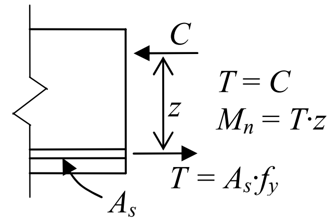
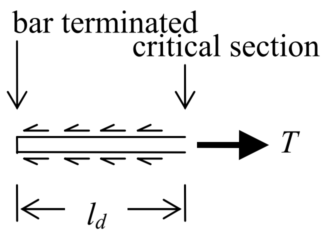
Anchorage and development of reinforcement



At the critical section for moment the tensile stresses in the tension bars are assumed to be f_y at the limit state moment M_n.

Thus, the tensile force in the a bar is $\boldsymbol{T} = \boldsymbol{A_s} \cdot \boldsymbol{f_y}$.

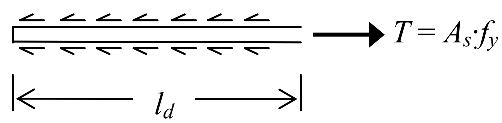


Tensile bars are typically terminated some where in the concrete. Thus a free body diagram shows that the bar force T must be transferred to the concrete surrounding the bar over the bar length l_d (as shown).

It should be clear that if the length l_d is very short (say a few inches) the bar will simply 'pull out' of the concrete before the full tensile force, $\boldsymbol{T} = \boldsymbol{A_s} \cdot \boldsymbol{f_y}$, can be developed.

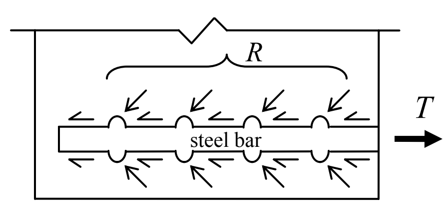
On the other hand, if the length l_d is quite long (say many feet) the full tensile force, $\boldsymbol{T} = \boldsymbol{A_s} \cdot \boldsymbol{f_y}$, most likely can be developed (assuming no other failure 'mode' occurs).

From the discussion above we can conclude that there must be a development length, l_d, which represents a minimum for full bar force development in tension.



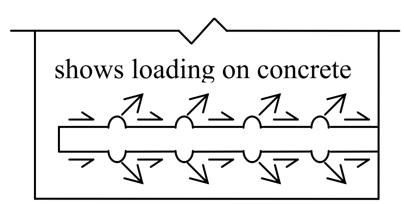
Equations for the required development length in tension have been determined by extensive testing. Unfortunately, many factors must be considered if the actual <u>minimum</u> l_d (with some factor of safety) is desired for a particular situation, bar size and bar spacing.

A brief summary of factors affecting l_d in tension



A deformed bar is held in place by forces and bond stresses applied to the perimeter of the bar.

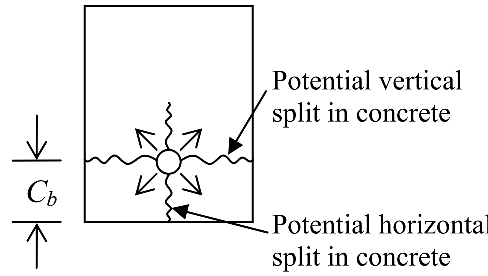
The deformations along the bar (lugs) hold the bar in place by a combination of 'bond stresses' along the perimeter area of the bar and forces developed at the lug-concrete interfaces. Thus, bar diameter (d_b), is an important factor controlling l_d.



d_b determines both area (A_b) and perimeter of bar as follows:

$$A_b = \frac{\pi d_b^{\,2}}{4}\text{, perimeter} = 2\pi db$$

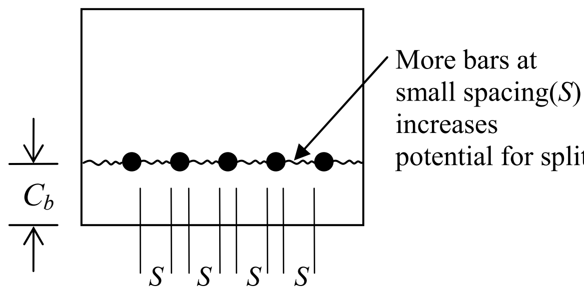
$$\frac{A_b}{Perimeter} = \frac{\pi d_b^{\,2}/4}{2\pi d_b} = \frac{d_b}{8} \cong \frac{T}{R}$$



Larger diameter bars require significantly longer l_d values.

The lugs in the bars (see figures) also produce splitting forces in the concrete. Splitting forces are resisted by concrete tensile strength ($f_t' \propto \sqrt{f_c'}$).

It is also clear that the actual maximum tensile load is proportional to f_y.

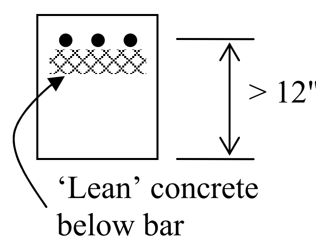


Thus fundamentally l_d depends on f_y, f_c', d_b, bar spacing (S) and coverage (c_b) to bar centerlines.

Other factors affecting l_d in tension

The most complex expression for l_d in tension also includes terms accounting for:

- Transverse reinforcement (stirrups tend to reduce splitting)
- Compressive loading (bearing at supports tends to reduce spitting)
- Light weight concrete (weaker in tension)
- Epoxy coated bars (bonding to concrete is reduced)
- Concrete cast with more than 12" below the bar to be developed (tends to have higher water/cement ratio).



When the stress which is actually needed to be developed (f_s) is less than f_y, l_d is reduced proportionally!

$$l_d' = l_d (f_s / f_y)$$

Simplified code formulas for development length in tension

For clear spacing of bars ≥ $2d_b$ and clear cover ≥ d_b; or clear spacing of bars ≥ d_b and clear cover ≥ d_b <u>and at least minimum code stirrups present</u>:

$l_d = \dfrac{f_y d_b}{25\sqrt{f_c'}}$	No. 6 and smaller bars	Basic l_d values but absolute l_d = 12"
$l_d = \dfrac{f_y d_b}{20\sqrt{f_c'}}$	No. 7 and larger bars	

If depth of concrete cast below bars is > 12" multiply basic l_d values by 1.30 (30% increase).

Note: No *Φ* factors apply to l_d computations!

Example of basic development length in tension (l_d)

Assume f_c' = 4,000 *psi*, f_y = 60,000 *psi*, and $f_s = f_y$ required.

Clear cover: $c_b \geq d_b$ and spacing S ≥ 2 d_b
or $c_b \geq d_b$ and at least minimum stirrups present.

l_d for #8 bar (d_b = 1.0")

$$l_d = \frac{60{,}000(1.0)}{20\sqrt{4{,}000}} = 47.4" = 3.95'$$

Note: generally basic $l_d = 47.4d_b$ for tension development if $f_s = f_y$.

Quick very accurate design aid:

l_d = (0.5)·(bar#) (gives l_d in feet)

The above aid gives l_d in feet for f_c' = 4 *ksi* and f_y = 60 *ksi*.

- If top bars with 12" concrete cast below: multiply by 1.3
- If $f_s < f_y$: multiply by f_s/f_y
- If bar size ≤ #6: multiply by 0.8 (if desired)

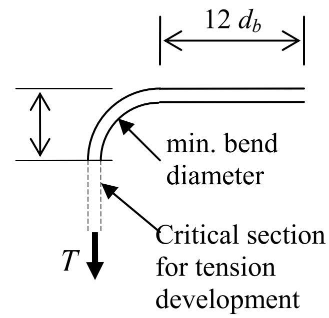
Second development length example

- #3 bar is to be developed for $f_s = 0.9\, f_y$.
- Assume that < 12" concrete is cast below the bar.
- f_c', f_y, c_b and S as in the previous example.

  l_d = (0.5)(3)(0.8)(0.9) = 1.08' = 13" > 12" (OK)

Standard ACI hooks for tension development

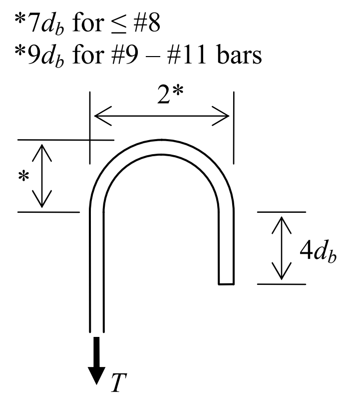
For main flexural steel bars a standard tension hook shall be:



A 90° bend plus 12 bar diameter extension.

Minimum bend diameter (inside of hook) is 6 d_b for ≤ No.8 bars and 8 d_b for #9 – #11 bars.

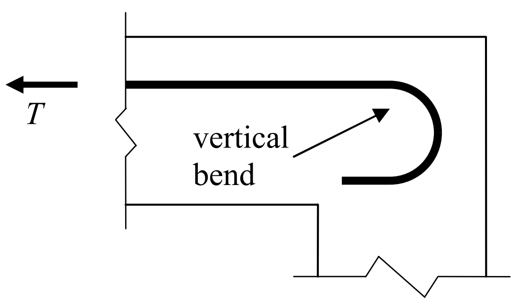
Or

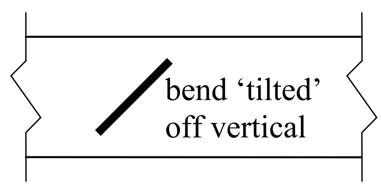


A 180° bend plus 4 bar diameter extension. Minimum bend diameter is as above.

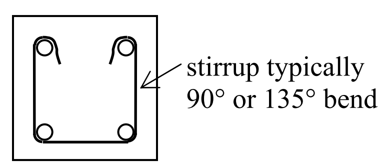
Most bends use the minimum bend diameter (standardized bending).

Note that if the 180° hook will not 'fit' within the depth a member (often true for slabs) the bend can be tilted off vertical as required.



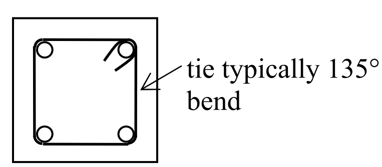


For stirrups and ties the bend diameters are reduced as compared to minimum bend diameters in main flexural bars.



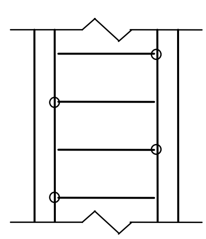
Typically these bars are #5 and smaller.

The inside diameter of the bend shall be ≥ 4 d_b. 4 d_b is generally assumed (standard).

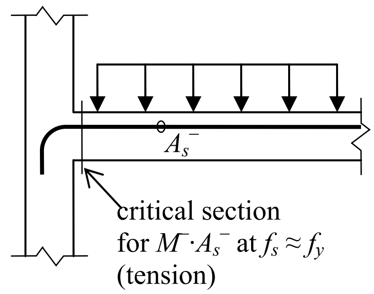


Typical stirrup and tie bends are either 90° or 135°.

Positions of column tie bends are alternated vertically.



Development of standard hooks in tension (l_{dh})



Hooks are used to develop tension reinforcement where inadequate length is available for developing such reinforcement as straight bar lengths.

A most common example is at critical sections for negative moment at exterior supports (see figure).

The straight bar development length beyond the face of the support will almost always be more than the column width available! Thus standard hooks (90° or 180°) are typically used at external supports for A_s^- steel.

The 'basic' tension development length for 'standard' hooks (no coating, normal weight concrete, no modifiers) per ACI code is:

$$l_{dh} = \frac{0.02 f_y d_b}{\sqrt{f_c'}} = \frac{f_y d_b}{50\sqrt{f_c'}}$$

Note that this is 2.5 times shorter than the basic length required for straight bars in tension.

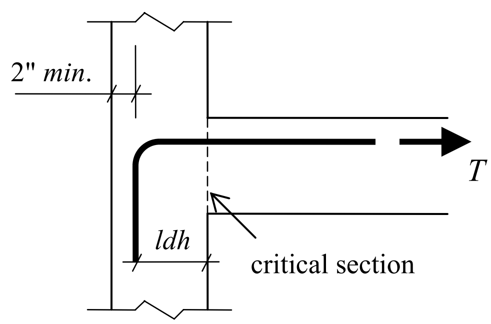
For f_c' = 4 *ksi* and f_y = 60 *ksi* a quick accurate 'design aid' for tension hook development in the 'basic' case is:

l_{dh} = (0.2)·(bar#) (give l_{db} in feet)

If f_s required < f_y: $l_{dh}' = (l_{dh})(f_s / f_y)$ (as for straight bars)

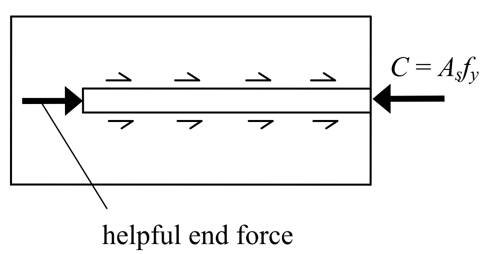
If ≥ 2" clear cover is provided for 90° hooks or if 2½" clear cover is provided for 180° hooks:

$l_{dh}' = (0.70)\cdot(l_{dh})$ (see figure).



If stirrups are provided perpendicular to the hooked bar extension further reductions in l_{dh} are possible (see code for details).

Basic development length for bars in compression



The required development length for bars in compression is denoted by l_{dc}.

Note that bars are 'easier' to develop in compression because the end of the bar bears on concrete.

'Basic' equation for l_{dc} if $f_s = f_y$:

$$l_{dc} = \frac{0.02 f_y d_b}{\sqrt{f_c'}} = \frac{f_y d_b}{50\sqrt{f_c'}}, \quad \text{but absolute minimum} = 8"$$

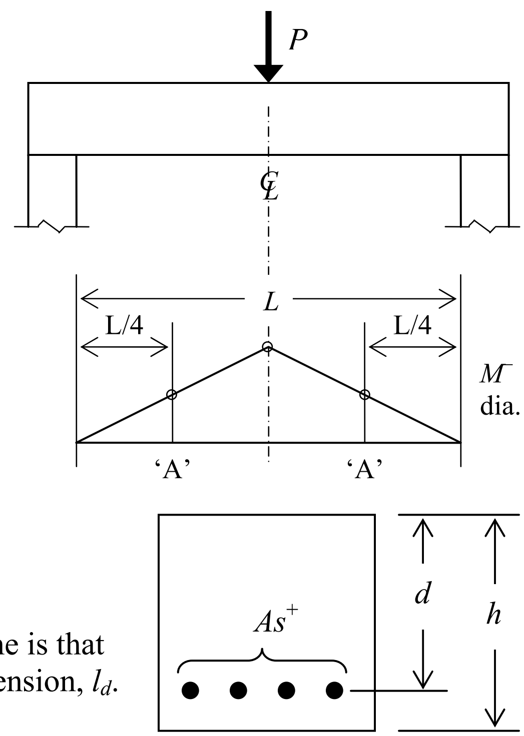
If fs < fy then $l_{dc}' = l_{dc}\left(\frac{f_s}{f_y}\right)$

Quick accurate 'design aid' for f_c' = 4 *ksi* and f_y = 60 *ksi*:

l_{dc} = (0.2)·(bar#) (gives l_{dc} in feet)

Note: Hooks provide no reduction in required compression development length. Hooks are <u>allowed</u> but do not reduce the l_{dc} required.

Theoretical bar cutoff points



Consider the simple beam loaded at the center line by a 'concentrated' load P (for simplicity, neglect beam self weight).

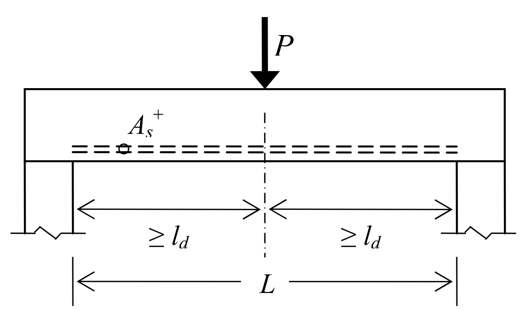
The 'critical section' for moment (maximum positive moment) is at the center line of the beam span.

For this example assume that four bars of equal size have been provided to resist the maximum moment, all bars stressed at $f_s \approx f_y$ at mid span.

If all four bars extend to the supports the basic requirement for bar length beyond the center line is that $L/2$ equal or exceed the development length intension, l_d.

It should be clear however that the full A_s^+ is required only at the critical section for maximum moment. For example, at locations A, shown on the moment diagram only ½ A^+ is required for flexural strength. Location A occurs at ¼ span ($L/4$) in this example.

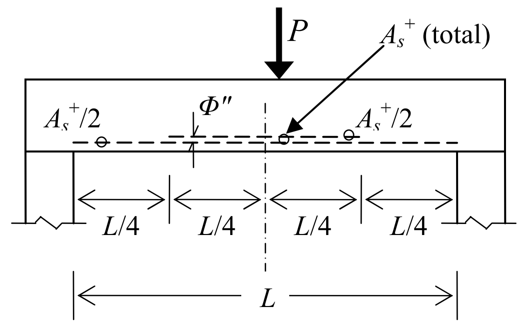
Let's denote location A as the theoretical cutoff (T.C.O) point for ½ A_s^+. Such points are also known as flexural cut off points.



All As+ extended 'to support'.

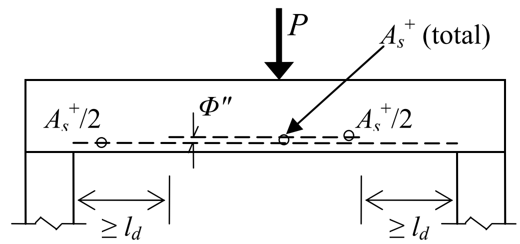
Theoretical issues for cutting off steel at exactly the flexural cutoff points

Suppose we cutoff ½ A_s^+ at exactly $L/4$ for the example beam, as shown in the figure to the right.



The steel stresses in the continuing steel beyond $L/4$ will again be at (theoretically) $f_s = f_y$!

Thus, 'new' critical sections occur at the steel cut off locations, and the steel with $f_s = f_y$ must be developed for a length ≥ l_d.



($L/4$ must be ≥ l_dA for cutoff of ΣAs^+ at $L/4$.)
Continuing steel is again at $f_s = f_y$.

Thus we observe that the bars must be extended from critical sections at least a distance of l_d' where l_d' is the adjusted tension development length. When tension bars are cut off new 'critical sections' must be considered to exist and analyzed with respect to development length requirements (the basic issue).

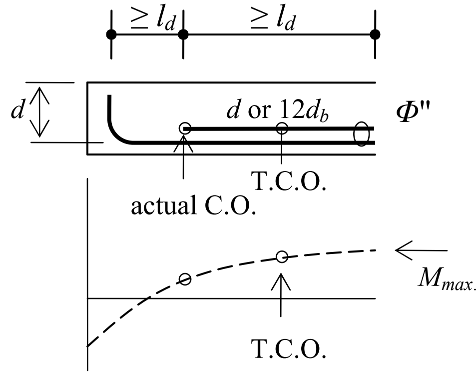
Unfortunately this is not the end of our consideration of required bar extensions and development length.

There are more than a dozen additional ACI code sections related to bar extension requirements and development length issues!

Some of these requirements are due to theoretical issues associated with uniform loads, some are related to practical issues of varying loading conditions which shift moment envelopes. Other issues are associated with concerns for structural 'integetry'.

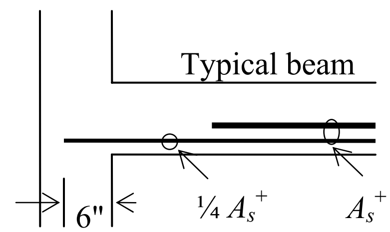
ACI rules requiring steel to be continued past theoretical flexural cutoff points

Critical sections for development of reinforcement are at points of maximum stress (maximum positive moment and negative moment locations) and at points where adjacent reinforcement is terminated. ACI rules for actual 'cutoff' points are given below:



Rule #1: Reinforcement shall extend beyond theoretical cut off points (T.C.O) a distance of d or 12 d_b (greater) (except at ends of simple spans and cantilevers). All steel, always, except as noted above.

This rule recognizes that flexural moment diagrams are approximate and may shift from the theoretical positions.



Rule #2: At least ¼ of A_s^+ required a mid–span must extend at least 6" into support(s) in continuous construction (1/3 of A_s^+ for simple spans).

Rule #3: Three additional rules relate to A_s^+ steel terminated in a tension zone (positive moment region of moment diagram).

These rules are somewhat 'complex' and involve extra stirrups, or providing extra flexural steel or very small shear loads.

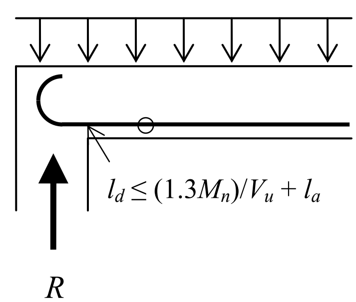
The ACI waives these rules (no consideration required) when steel is terminated beyond or at inflection points. Thus we will always terminate A_s^+ steel (if any is cutoff) at inflection points or beyond (in negative moment regions).

Note: Steel cutoff in tension zones produce premature shear–flexure cracking. This is the reason for the 'special' ACI rules.

Rule #4: This rule also applies to A_s^+ steel (positive moment reinforcement).

At simple supports and at points of inflection (Note: both are points of zero moment):

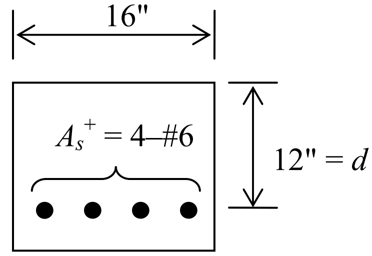
$$l_d \leq \frac{M_n}{V_u} + l_a \quad \text{(the requirement for rule \#4)}$$



rule #4 at simple support

M_n = calculated section capacity assuming continuing A_s^+ at $f_s = f_y$.
V_u = factored shear at point of zero moment.
l_a = greater of d or 12 d_b at points of inflection.

This rule has the indirect effect of limiting bar size in some situations.



Example application of rule #4:

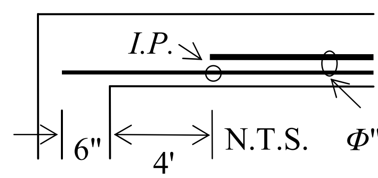
$f_c' = 4$ ksi, $f_y = 60$ ksi, A_s^+ continuing past inflection point = 2– #6 = 2(0.44) = 0.88 in^2. Given: V_u at inflection point = 21.5 kips.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \qquad a = \frac{A_s f_y}{(0.85) f_c' b} = \frac{(0.88)(60)}{(0.85)(4)(16)} = 0.97"$$

$$M_n = (0.88)(60)\left(12 - \frac{0.97}{2}\right) = 608 \;\; k-in = 50.7 \;\; k-ft$$

$$\frac{M_n}{V_u} + l_a = \frac{(50.7)(12)}{21.5} + 12 = 40.3"$$

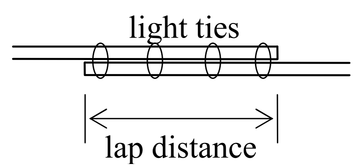
For #6 bars: $l_d = (0.5)(6) = 3' = 36"$
$l_d = 36" \leq 40.3"$ Rule is satisfied (OK)



Before we give rule #5 for A_s^+ steel we need to discuss bar lengths stocked and splices.

#5 – #18 bars are typically stocked in 60' lengths
#3 – #4 bars are typically stocked in 20' – 40' lengths

Lap splices are generally required when reinforcement must be continuous for distances greater than the bar lengths available.



Lap splices are typically made by overlapping bars by the required distance and lightly wiring them together. Splices at locations of maximum bar stress should be avoided 'as much as possible'.

Lap splices in tension (two classes, 'A' and 'B')

Class 'A' splice requires lap of (1)(l_d) (larger bar controls)
Class 'B' splice requires lap of (1.3)(l_d) (larger bar controls)
Class 'B' splices is required unless code states otherwise

Lap splices in compression

Lap length required = $0.0005 f_y d_b$

Typical compression splices occur at column to footing dowels and above floors for bottom story column steel to next story column steel.

Design aid for compression splices of $f_y = 60\ ksi$ bars:

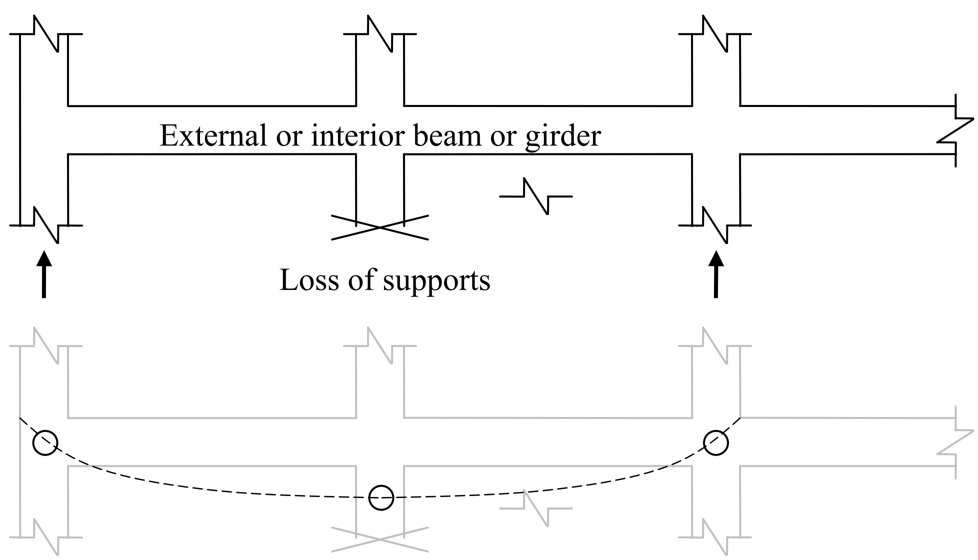
$$(0.0005)(60{,}000)d_b = 30d_b$$

Example: For #8 bar ($d_b = 1.0''$) compression splice lap requirements are:

Class A: $30d_b = 30''$, Class B: $(1.3)(30)(1.0) = 39''$

Structural integrity provision of ACI code for A_s^+

1. The structural integrity provisions are provided to afford some protection if vertical support is lost for a beam (or girder) due to a catastrophic event (earthquake or external or internal explosion).

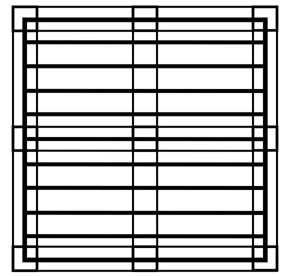


2. Rule #5: for both perimeter beams and girders and non–perimeter beams and girders, $A_s^{\pm}$ steel:

- ≥ ¼ of A_s^+ steel required a mid span (max. A_s^+), but not less than two bars must be continuous or spliced over or near supports by a class 'A' splice.

- At non–continuous ends terminate required bars with standard Hooks.

- In one–way **joist** construction at least one bottom bar should be continuous or spliced at or near the supports with a class 'A' splice. At discontinuous ends the 'tie' bars should terminate with a standard Hook.



Note: the intent here is to enclose the entire perimeter of the structure and framing lines across the structure with continuous A_s^+ steel. A_s^- steel is discussed next.

Discussion of requirements for continuing A_s^- steel beyond theoretical cutoff points (T.C.O.)

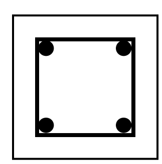
Rule #6: At least 1/3 of the total A_s^- reinforcement provided for negative moment at a support shall continue beyond points–of–inflection (I.P.) not less than the greater of: d, $12d_b$ or $l_n/16$.

This rule provides for possible shifts in the negative moment diagram.

Note also that rule #1 also applies to A_s^- reinforcement.

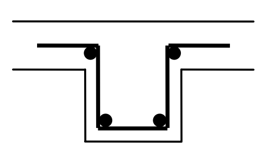
Rule #7 is a structural integrity provision for A_s^- reinforcement. This rule is indented to provide some structural continuity to perimeter beams and girders in addition to that provided by rule #5 for A_s^+.

Rule #7: In addition to rule #6 all perimeter beams (and girders) must have ≥ 1/6 of the total A_s^- provided at the supports continuous or class 'A' spliced at or near mid span. Minimum of 2 bars.



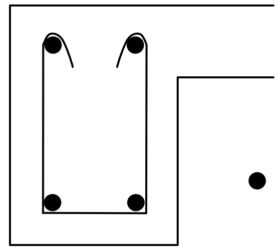
These bars plus the minimum of 2 A_s^+ bars provided for continuity shall be enclosed in 'one–piece' stirrups or stirrups with 135° hooks at upper end. (Minimum A_x at maximum spacing $d/2$ is OK!)

Note: we will discuss 'one piece stirrups' in class. Some stirrup stiles are not allowed for torsion which is commonly required for reinforcement of perimeter members!

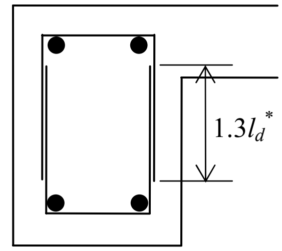


90° hooks NOT adequate for perimeter beam stirrups.

It should be quite clear that given the many rules (stated in more than a dozen ACI subsections) choosing exact and optimal bar cutoff locations for flexural members is complex and tedious!



135° hooks



one–piece with required lap

Fortunately there is an easier way in many common situations. If the flexural member (slab, beam, joist) qualifies for design by ACI coefficients and L/d ≥ 10 for a beam or L/d ≥ 18 for a slab (typical cases) designers typically use 'standard' cutoff locations. These 'standard' solutions are presented in Appendix A page 11.

The 'standard' cutoff points generally satisfy all the ACI rules except for rule #4 (the bar size rule) which theoretically should be checked for the bars continuing past the beam inflection points.

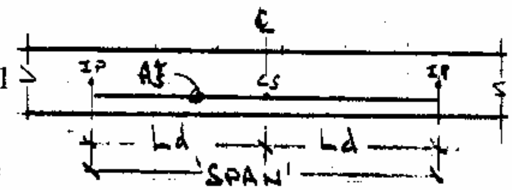
A UNIQUE DEVELOPMENT LENGTH RULE FOR POSITIVE MOMENT TENSILE REINFORCEMENT IN 'BEAM'

Basic 'ground rules' for the discussion which follows:

(1) Assume the stress, f_s, in the beam flexural reinforcement at any section is proportional to the moment, M, at that section. (This is very close to 'correct', the stress 'drops off' just a 'little bit' less than the moment).

(2) The critical section for positive moment is at the span centerline. At this critical section the flexural steel is fully stressed (f_s=f_y). This is correct if the flexural steel bars were selected such that the computed $A^+_{s,required}$ is equal to the A^+_s provided.

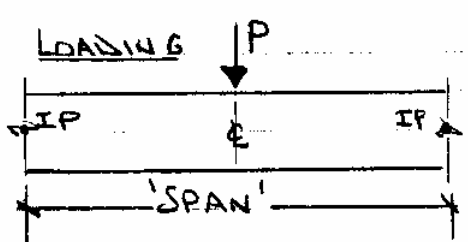
(3) The beam span between the inflection points (or simple supports) is exactly equal to two times the development length (L_d) of the flexural reinforcement bars which have been chosen. Thus these bars are adequately developed at the critical section (centerline of the span).

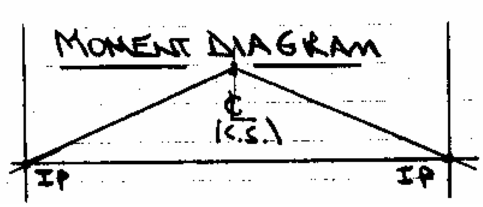


The <u>BASIC QUESTION</u> to be answered is: If the bars are adequately developed at the centerline (the critical section) does this assure that they are properly developed at every <u>other</u> section along the span?

A.) First consider the case of the span between the inflection points loaded by a <u>point load at the centerline</u> (for simplicity neglect beam self weight). The moment diagram for this load case 'drops off' <u>linearly</u> from an maximum at the centerline to zero at the inflection points. (see Figs. below)

By assumption (1) above the stress (f_s) in the flexural reinforcement also decreases in the same linear manner as the moment. Since the required development length (L_d') for the steel bars is proportional to the bar stress [$L_d' = L_d \cdot (f_s / f_y)$] since the flexural bars were adequately developed at the centerline (critical section) they will be properly developed at every section along the span! Thus the answer to the 'basic question' posed above is 'YES' for the case of point loading at the span centerline. In Fact in for this load case the bar length at every section is <u>EXACTLY EQUAL TO THE REQUIRED DEVELOPMENT LENGTH, NO MORE and NO LESS</u>!





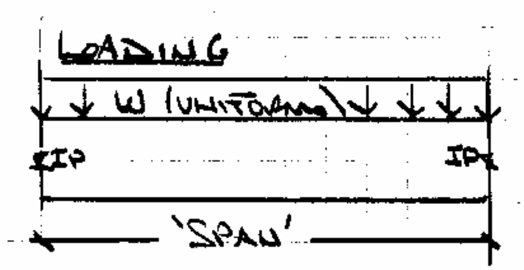
B) Next, consider the same system with uniformly applied loading, w, on the span. The moment diagram is now a parabola with maximum moment at the span centerline (this is still the critical section). The moment diagram now 'drops off' very slowly near the span centerline (see Figs. below).

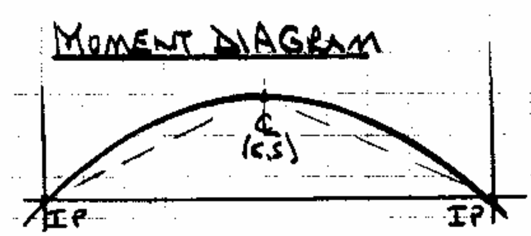
The observation above tells us that the stresses (f_s) in the reinforcing bars are at every section of the uniformly loaded beam are larger that those in the point load case with the same maximum moment except at the centerline (critical section) and at the end points (inflection points).

We next observe that since all sections in the point load case had exactly the correct development length (L_d') this means that at every section along the span in the uniform load case the development length is NOT ADEQUATE with the exception of the centerline and end sections! The answer to the 'basic question' is 'NO' and clearly we have a potential development length 'problem' if our only 'rule' is to require development at the positive moment critical sections..

We also observe that the 'problem' is associated with the convex shape, and the high 'slope' at the inflection point, of the moment diagram. The problem could be 'solved' by computation of the exact bar length for development at every section along the span, then finding the worst case and extending the bars appropriately from that critical location. This as 'solvable' problem but would be quite 'tedious' for any but 'simple' loading patterns! Fortunately, the ACI code provides us with a development length 'Rule' which lets us avoid the computations described in this paragraph.

Before we give the new ACI 'rule' lets recall that the slope of the moment diagram (dM/dX) at any section is related to the shear load (V) by the relation dM/dX = V. This is easily derived from 'statics' using a simple free body diagram of a beam flexural section.





Development Length 'RULE' paraphrased from ACI 12.11.3:
Note: I call this the 'MAD' rule. This is Rule 4 in the noteset, Chapter 8.

"At points of inflection, positive moment tension reinforcement shall be limited to such a diameter that L_d computed for f_s equal to f_y satisfies:

$$L_d < (M_n / V_u) + L_a$$

in which:

M_n = Nominal moment strength, just past the point if inflection, computed assuming the flexural reinforcement (A_s) which continues past the inflection point is stressed at $f_s = f_y$.
(Note: $Mn = A_s \cdot f_y \cdot [d-('a'/2)]$ and $'a' = f_s \cdot f_y / .85 f_c' b$)

V_u = Factored shear force at the inflection point under consideration.

L_a = The larger of **d or $12d_b$**
(unless computed L_a is greater than bar extension which actually exists!).

The ACI Code has contained this development length rule for many years to address the 'problem' we discussed on the previous two pages. Observe that the 'M_n / V_u' term is a 'length' and is related to the slope of the moment diagram at the point of inflection.

Final note: This rule also applies, with some modifications, at the supports of simple beams. See code 12.11.3 and next few sections for details.