Behavior and Design of Steel Anchor Systems in Cementitious Concrete

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1.0 Scope

There are many facets to the concept of connecting structural elements to concrete with steel anchors. The designer must consider anchor type, applied forces, condition of the concrete, and proximity to other anchors.

There are four major types of anchors, as seen in Figure 1, including Headed (a), Keying (b), Bonding (c), and Friction (d). Keying and headed anchors undercut the concrete to provide mechanical resistance, bonding anchors use a chemical adhesive to provide resistance, and friction anchors generally expand near the tip to provide frictional resistance. Headed anchors are cast in situ, while the other three types are applied to hardened concrete. The force being applied to the fasteners are characterized as tension, shear, or both. And the concrete may be cracked or uncracked at the fastener location.

Meanwhile, the researcher (and code-writer) must consider the failure mode for each of the above cases, and its associated mechanics and limitations.

Each of the above anchor types, in tension and shear, and in cracked and uncracked concrete, are addressed in the ACI Code, and have been examined using fracture mechanics based 3-D finite element analysis [1,2,3]. In an attempt to limit the complexity of the subject, this paper will focus primarily on headed anchor studs, both

![Figure 1: Types of Anchorages](image)

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Figure 1: Types of Anchorages
single and in groups, subjected to tension only in uncracked concrete. This, and the corresponding shear cases, are the predominant conditions in new construction, where large numbers of connections may be required, requiring economically optimized (i.e. not over-conservative) connections.

2.0 History

The current accepted code, referenced by the 2003 International Building Code (IBC), for the design of headed anchor studs is Appendix D of ACI 318-02. This document describes the design procedure, which is covered in section 3.0 of this paper and references materials that support the method.

There was a flurry of activity in the mid 90’s, which attempted to (and eventually succeeded in) replacing the then current ACI 349 code (1985), which was based on a method developed in the mid 70’s. Eligehausen, who led the charge, conducted most of the experimental research in Germany, and based the new method, called the Concrete Capacity Design (CCD) [4], on the European version (kappa method). One of the few very vocal dissenting voices was Cannon [5], a practicing engineer in Tennessee.

The CCD approach was primarily based on extensive testing, unavailable when the previous code was written [6], with consideration given to the principles of fracture mechanics.

The interesting aspect of the CCD method is that its development has been somewhat backwards of the normal code development. Instead of being an analytic theory verified by increasingly complex and encompassing experimental results (like plastic design for example), it has experienced the opposite. As finite-element simulations of materials subjected to elasto-plastic fracture mechanics have improved (along with a plunge in the cost of computing power), the CCD method has been verified by multiple researchers, including Fox [7] at the University of Colorado. The exception, discussed in Section 4.0, is for cases difficult to test, such as deep anchors in large masses of concrete or anchors with relatively large heads.

3.0 ACI Design Method (CCD)

3.1 Single Anchor

The ACI Code [8] specifies that the nominal breakout strength of an anchor is:

\[
N_{cb} = \frac{A_N}{A_{NO}} \cdot \Psi_2 \Psi_3 N_p
\]  

(3.1)

where:
AN = Projected area of the failure surface
ANO= Projected area of the failure surface of a single anchor remote from edges
Ψ₂ = Modification for edge effects
Ψ₃ = Modification for Cracked Concrete
N_b = basic concrete breakout strength for a single anchor in cracked concrete:

\[ N_b = 24\sqrt{f'_c h_{ef}}^{1.5} \]  

where:

\[ h_{ef} = \text{effective anchor embedment depth (in)} \]
\[ f'_c = \text{Concrete compressive strength (psi)} \]

At first glance, this equation does not seem to be based on anything except empirical data. However, a review of the Code background paper [4] reveals differently. That paper defines the breakout strength as:

\[ N_b = k_{mc}\sqrt{f'_c h_{ef}}^{1.5} \]  

where

\[ k_{mc} = k_1 \cdot k_2 \cdot k_3 \]  

\[ N_b = k_1\sqrt{f'_c} \cdot k_2 h_{ef}^{2} \cdot k_3 \cdot 1 \sqrt{h_{ef}} \]  

The first coefficient and term \( k_1\sqrt{f'_c} \) represents the nominal tensile strength of the concrete. The second coefficient and term \( k_2 h_{ef}^{2} \) is the failure area, shaped like a pyramid, and discussed later (figure 3). And the last term \( k_3 \frac{1}{\sqrt{h_{ef}}} \) incorporates size effect.

The authors further explain that since the strain gradient for concrete in fastenings is very large, the size effect is maximized, and thus essentially behaves in a Linear Elastic Fracture Mechanics (LEFM) fashion. This in turn means that the stress (and the final force) is proportional to the square root of the size, here represented by the effective depth. Thus, very large embedded anchors will have a much smaller tensile stress.

Fuchs et al. go on to compare the CCD method with, ACI 349 and a large array of test data. The older code uses a similar method of assuming a breakout shape (in
In this case, a cone) and multiplying the surface area by a tensile stress capacity. This curve was fitted to available data back in the 1970’s such that it formed a lower bound of the test data. However, there was no fracture mechanics component to account for size effect. The results of this can be seen in Figure 2. As the effective depth becomes larger, ACI 349 becomes unconservative.

![Figure 2: Concrete Breakout Load for Cast-In Headed Studs.](image)

The authors of the Code background paper found that $k_{nc} = 40$ provided good agreement with the data. It seems that the ACI committee felt this to be too risky, and they lowered the number to $k_{nc} = 30$ for headed anchor studs in uncracked concrete, along with a strength reduction factor, $\phi$, generally around 0.7, depending on the anchor type and load condition.

Once $N_b$ has been found, the concrete breakout strength can be computed using equation 3.1. The first term of this equation ($A_N/A_{nc}$) infers that the strength of a fastener varies linearly with the ratio of its projected area to the area of a remote
fastener (unaffected by edges or other anchors). The area of a remote fastener is defined as:

\[ A_{NO} = 9h_{ef}^2 \] (3.6)

which comes from Figure 3. The angle of 35 degrees, which is the most important factor in determining \( A_{NO} \) is “understood”, and was not proven by any analytical means. This is discussed further in Section 4.0. This linear relationship has generally been found to be true, with the exception of edge effects and the influence of cracks, for which the modification factors (\( \Psi \)) are less than 1.0.

### 3.2 Multiple Anchors

The equation for the nominal concrete breakout strength of multiple anchors in a group is almost the same as that for a single anchor (Eq 3.1), with the addition of a modification factor for eccentrically loaded groups (\( \Psi_1 \)):

\[ Ncb = \frac{A_N}{A_{NO}} \cdot \Psi_1 \Psi_2 \Psi_3 N_b \] (3.7)

The surprising aspect about this equation is that there is no modification factor for closely spaced anchors. The reduced strength of a tightly spaced group of anchors is entirely captured in the linear relationship between \( A_N \) and \( A_{NO} \). Figure 4 shows an example of how \( A_N \) is calculated.
While the fracture mechanics based approach yielded somewhat better results for case of a single anchor with large effective depths, as discussed above, the CCD method was found to predict tensile capacities far better than ACI 349 for anchor groups. This can be seen in Figure 5.

**Figure 4: Nominal Area ($A_N$) of a Group of Anchors**

$$A_N = (c_1 + s_1 + 1.5h_{ef})(c_2 + s_2 + 1.5h_{ef})$$
if $c_1$ and $c_2 < 1.5h_{ef}$
and $s_1$ and $s_2 < 3h_{ef}$

**Figure 5: Actual and Predicted Loads for Cast-In Headed Studs as a function of distance between outermost anchors**

- $f_c = 25$ N/mm$^2$ (3.6 ksi)
- $h_{ef} = 185$ mm (7.3 in)
- $n$ Symbol:
  - 1 +
  - 4 ○
  - 16 △
  - 36 □
  - Mean value of $c$ series

$N_a$ [kN] vs. $s_1$ [mm] or [in] with $h_{ef}$ as a parameter.
4.0 FEM Simulations

There have been several independent researchers that have undertaken the 3-D finite element analysis of a single anchor [3,9,10], including the author of the original CCD proposal [1,11]. Much of this work was done with the nuclear industry in mind, which uses these types of connections in critical spots more than other industries. Early attempts at modeling concrete breakout with fracture mechanics contained rigorous analysis with the attempt of finding a simplified design method. On example is presented below.

More recent research has focused on investigating the myriad of geometric and material conditions, the physical testing of which would be too costly and time consuming. This is done with the purpose expanding and revising the current code (CCD method). One such example is presented in the following sections.

4.1 Two-Domain Boundary Element Pull-out Test

Chahrour and Ohtsu [9] give an extremely in depth description of their analysis of different geometries using the Two-Domain Boundary Element. A Two-domain boundary element is a formulation of two elastic domains with a boundary around each, and shared in one area (Figure 6). The interface boundary maintains the compatibility condition and thus the traction and displacement can be related:

\[ u_i^j(Q) = u_i^j(Q) \]  \hspace{1cm} (4.1)
\[ t_i^j(Q) = -t_i^j(Q) \]  \hspace{1cm} (4.2)

These can be discretized along the boundary, and then numerically integrated over 6 gauss points. The relationship (now in matrix form) can then be converted into a standard finite element formulation to find the nodal forces and displacements.

Once the analysis is complete, the stress intensity factors, \( K_I \) and \( K_{II} \) at the crack tip are computed from the relative displacements of A and B (Smith’s one point formulae), as seen in Figure 7a. And then the direction of the crack propagation is determined from the maximum circumferential stress (Figure 7b). The continuum is then re-meshed and the process continued.
This method was used to analyze specimens such as the one shown in Figure 8. Using the results of the simulations, Chahrou and Ohtsu proposed the following formula for the capacity of anchor bolts:

\[ F_u = q \cdot b \left( E G_p \right)^{1/2} d^{1.5} \left( \frac{a}{d} \right)^{\gamma} \]  \hspace{1cm} (4.3)

Where:

\( F_u \) = pullout capacity  \\
\( b \) = specimen thickness  \\
\( d \) = diameter of bolt  \\
\( a \) = distance to support  \\
\( q \) and \( \gamma \) = constants depending on restraints (given in 9)

This equation was never adopted due to its complexity and lack of generality, but one can easily see that it is fracture mechanics based, and it illustrates that solutions to such complex problems can be proposed using only analytical tools.
4.2 3D FE Analysis of Anchor Bolts with Large Embedment Depths

Ozbolt et al. [11] conducted numerical testing to examine the effects of larger heads for cast in place studs, and for very long embed depths. The finite element code used is called MASA. The code uses the microplane model and a smeared crack approach. Steps were taken to relate the total energy consumption to the concrete fracture energy, thus make the meshing size independent.

Only one quarter of the anchor bolt is modeled due to symmetry (Figure 9). The results of multiple analyses using this method have been compared with the CC-method (same as the CCD...
method, metric units in this case). These results generally agrees with the conclusions drawn above, that the CCD method accurately predicts the tensile capacity of the anchors. In addition, the resulting FEM failure planes (figure 9) support the "assumption" that the failure cone is at 35 degrees. One discovery of this analysis, however, is that the CCD method is overly conservative for anchors with large heads. This can be seen in Figure 10 and Table 2.

![Figure 10: Relative Concrete Cone Resistance as a Function of the Embedment Depth](image)

As noted in Section 3, the CCD method is based on size effect and behaves in an LEFM manner. The authors give a fairly simple explanation for why the small anchor heads agree with CCD predictions, and large ones don't:

The reason why for fasteners with small anchor head size the size effect agrees well with the size effect prediction according to LEFM is
due to the fact that for all embedment depths the crack patterns at peak load are similar – the crack length is relatively small and approximately proportional to the embedment depth. The main assumption of LEFM...is fulfilled and therefore size effect is maximal. On the contrary, for fasteners with larger heads the crack pattern for different embedment depths is not proportional, This is the case for both the crack length at peak load, as well as for the shape of the failure cone. Consequently, the size effect on the concrete cone failure load is smaller.

And thus, larger headed anchor studs move up the size-effect curve, and are better described by Non-Linear Fracture Mechanics (NLFM). The authors also propose the following addition to the CCD method (metric):

\[ P_u = 15.5 \sqrt{f'_c} h_{cf}^{1.5} \] (4.4)

\[ P_u = 15.5 \cdot \gamma^{0.35} \cdot \sqrt{f'_c} h_{cf}^{1.5} \] (4.5)

where:

\[ \gamma = \frac{A}{A_0} ; A_0 = \frac{P_u}{15f'_c} \] (4.6)

and:

A= bearing area under the head of the anchor
A₀= bearing area under the head of the anchor such that for \( P_u \) from Eq 4.4, the ratio

\[ \frac{P_u}{A} = 15 \]

Ozbolt et al. claim that this modification accounts for the discrepancy. Note that Eq 4.4, valid for small heads is equal to Eq 3.3. This equation has yet to be adopted by an American code, and studs with larger heads are not addressed.

5.0 Further Study

One very important factor, though not directly related to fracture mechanics, to the strength of multiple anchor connections (such as column baseplates), is the relative stiffness of the plate that connects the anchorages. When a moment is applied to such a system, the connecting element can not be too stiff or too flexible. This will result in a lessening of the moment arm and an increase in the force on the anchors, possibly causing failure [12]. Thus, the designer must consider that the tensile capacity of the anchorages alone may not necessarily govern the connection. The relationship between
the anchorages, their connecting element, and the corresponding strengths and stiffnesses is an important one and has not been studied in depth.

Another area that has been as yet neglected is that of fatigue behavior of anchorages. With continuing sophistication of the models, and the current trend of investigating the capacity of anchorages subject to high temperatures, there is no doubt that this problem will be addressed by fracture mechanics based finite element simulation in the not too distant future.
References


8 Building Code Requirements for Structural Concrete (ACI-318-02). Appendix D.


