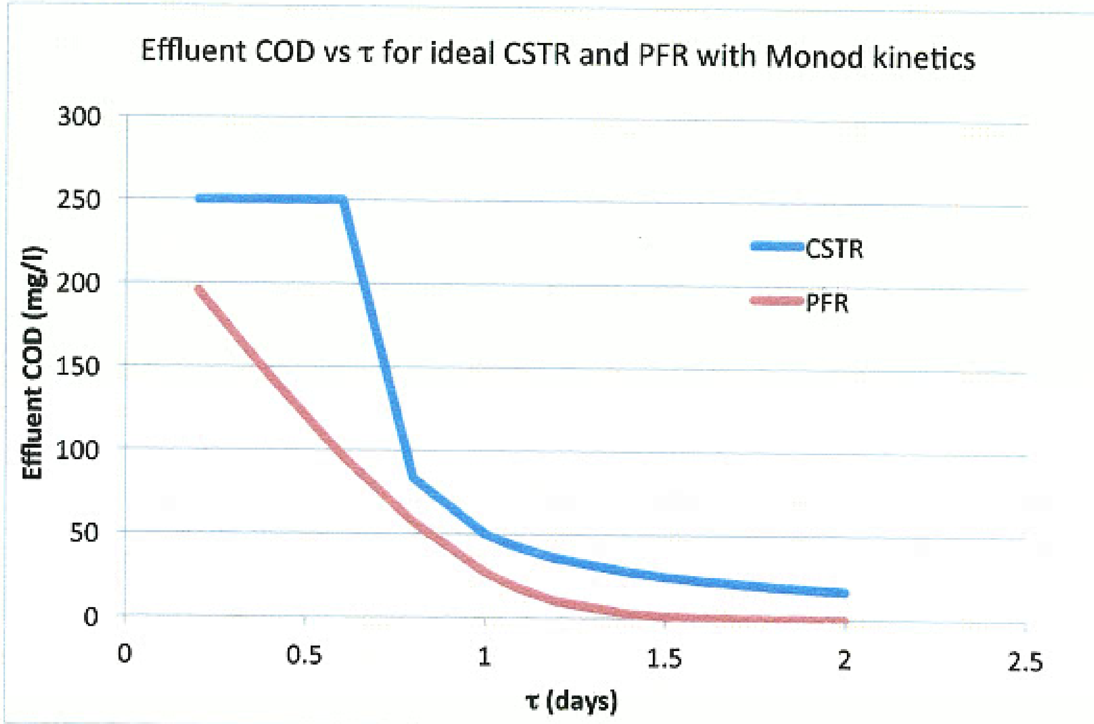
$$0 = \frac{Y}{\hat{\mu} X}\left(\quad\right) - \tau$$

CVEN 5534 Assignment 3 solution

Problem 1d. PFR with Monod kinetics compared with CSTR

τ (d-1)	Ss(CSTR) mg/l COD	Ss(PFR) mg/l COD	
0			
	CSTR	PFR	
0.2	250	195.6	4.6687E-05
0.4	250	144.2	6.6925E-06
0.6	250.0	97.2	0.00014206
0.8	83.3	57.0	0.00059994
1	50.0	27.3	0.00070216
1.02	48.1	25.0	0.00048448
1.1	41.7	17.2	5.0881E-05
1.2	35.7	10.2	8.8179E-05
1.4	27.8	3.1	0.00066243
1.5	25.0	1.6	0.00052043
1.6	22.7	0.8	0.00055712
1.8	19.2	0.2	-0.0005136
2	16.7	0.1	-0.0002967

c.



d. tau for 90% COD removal in ideal PFR = 1.02d.
VPFR = 10,000m3/d*1.02 d = 10,200 m3

VCSTR for 90% removal = 15,000 m3, 50% larger than PFR

Problem 2

Influent COD = 300 mg/l

$r_S = -k*S$, $k = 5\ d^{-1}$

Values for τ (day)

COD Removal	CSTR	2-tank Cascade	4-tank cascade	PFR
0.85	1.13	0.63	0.49	0.38
0.99	19.80	3.60	1.73	0.92

formulas

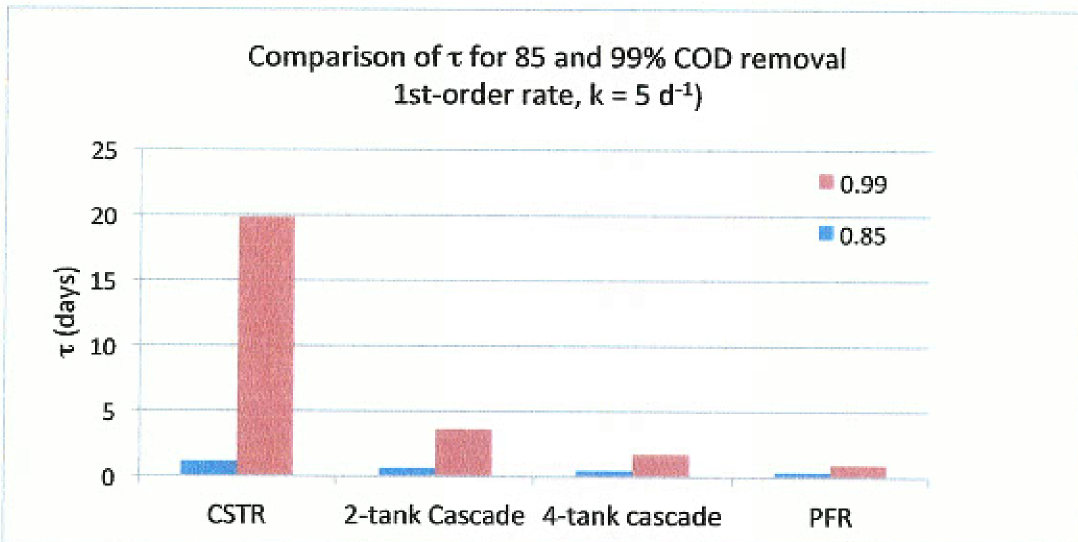
CSTR and Cascade

nτ = (n/k)((So/S)^(1/n)-1)

n = 1, 2, and 4

PFR

τ = -(1/k)*ln(S/So)



$\frac{dS}{d\tau} = -kS$

$\frac{S_s}{S_{s0}} = e^{-k\theta}$

$\frac{C_n}{C_{A0}} = \frac{1}{(1+k\tau)^n}$

3. mb on cells, X st.st

$$-Q_W X + V r_g = 0 \qquad r_g = (\mu - b) X$$

$$Q_W = V(\mu - b)$$

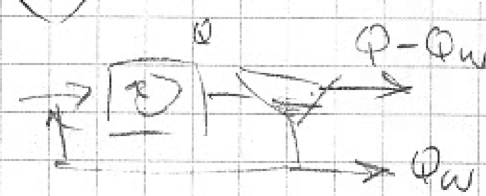
$$\frac{1}{\Theta} = \mu - b = \frac{\hat{\mu} S_S}{K_S + S_S} - b$$

$$\frac{\hat{\mu} S_S}{K_S + S_S} = \left(\frac{1}{\Theta} + b\right) = \left(\frac{1 + b\Theta}{\Theta}\right)$$

$$\Theta \hat{\mu} S_S = (K_S + S_S)(1 + b\Theta)$$

$$S_S(\Theta \hat{\mu} - (1 + b\Theta)) = K_S(1 + b\Theta)$$

$$S_S = \frac{K_S(1 + b\Theta)}{(\Theta \hat{\mu} - (1 + b\Theta))} \qquad (i)$$



ii mb on COD-S

$$Q S_0 - (Q - Q_W) S_S - Q_W S_S + V r_S = 0$$

$$Q(S_{S0} - S_S) + V r_S = 0$$

$$-r_S = \frac{S_{S0} - S_S}{\tau} = \mu \frac{X}{Y}$$

$$X = Y\frac{(S_{S0} - S_S)}{\mu \tau}, \qquad \mu = \frac{1}{\Theta} + b$$

$$X = \frac{Y(S_{S0} - S_S)}{\tau\left(\frac{1}{\Theta} + b\right)} = \left(\frac{\Theta}{\tau}\right)\frac{Y(S_{S0} - S_S)}{(1 + b\Theta)}$$

b. $Q_W = 300\ m^3/d$

from (i) $$25 = \frac{50(1+0.1\theta)}{(2\theta - (1+0.1\theta))}$$

$$\theta \hat{\mu} - (1+b\theta) = \frac{K_S}{S_S}(1+b\theta)$$

$$\theta\hat{\mu} - b\theta - \left(\frac{K_S}{S}\right)b\theta = \frac{K_S}{S} + 1$$

$$\theta\left(\hat{\mu} - b - \frac{K_S}{S}b\right) = \frac{K_S + S}{S}$$

$$\theta = \frac{K_S + S_S}{S_S\left(\hat{\mu} - b\left(1 + \frac{K_S}{S_S}\right)\right)}$$

$$\theta(S_S = 25) = \frac{50+25}{\left(25\left(2 - 0.1\left(\frac{50+25}{25}\right)\right)\right)}$$

$$\theta = 1.76\ d$$

$$V = Q_W\theta = 300\frac{m^3}{d}(1.76\ d) = \underline{528\ m^3}$$

$$\tau = \frac{528\ m^3}{10^4\ m^3/d} = 0.053\ d \text{ or } 1.3 \text{ hours}$$

$$X = \frac{0.6(250-25)}{(1+0.1(1.76))}\left(\frac{1.76}{0.053}\right) = 3800 \frac{mg\ COD_X}{L}$$

$$= 2{,}700 \frac{mg\ VSS}{L}$$

c.

	V (90%) (m^3)	X (90%) (mg/L VSS)	X (90%) mg/L COD
CSTR no recycle	15,000	95	135
CSTR recycle	528	2700	3800

$$\frac{V_{recycle}}{V_{norecycle}} = 0.035 \qquad \frac{X_{recycle}}{X_{norecycle}} = 28.$$