AGU PUBLICATIONS

Water Resources Research

RESEARCH ARTICLE

10.1002/2015WR017677

Key Points:
- Two types of change-point in multivariate hydrological series are distinguished
- The copula-based likelihood-ratio method is applied to multivariate case
- Dependence structure of flood features of the Upper Hanjiang River changed

Correspondence to:
L. Xiong,
xionglh@whu.edu.cn

Citation:
Xiong, L., C. Jiang, C.-Y. Xu, K.-x. Yu, and S. Guo (2015), A framework of change-point detection for multivariate hydrological series, *Water Resour. Res.*, *51*, 8198–8217, doi:10.1002/2015WR017677.

Received 9 JUN 2015
Accepted 9 SEP 2015
Accepted article online 21 SEP 2015
Published online 17 OCT 2015

A framework of change-point detection for multivariate hydrological series

Lihua Xiong[1], Cong Jiang[1], Chong-Yu Xu[1,2], Kun-xia Yu[3], and Shenglian Guo[1]

[1]State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, China, [2]Department of Geosciences, University of Oslo, Oslo, Norway, [3]State Key Laboratory Base of Eco-hydraulic Engineering in Arid Area, Xi'an University of Technology, Xi'an, China

Abstract Under changing environments, not only univariate but also multivariate hydrological series might become nonstationary. Nonstationarity, in forms of change-point or trend, has been widely studied for univariate hydrological series, while it attracts attention only recently for multivariate hydrological series. For multivariate series, two types of change-point need to be distinguished, i.e., change-point in marginal distributions and change-point in the dependence structure among individual variables. In this paper, a three-step framework is proposed to separately detect two types of change-point in multivariate hydrological series, i.e., change-point detection for individual univariate series, estimation of marginal distributions, and change-point detection for dependence structure. The last step is implemented using both the Cramér-von Mises statistic (CvM) method and the copula-based likelihood-ratio test (CLR) method. For CLR, three kinds of copula model (symmetric, asymmetric, and pair-copula) are employed to construct the dependence structure of multivariate series. Monte Carlo experiments indicate that CLR is far more powerful than CvM in detecting the change-point of dependence structure. This framework is applied to the trivariate flood series composed of annual maxima daily discharge (AMDD), annual maxima 3 day flood volume, and annual maxima 15 day flood volume of the Upper Hanjiang River, China. It is found that each individual univariate flood series has a significant change-point; and the trivariate series presents a significant change-point in dependence structure due to the abrupt change in the dependence structure between AMDD and annual maxima 3 day flood volume. All these changes are caused by the construction of the Ankang Reservoir.

1. Introduction

As pointed out in the summary of the Science Plan of the new Scientific Decade 2013–2022 of IAHS, entitled "Panta Rhei-Everything Flows," hydrological systems are changing under changing environments [*Montanari et al.*, 2013]. As a result, hydrological series in many regions throughout the world have been found to present obvious nonstationarities, in the form of either change-point or trend [*Douglas et al.*, 2000; *Burn and Hag Elnur*, 2002; *Xiong and Guo*, 2004; *Villarini et al.*, 2010; *Schmocker-Fackel and Naef*, 2010; *López and Francés*, 2013]. The assumption of stationarity, a basic premise behind conventional hydrological frequency analysis, is often necessary in hydrologic practice such as hydrologic design but may be invalid for nonstationary hydrological series [*Khaliq et al.*, 2006]. Consequently, it is necessary to diagnose the nonstationarity in hydrological series to reveal or validate the impacts of changes in hydrological systems, and help understanding the nature of changes in hydrological systems.

Up to now, a number of methods have been employed to analyze the nonstationarity in univariate hydrological series [*Kendall*, 1975; *Chiew and McMahon*, 1993; *Moraes et al.*, 1998; *Perreault et al.*, 1999; *Rasmussen*, 2001; *Strupczewski and Kaczmarek*, 2001; *Strupczewski et al.*, 2001a,b; *Yue et al.*, 2002; *Wong et al.*, 2006; *El Adlouni et al.*, 2007; *Villarini et al.*, 2009; *Villarini and Serinaldi*, 2012; *Xie et al.*, 2014; *Du et al.*, 2015]. However, the nonstationarity in multivariate hydrological series has just begun to attract some attention only recently [e.g., *Ben Aissia et al.*, 2012; *Chebana et al.*, 2013; *Ben Aissia et al.*, 2014; *Bender et al.*, 2014; *Jiang et al.*, 2015].

The nonstationarity in multivariate series should be more complex than that in univariate series since multivariate series can be treated as a composition of several univariate series that could be dependent on each other. Multivariate series might hide two types of nonstationarity: one is the nonstationarity in some individual univariate series or marginal distributions, and the other in dependence structure among individual univariate series [*Quessy et al.*, 2013]. These two types of nonstationary in many cases may be intertwined with one another.

Some methods have been proposed to detect trend or change-point in multivariate time series, however, without making clear distinction between the two types of nonstationarities mentioned above. For the trend case, *Chebana et al*. [2013] employed the nonparametric Mann-Kendall type and Spearman's rho type tests to detect the trends in multivariate hydrological series. As for the change-point case, *Gombay and Horváth* [1999] proposed a nonparametric approach based on the Kolmogorov-Smirnov statistic to detect the change-points in the distributions of both univariate and multivariate series. On the basis of the approach proposed by *Gombay and Horváth* [1999], *Holmes et al*. [2013] developed a new nonparametric test for detecting the change-points in multivariate series based on Cramér-von Mises (CvM) statistic, which was found to be more powerful than the Kolmogorov-Smirnov statistic [*Bücher et al*., 2014]. *Ben Aissia et al*. [2014] used the Bayesian method of multiple change-point detection in multiple linear regressions to detect the change-points in the bivariate series with a dependence structure.

In detecting the nonstationarity in multivariate hydrological series, clear distinction between the two types of nonstationarity is very essential in order to gain a deep understanding of the physical causes behind the nonstationarity in multivariate hydrological series [*Bender et al*., 2014]. As discussed above, many methods have been available for detecting nonstationarity in univariate series. As far as the detection of (abrupt) changes in the dependence structure among individual univariate series, the methods are very limited and nearly all seem to be closely linked to the methodology of copula, which describes the shape and strength of the dependence structure via a copula function and its parameter(s), respectively [*Nelsen*, 2006; *Salvadori et al*., 2007]. For example, *Dias and Embrechts* [2004] combined the theories of likelihood-ratio test [*Csörgő and Horváth*, 1997] and copulas [*Nelsen*, 2006] to propose the copula-based likelihood-ratio test (CLR) method to detect the change-point in dependence structure of multivariate financial series. *Bouzebda and Keziou* [2013] also presented the detail of the copula-based likelihood-ratio test method proposed by *Dias and Embrechts* [2004]. In hydrology, *Bender et al*. [2014] applied a 50 year moving time window to present the nonstationarities in both marginal distribution parameters and copula parameter of the bivariate flood series of the Rhine River. *Jiang et al*. [2015] proposed a framework based on time-varying copula model to describe the temporal variation in a bivariate low-flow series. However, the application of these copula-based methods for detecting the nonstationarity in the dependence structure of multivariate time series is still at the very early stage, and is usually limited to bivariate cases in both hydrology and some other fields such as economics and finance [*Guégan and Zhang*, 2010; *Manner and Candelon*, 2010; *Ye et al*., 2012; *Boubaker and Sghaier*, 2013; *Bender et al*., 2014; *Jiang et al*., 2015].

In this paper, a framework is proposed for detecting change-points in multivariate hydrological series by distinguishing the two types of change-points in both marginal distribution and dependence structure, which includes three steps. First, detect the change-point in each univariate hydrological series. Second, estimate the marginal distributions based on the results of change-point detection for each univariate hydrological series in the first step. Third, detect the change-point in the dependence structure of multivariate hydrological series. The third step is the most crucial step in the proposed framework and implemented by two approaches: one is the CvM method and the other the CLR method. In practice, a complete description of hydrological events might involve three or more physical variables, whereas most applied copulas are bivariate (except symmetric copula) [*Joe*, 1997; *Nelsen*, 2006]. Thus, choosing a proper copula model to construct the dependence structure of these variables is very crucial before applying the CLR method to multivariate (more than two random variables) hydrological series. As far as we know, there have been at least three kinds of copula model that can be applied to construct the dependence structure of multivariate hydrological series, i.e., symmetric copula [*Joe*, 1997; *Nelsen*, 2006; *Zhang and Singh*, 2007; *Aas and Berg*, 2009], asymmetric copula [*Joe*, 1997; *Nelsen*, 2006; *Grimaldi and Serinaldi*, 2006; *Serinaldi and Grimaldi*, 2007; *Aas and Berg*, 2009], and pair-copula [*Aas et al*., 2009; *Aas and Berg*, 2009; *Xiong et al*., 2014]. In this study, these three kinds of copula model will be investigated for their suitability to capture the dependence structure in multivariate series when the CLR method is used to detect change-point in the dependence structure of multivariate series.

To perform a case study by the proposed framework, we select the trivariate flood series composed of annual maxima daily discharge (AMDD), annual maxima 3 day flood volume, and annual maxima 15 day flood volume observed at the Ankang hydrological station at the Upper Hanjiang River, China. In China, these three series are all important flood features required for deriving design flood hydrograph of reservoirs [*Zhan and Ye*, 2000; *Li et al*., 2013]. Previous study has found that the low flow of the Ankang hydrological station has been influenced by the reservoirs in the basin [*Jiang et al*., 2015]. However, the nonstationarity in the trivariate flood series has not yet been investigated, even though it is very possible that the trivariate flood series would also have been influenced by the reservoirs.

The remainder of the paper is organized as follows: in next section, the methods used in this study are described. Section 3 presents the performance of the CvM and CLR methods in detecting the change-point in dependence structure of multivariate series. The results of a case study are presented in section 4. Finally, the main conclusions and discussion are summarized in section 5.

2. Methods

2.1. Methodological Framework

In this section, the outline of the proposed framework for detecting the change-points in multivariate hydrological series is briefly described. First, the change-point in each marginal distribution or individual univariate series is detected by the CvM method. Then the marginal distributions are estimated based on the results of change-point detection for univariate series. Finally, both the CvM and CLR methods are employed to detect the change-point in the dependence structure of multivariate series. For the purpose of applying the CLR method to the multivariate series consisting of more than two variables, three kinds of copula model (symmetric copula, asymmetric copula, and pair-copula) are considered to construct the multivariate dependence structure. All the methods mentioned above will be described in detail below.

2.2. The CvM Method for Change-Point Detection

Let $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ be a time series of independent d-dimensional random vectors ($d \geq 1$), the distributions of which are $F_1(\mathbf{x}_1), F_2(\mathbf{x}_2), \ldots, F_n(\mathbf{x}_n)$, respectively, where $\mathbf{X}_i = (X_{1,i}, X_{2,i}, \ldots, X_{d,i})$ are the random variables at time i ($i = 1, 2, \ldots, n$), $\mathbf{x}_i = (x_{1,i}, x_{2,i}, \ldots, x_{d,i})$ are the observation values or realizations of $\mathbf{X}_i$, and n is the size of the time series. In the change-point detection procedure, the corresponding null hypothesis of no change in the distributions of $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ is given as follows:

$H_0 : F_1(\mathbf{x}) = F_2(\mathbf{x}) = \cdots = F_n(\mathbf{x})$ for all $\mathbf{x} \in R^d$.

Against H_0, the alternative hypothesis is that

$H_1 : \exists k^*$ and $1 \leq k^* \leq n-1$, such that $F_1(\mathbf{x}) = \cdots = F_{k^*}(\mathbf{x})$, $F_{k^*+1}(\mathbf{x}) = \cdots = F_n(\mathbf{x})$ and $F_{k^*}(\mathbf{x}_0) \neq F_{k^*+1}(\mathbf{x}_0)$ with some $\mathbf{x}_0$.

For $k = 1, 2, \ldots, n-1$, we define

$$D(k, \mathbf{x}) = \frac{k(n-k)}{n^{3/2}} \left[\hat{F}_k(\mathbf{x}) - \hat{F}^*_{n-k}(\mathbf{x}) \right], \tag{1}$$

where

$$\hat{F}_k(\mathbf{x}) = \frac{1}{k} \sum_{i=1}^{k} \mathbf{1}(\mathbf{X}_i \leq \mathbf{x}), \tag{2a}$$

$$\hat{F}^*_{n-k}(\mathbf{x}) = \frac{1}{n-k} \sum_{i=k+1}^{n} \mathbf{1}(\mathbf{X}_i \leq \mathbf{x}), \tag{2b}$$

are empirical distribution functions, which are computed from $\mathbf{X}_1, \ldots, \mathbf{X}_k$ and $\mathbf{X}_{k+1}, \ldots, \mathbf{X}_n$, respectively. For the $n-1$ possible change-points, the statistics are defined as follows:

$$S_k = \frac{1}{n} \sum_{i=1}^{n} [D(k, \mathbf{X}_i)]^2. \tag{3}$$

It can be seen that S_k is a statistic in the form of Cramér-von Mises statistic [*Genest and Favre*, 2007]. Then the test statistic for detecting the change-point in the series is given as follows:

$$\hat{S}_n = \max_{1 \leq k \leq n-1} S_k. \tag{4}$$

The null hypothesis H_0 is rejected if $\hat{S}_n$ is large, and the change-point of the series is given by

$$k^*=\underset{1\le k\le n-1}{\operatorname{argmax}} S_k, \tag{5}$$

where the symbol "argmax" stands for a function which returns the location of the maxima among S_k ($1\le k\le n-1$). The approximate p value of $\hat{S}_n$ is estimated by the bootstrap method [*Holmes et al.*, 2013]. In this paper, the significance level of the change-point test is taken 5%, that is, if the p value of $\hat{S}_n$ is smaller than 5%, H_0 will be rejected.

It is worth noting that *Holmes et al.* [2013] proposed the CvM method and applied it to detect the change-point in multivariate series but without distinguishing the two types of change-points underlying in marginal distributions and dependence structure. In this study, the CvM method is separately employed to detect the two types of change-points in multivariate series.

2.3. Change-Point Detection for Marginal Distribution

In this paper, in order to separately investigate the two types of change-point in $\mathbf{X}_1,\mathbf{X}_2,\ldots,\mathbf{X}_n$, the change-point in marginal distribution of each univariate series is first detected by using the CvM method described above. The individual univariate series is denoted by $\mathbf{X}'_m=(X_{m,1},X_{m,2},\ldots,X_{m,n})$ ($m=1,2,\ldots,d$), where $\left(\mathbf{X}_1'^{\mathrm{T}},\mathbf{X}_2'^{\mathrm{T}},\ldots,\mathbf{X}_d'^{\mathrm{T}}\right)^{\mathrm{T}}=(\mathbf{X}_1{}^{\mathrm{T}},\mathbf{X}_2{}^{\mathrm{T}},\ldots,\mathbf{X}_n{}^{\mathrm{T}})$ and the symbol "T" means matrix transpose.

2.4. Estimation of Marginal Distribution With Change-Point

According to the results of change-point detection for each individual univariate series $\mathbf{X}'_m$ ($m=1,2,\ldots,d$), the marginal probabilities corresponding to $\mathbf{X}'_m$, denoted by $\mathbf{U}'_m=(U_{m,1},U_{m,2},\ldots,U_{m,n})$, are estimated by Gringorten plotting position formula [*Gringorten*, 1963], which has been widely used in estimating nonexceedance probability of flood series [*Guo*, 1990; *Grimaldi and Serinaldi*, 2006; *Salvadori et al.*, 2007; *Zhang and Singh*, 2007]. If the univariate hydrological series has no change-point, the marginal distribution is given as follows:

$$u_{m,i}=F_{X_m}(x_{m,i})=\frac{1}{n+0.12}\left[\sum_{j=1}^{n}\mathbf{1}\left(X_{m,j}\le x_{m,i}\right)-0.44\right], \tag{6}$$

where $i=1,2,\ldots,n$, and $u_{m,i}$ is the marginal probability of $x_{m,i}$.

If the univariate hydrological series has a significant change-point k_m^*, the marginal distribution is estimated as follows:

$$u_{m,i}=F_{X_m}(x_{m,i})=\frac{1}{k_m^*+0.12}\left[\sum_{j=1}^{k_m^*}\mathbf{1}\left(X_{m,j}\le x_{m,i}\right)-0.44\right],\quad i=1,2,\ldots,k_m^*, \tag{7a}$$

$$u_{m,i}=F_{X_m}(x_{m,i})=\frac{1}{n-k_m^*+0.12}\left[\sum_{j=k_m^*+1}^{n}\mathbf{1}\left(X_{m,j}\le x_{m,i}\right)-0.44\right],\quad i=k_m^*+1,\ldots,n. \tag{7b}$$

2.5. Change-Point Detection for Dependence Structure in Multivariate Series

Since the marginal distributions are estimated based on the results of change-point detection, the empirical probabilities $\mathbf{U}'_m$ ($m=1,2,\ldots,d$), as the mappings of the observed hydrological series $\mathbf{X}'_m$ ($m=1,2,\ldots,d$), should be independently, identically, and uniformly distributed on [0,1]. Therefore, the influence of the change-points, if any, on marginal distributions has been removed, and the task of detecting the change-point in dependence structure of $\mathbf{X}_1,\mathbf{X}_2,\ldots,\mathbf{X}_n$ has actually become the task of detecting the change-point in $\mathbf{U}_1,\mathbf{U}_2,\ldots,\mathbf{U}_n$, where $(\mathbf{U}_1{}^{\mathrm{T}},\mathbf{U}_2{}^{\mathrm{T}},\ldots,\mathbf{U}_n{}^{\mathrm{T}})=\left(\mathbf{U}_1'^{\mathrm{T}},\mathbf{U}_2'^{\mathrm{T}},\ldots,\mathbf{U}_d'^{\mathrm{T}}\right)^{\mathrm{T}}$. For such a task, there are two methods that can be employed, i.e., the CvM and CLR methods.

2.5.1. The CvM Method

The change-point in the dependence structure of $\mathbf{X}_1,\mathbf{X}_2,\ldots,\mathbf{X}_n$ can be determined via detecting the change-point in $\mathbf{U}_1,\mathbf{U}_2,\ldots,\mathbf{U}_n$ by using the CvM method described above, i.e., equations (1)–(5).

2.5.2. The CLR Method

Copula is a very effective tool to construct the joint distribution of multivariate series and has been widely used in multivariate hydrological frequency analysis [*Salvadori and De Michele*, 2004; *Favre et al.*, 2004; *Grimaldi and Serinaldi*, 2006; *Serinaldi and Grimaldi*, 2007; *Zhang and Singh*, 2007; *Genest and Favre*, 2007; *Salvadori et al.*, 2007; *Kao and Govindaraju*, 2008; *Salvadori and De Michele*, 2010; *Mediero et al.*, 2010; *Volpi and Fiori*, 2012; *Xiong et al.*, 2014]. Through a copula, the joint distribution of a multivariate hydrological series can be characterized by two components, i.e., the individual univariate distributions (i.e., marginal distributions), and the dependence structure represented by a copula function and its parameter(s), which describe the shape and strength of the dependence structure, respectively. With these two components consisting of a multivariate series being clearly identified, the change in each component can be separately analyzed, offering a deep physical insight into the change in multivariate hydrological series [*Jiang et al.*, 2015].

For a d-dimensional hydrological series $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$, the joint distribution can be constructed by a copula as follows:

$$H(\mathbf{X}_i)=C(\mathbf{u}_i|\boldsymbol{\theta}_i), \tag{8}$$

where $i=1, 2, \ldots, n$; $\mathbf{u}_i$ is the marginal probability vector of $\mathbf{X}_i$ and expressed by equations (6) and (7); $C(\cdot)$ stands for a copula function, and $\boldsymbol{\theta}_i$ is the copula parameter vector.

Assuming the type of copula is constant, the null hypothesis of no change in the dependence structure of $\mathbf{X}_i$ ($i=1, 2, \ldots, n$) is given as follows:

$$H_0 : \boldsymbol{\theta}_1=\boldsymbol{\theta}_2=\cdots=\boldsymbol{\theta}_n=\boldsymbol{\eta}_0.$$

The alternative hypothesis is that

$$H_1 : \exists\,\lambda^* \text{ and } 1 \leq \lambda^* \leq n-1, \text{ such that } \boldsymbol{\theta}_1=\cdots=\boldsymbol{\theta}_{\lambda^*}=\boldsymbol{\eta}_1,\ \boldsymbol{\theta}_{\lambda^*+1}=\cdots=\boldsymbol{\theta}_n=\boldsymbol{\eta}_2 \text{ and } \boldsymbol{\eta}_1 \neq \boldsymbol{\eta}_2.$$

If the null hypothesis H_0 is rejected, λ^* is the so called change-point in dependence structure. Assuming that the change-point $\lambda^*=\lambda$ is known, the null hypothesis H_0 should be rejected for a small value of the test statistic of copula-based likelihood-ratio Λ_λ [*Dias and Embrechts*, 2004; *Bouzebda and Keziou*, 2013], which is given by

$$\Lambda_\lambda=\frac{L_n(\hat{\boldsymbol{\eta}}_0)}{L_\lambda(\hat{\boldsymbol{\eta}}_1)L^*_{n-\lambda}(\hat{\boldsymbol{\eta}}_2)}=\frac{\prod\limits_{1\leq i\leq n} c(\mathbf{u}_i|\hat{\boldsymbol{\eta}}_0)}{\prod\limits_{1\leq i\leq \lambda} c(\mathbf{u}_i|\hat{\boldsymbol{\eta}}_1)\prod\limits_{\lambda+1\leq i\leq n} c(\mathbf{u}_i|\hat{\boldsymbol{\eta}}_2)}, \tag{9}$$

where $L_n(\cdot)$ stands for likelihood function of the whole series under the null hypothesis; $L_\lambda(\cdot)$ and $L^*_{n-\lambda}(\cdot)$ are likelihood functions of the series up to and after the change-point λ^*, respectively; $c(\cdot)$ is the density function of copula; and $\hat{\boldsymbol{\eta}}_0$, $\hat{\boldsymbol{\eta}}_1$ and $\hat{\boldsymbol{\eta}}_2$ are the maximum likelihood estimations for $\boldsymbol{\eta}_0$, $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$, respectively. Then the statistic of likelihood-ratio can be written as the logarithmic form as follows:

$$-2\ln(\Lambda_\lambda)=2\{\ln[L_\lambda(\hat{\boldsymbol{\eta}}_1)]+\ln\left[L^*_{n-\lambda}(\hat{\boldsymbol{\eta}}_2)\right]-\ln[L_n(\hat{\boldsymbol{\eta}}_0)]\}. \tag{10}$$

On the condition that the change-point λ is unknown, H_0 will be rejected if the value of

$$Z_n=\max_{1\leq\lambda\leq n-1}\{-2\ln(\Lambda_\lambda)\} \tag{11}$$

is large. The asymptotic distribution of the likelihood-ratio statistic $Z_n^{1/2}$ can be obtained by using Monte Carlo simulation method. But there is a more convenient way to approximate the distribution of $Z_n^{1/2}$, which is given as follows:

$$P\left(Z_n^{1/2}\geq z\right)=\frac{z^p\exp(-z^2/2)}{2^{p/2}\Gamma(p/2)}\cdot\left[\ln\frac{(1-h)(1-l)}{hl}-\frac{p}{z^2}\ln\frac{(1-h)(1-l)}{hl}+\frac{4}{z^2}+O\left(\frac{1}{z^4}\right)\right], \tag{12}$$

as $z\to\infty$, where $h=l=[\ln(n)]^{3/2}/n$ [*Gombay and Horváth*, 1996, equation (3.8), p. 129; *Csörgő and Horváth*, 1997, equation (1.3.26), p. 25; *Dias and Embrechts*, 2004]; p is the number of copula parameters with a change-point. If the p value of $Z_n^{1/2}$ is less than 5%, the null hypothesis H_0 will be rejected and the change-point is

$$\lambda^*=\operatorname*{argmax}_{1\leq\lambda\leq n-1}\{-2\ln(\Lambda_\lambda)\}. \tag{13}$$

Before applying the CLR method described above to the multivariate series consisting of more than two random variables, a crucial issue should be considered, that is, choosing a proper copula model to construct the dependence structure of the multivariate series. For the high-dimensional hydrological series, i.e., $d>2$,

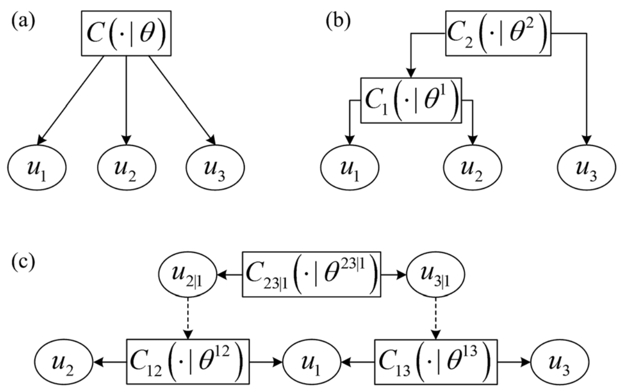

Figure 1. Structures of (a) symmetric copula, (b) asymmetric copula, and (c) pair-copula for trivariate case. In Figure 1c, $u_{2|1}=F(u_2|u_1)$ and $u_{3|1}=F(u_3|u_1)$, which are expressed as equations (20) and (21), respectively.

the joint distribution can be built by at least three copula models, i.e., symmetric copula [*Joe*, 1997; *Nelsen*, 2006; *Zhang and Singh*, 2007; *Aas and Berg*, 2009], asymmetric copula [*Joe*, 1997; *Nelsen*, 2006; *Grimaldi and Serinaldi*, 2006; *Serinaldi and Grimaldi*, 2007; *Aas and Berg*, 2009], and pair-copula [*Aas et al.*, 2009; *Aas and Berg*, 2009; *Xiong et al.*, 2014]. Figure 1 displays the structures of symmetric copula, asymmetric copula and pair-copula for trivariate case. The application of the CLR method to multivariate case based on the construction of multivariate distribution by the three copula models mentioned above will be described below.

2.5.3. Application of the CLR Method via Symmetric and Asymmetric Copulas

For any $d>2$, bivariate Archimedean copulas can be extended to the d-dimensional symmetric copulas and written as follows:

$$C(\mathbf{u}_i)=\varphi^{-1}\{\varphi(u_{1,i})+\varphi(u_{2,i})+\ldots+\varphi(u_{d,i})\}, \tag{14}$$

where $\varphi(\cdot)$ is a decreasing function known as the generator of the copula, and $\varphi^{-1}(\cdot)$ denotes its inverse. In this study, only the one-parameter symmetric Archimedean copulas are considered, and they can be directly substituted into equation (9) to detect the change-point in the dependence structure of multivariate hydrological series.

It can be seen from equation (14) the construction of multivariate distribution by the symmetric copula is fairly simple. But this model has obvious limitations that it can describe only positive dependence, and all mutual dependence structures among the multivariate series should be identical so that the asymmetric dependence structure cannot be characterized [*Nelsen*, 2006; *Grimaldi and Serinaldi*, 2006; *Aas and Berg*, 2009].

In order to capture the asymmetric dependence structure in multivariate hydrological series, asymmetric copula has been considered. Based on bivariate Archimedean copulas, the asymmetric copula can be written in the form

$$\begin{aligned}C(\mathbf{u}_i)&=\varphi_{d-1}^{-1}\{\varphi_{d-1}(u_{d,i})+\varphi_{d-1}\mathrm{O}\varphi_{d-2}^{-1}\{\varphi_{d-2}(u_{d-1,i})+\varphi_{d-2}\mathrm{O}\ldots\mathrm{O}\varphi_1^{-1}\{\varphi_1(u_{2,i})+\varphi_1(u_{1,i})\}\}\}\\&=C_{d-1}\{u_{d,i},C_{d-2}\{u_{d-1,i},\ldots C_1(u_{2,i},u_{1,i}|\theta_i^1)|\theta_i^{d-2}\}|\theta_i^{d-1}\},\end{aligned} \tag{15}$$

where symbol "O" stands for composition of functions; $\theta^1,\theta^2,\ldots,\theta^{d-1}$ are the parameters in the bivariate copulas $C_1(\cdot),C_2(\cdot),\ldots,C_{d-1}(\cdot)$, respectively. Actually, equation (14) is a special case of equation (15) when $\varphi_1(\cdot)=\varphi_2(\cdot)=\ldots=\varphi_{d-1}(\cdot)$. As for trivariate series $\mathbf{X}_1',\mathbf{X}_2',\mathbf{X}_3'$, equation (15) can be written as follows:

$$C(u_{1,i},u_{2,i},u_{3,i}|\theta_i^1,\theta_i^2)=C_2\{u_{3,i},C_1(u_{2,i},u_{1,i}|\theta_i^1)|\theta_i^2\}. \tag{16}$$

As shown in equation (16) and Figure 1b, the trivariate asymmetric copula has a hierarchical structure with two levels, thus the change-point detection of dependence structure for $\mathbf{X}_1',\mathbf{X}_2',\mathbf{X}_3'$ by the CLR method is performed via detecting the change-points in two copula parameters, with θ^1 being detected first and θ^2 then second. It is worth noting that each parameter in asymmetric copula measures the dependence strength of two variables, consequently, the change-point detection based on asymmetric copula is essentially the change-point detection for bivariate.

While asymmetric copula makes an obvious improvement compared to the symmetric copula, it still has many deficiencies. In general, the d-dimensional series can be coupled in $d(d-1)/2$ ways, but the asymmetric copula can only allow for the modeling of up to $d-1$ copulas. On the other hand, the degree of dependence in asymmetric copula must decrease with the level of nesting, i.e., $\theta^1\geq\theta^2\geq\ldots\geq\theta^{d-1}$. For more information about symmetric and asymmetric copulas, together with their specific expressions of the d-

dimensional distributions, readers are referred to *Joe* [1997], *Nelsen* [2006], *Grimaldi and Serinaldi* [2006], and *Aas and Berg* [2009].

2.5.4. Application of the CLR Method via Pair-Copula

Pair-copula is a very effective way to construct the joint distribution of $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ by decomposing the multivariate distribution into $d(d-1)/2$ bivariate copulas [*Aas et al.*, 2009]. Compared with asymmetric copula, pair-copula is more flexible and has the freedom to be built not only on Archimedean class of copulas but also on any copula classes. The probability density function of vector $\mathbf{X}_i$ ($i=1,2,\ldots,n$) is denoted by $f\{F_{X_1}(x_{1,i}), F_{X_2}(x_{2,i}), \ldots, F_{X_d}(x_{d,i})\}$, which can be factorized as

$$\begin{aligned} &f\{F_{X_1}(x_{1,i}), F_{X_2}(x_{2,i}), \ldots, F_{X_d}(x_{d,i})\} = f(u_{1,i}, u_{2,i}, \ldots, u_{d,i}) \\ &= f_d(u_{d,i}) \cdot f(u_{d-1,i}|u_{d,i}) \cdot f(u_{d-2,i}|u_{d-1,i}, u_{d,i}) \cdots f(u_{1,i}|u_{2,i}, \ldots, u_{d,i}). \end{aligned} \tag{17}$$

Each term in equation (17) can be decomposed into the appropriate pair-copula times a conditional marginal density with the general formula

$$f(u|\mathbf{v}) = c_{uv_j|\mathbf{v}_{-j}}\{F(u|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})\} \cdot f(u|\mathbf{v}_{-j}), \tag{18}$$

where v_j is one arbitrarily chosen component of $\mathbf{v}$, and $\mathbf{v}_{-j}$ denotes the v-vector excluding this component.

Since there are a large number of possible forms of pair-copula constructions for the high-dimensional distributions, two regular vine trees denoted as the canonical vine (C-vine) and drawable vine (D-vine) are usually used to identify the different pair-copula decompositions [*Aas et al.*, 2009]. With the special structure, the C-vine is more suitable when there is a key variable that governs interactions in the data set, while D-vine more resembles independence graphs than that of the C-vine. In addition, each decomposition for the trivariate series is both a C-vine and a D-vine.

Taking the trivariate series $\mathbf{X}_1', \mathbf{X}_2', \mathbf{X}_3'$ as an example, the joint distribution built by the C-vine structure with $\mathbf{X}_1'$ chosen as the key variable is expressed as

$$\begin{aligned} f\{F_{X_1}(x_{1,i}), F_{X_2}(x_{2,i}), F_{X_3}(x_{3,i})\} &= f(u_{1,i}, u_{2,i}, u_{3,i}) \\ &= f_1(u_{1,i}) \cdot f_2(u_{2,i}) \cdot f_3(u_{3,i}) \cdot \\ c_{12}(u_{1,i}, u_{2,i}|\theta_i^{12}) \cdot c_{13}(u_{1,i}, u_{3,i}|\theta_i^{13}) &\cdot c_{23|1}\{F(u_{2,i}|u_{1,i}), F(u_{3,i}|u_{1,i})|\theta_i^{23|1}\} \\ = c_{12}(u_{1,i}, u_{2,i}|\theta_i^{12}) \cdot c_{13}(u_{1,i}, u_{3,i}|\theta_i^{13}) &\cdot c_{23|1}\{F(u_{2,i}|u_{1,i}), F(u_{3,i}|u_{1,i})|\theta_i^{23|1}\}, \end{aligned} \tag{19}$$

where

$$F(u_{2,i}|u_{1,i}) = \frac{\partial C_{12}(u_{1,i}, u_{2,i}|\theta_i^{12})}{\partial u_{1,i}}, \tag{20}$$

$$F(u_{3,i}|u_{1,i}) = \frac{\partial C_{13}(u_{1,i}, u_{3,i}|\theta_i^{13})}{\partial u_{1,i}}. \tag{21}$$

In equation (19), $f_m(u_{m,i})=1$ ($m=1,2,3$, $i=1,2,\ldots,n$), since $u_{m,i}$ is uniformly distributed on [0,1]. Thus, the dependence structure of $\mathbf{X}_1', \mathbf{X}_2', \mathbf{X}_3'$ is characterized by the pair-copula with parameters θ^{12}, θ^{13}, and $\theta^{23|1}$, where θ^{12} and θ^{13} measure the dependence strength of the pairs ($\mathbf{X}_1'$, $\mathbf{X}_2'$) and ($\mathbf{X}_1'$, $\mathbf{X}_3'$), respectively, and $\theta^{23|1}$ quantifies the dependence strength of the pair ($\mathbf{X}_2'$, $\mathbf{X}_3'$) conditioned on $\mathbf{X}_1'$. Similar to asymmetric copula, the structure of the trivariate pair-copula is hierarchical with two levels, and each of the three copula parameters θ^{12}, θ^{13} and $\theta^{23|1}$ describes the relationship between the corresponding two variables (as shown in Figure 1c). Thus, the change-point detection for multivariate series can be decomposed into the issue for bivariate, and includes two steps: (i) detecting the change-points in θ^{12} and θ^{13}, (ii) detecting the change-point in $\theta^{23|1}$ on the basis of the results in step (i).

Finally, the goodness-of-fit test for symmetric copula, asymmetric copula, and pair-copula is performed by the probability integral transformation method (PIT) [*Breymann et al.*, 2003; *Aas et al.*, 2009]. As shown in Figure 1, symmetric copula with only one parameter seems to be the simplest among the three models, while pair-copula with three parameters is the most complex one. In order to select a proper model to build the joint distribution of multivariate hydrological series, the Akaike information criterion (AIC) [*Akaike*, 1974; *Zhang and Singh*, 2007], which penalizes the models with more parameters, is used to judge the fitting effect of each model.

Table 1. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Bivariate Time Series of Lengths $n \in \{50, 100, 200\}$, Which Are Generated Under the Null Hypothesis[a]

		CvM			CLR		
n	τ	GH	Fr	Cl	GH	Fr	Cl
50	0.3	5.1	5.1	4.5	4.5	5.5	4.7
50	0.5	3.6	5.8	6.0	3.4	5.8	5.4
50	0.7	5.2	4.7	5.0	5.6	4.2	5.2
100	0.3	5.1	3.6	5.1	4.8	4.6	4.3
100	0.5	5.3	5.6	4.9	5.0	6.1	5.4
100	0.7	5.8	5.0	5.1	5.6	4.8	5.1
200	0.3	4.3	4.6	5.2	5.0	5.1	4.8
200	0.5	5.4	7.6	5.2	5.3	5.3	6.9
200	0.7	5.9	6.4	4.8	5.3	5.9	6.0

[a]GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

3. Performance of the CvM and CLR Methods for Detecting Change-Points in Dependence Structure

In this study, the change-point detection in the marginal distribution of each individual univariate hydrological series is performed by the CvM method. Given that the change-point detection for univariate has been thoroughly studied in the literatures [*Perreault et al.*, 1999; *Xie et al.*, 2014; *Serinaldi and Kilsby*, 2015a], and thus it is not the study focus of this paper. For the performance of the CvM method in detecting change-point of univariate series, readers are referred to *Holmes et al.* [2013].

Since the detection of change-point in dependence structure (and/or degree of dependence) is a more difficult problem than that of changes in marginal distribution [*Holmes et al.*, 2013], it is a key aspect in the analysis of multivariate hydrological series. In this section, the performance of both the CvM and CLR methods in detecting the change-point in dependence structure of both bivariate and trivariate series is tested by large-scale Monte Carlo experiments.

3.1. Test of Ability to Hold the Nominal Level

In order to test how well both the CvM and CLR methods can hold the nominal level under the null hypothesis that there is no change in dependence structure, large-scale Monte Carlo experiments are carried out to estimate the percentage of rejection of the null hypothesis out of 1000 samples generated under the null hypothesis [*Holmes et al.*, 2013; *Bücher et al.*, 2014]. The one-parameter Gumbel-Hougaard (GH), Frank, and Clayton copulas, which all belong to Archimedean class of copulas [*Nelsen*, 2006], are used to generate both bivariate and trivariate random samples with uniform margins of [0,1]. Since the length of observed hydrological series usually varies from 50 to 200 [*Douglas et al.*, 2000; *Burn and Hag Elnur*, 2002; *Xiong and Guo*, 2004; *Favre et al.*, 2004; *Salvadori and De Michele*, 2004; *Grimaldi and Serinaldi*, 2006; *Serinaldi and Grimaldi*, 2007; *Zhang and Singh*, 2007; *Villarini et al.*, 2009, 2010; *Schmocker-Fackel and Naef*, 2010; *López and Francés*, 2013; *Bender et al.*, 2014; *Jiang et al.*, 2015], three respective sample lengths, i.e., $n \in \{50, 100, 200\}$ are selected. For trivariate case, symmetric copula, asymmetric copula, and pair-copula are employed to generate the random samples, respectively. For each copula parameter, three values are chosen to match the given levels of weak, medium, and strong dependence strength as measured by Kendall's τ of 0.3, 0.5, and 0.7, respectively. The significance level (i.e., the given nominal level) for both the CvM and CLR methods is set to 5%.

Tables (1–4) report the percentage of rejection of the null hypothesis when the null hypothesis holds. It can be seen that both the CvM and CLR methods perform well in holding the given nominal level (5%) for both bivariate and trivariate cases with minor fluctuations depending on sample length, dependence strength, sample dimension, and copula type (i.e., shape of dependence structure).

3.2. Power Test

In this section, the power of both the CvM and CLR methods in detecting the change-point in dependence structure is examined by estimating the percentage of rejection of the null hypothesis out of 1000 samples generated under the alternative hypothesis that there is a change-point in dependence structure. The power test has the same selections of the sample length, copula type, and significance level as the nominal

Table 2. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Trivariate Time Series of Lengths $n \in \{50, 100, 200\}$, Which Are Generated by the Trivariate Symmetric Copula Under the Null Hypothesis[a]

		CvM			CLR		
n	τ	GH	Fr	Cl	GH	Fr	Cl
50	0.3	3.0	4.4	4.3	2.5	4.5	4.1
50	0.5	4.6	5.2	5.2	4.7	4.7	4.2
50	0.7	5.8	4.8	5.2	3.4	4.5	3.8
100	0.3	4.7	5.9	4.4	5.0	3.5	4.6
100	0.5	5.3	4.5	4.7	5.7	5.5	4.5
100	0.7	3.9	4.2	5.2	5.0	4.6	4.8
200	0.3	4.4	4.5	4.3	4.7	5.3	4.8
200	0.5	5.5	4.4	3.9	4.9	4.6	4.9
200	0.7	4.6	4.2	4.3	4.5	4.9	5.5

[a]GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

level test performed in section 3.1. The abrupt changes in each copula parameter is taken to occur at the location of $n/2$, $n/4$, and $n/10$, and the change ranges of each copula parameter are defined to match the change ranges of dependence as measured by Kendall's τ of (0.3, 0.5), (0.3, 0.7), and (0.5, 0.7), respectively.

The results of the power test for both the CvM and CLR methods are shown in Tables (5–8). In general, the CLR method has a far larger rejection rate of the null hypothesis than that of the CvM method for all change-point scenarios. It means that the CLR method is more powerful than the CvM method in detecting the change-point in dependence structure. This finding matches the results presented by *Holmes et al.* [2013],

Table 3. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Trivariate Time Series of Lengths $n \in \{50, 100, 200\}$, Which Are Generated by the Trivariate Asymmetric Copula Under the Null Hypothesis[a]

					CLR					
		CvM			GH		Fr		Cl	
n	τ	GH	Fr	Cl	θ^1	θ^2	θ^1	θ^2	θ^1	θ^2
50	0.3	5.2	5.0	5.0	3.9	3.3	3.4	3.4	3.9	3.8
50	0.5	5.6	3.9	5.2	5.7	4.1	5.2	5.7	4.3	4.0
50	0.7	5.3	5.2	5.6	4.1	6.0	5.5	4.6	5.7	4.1
100	0.3	4.2	4.5	5.2	4.4	4.6	4.4	6.6	8.6	4.4
100	0.5	4.5	4.9	4.7	4.5	4.7	4.3	4.4	5.8	3.9
100	0.7	3.8	4.2	4.2	3.9	4.1	4.7	5.2	5.2	5.3
200	0.3	4.6	4.4	5.2	5.0	5.3	4.8	4.6	5.0	4.2
200	0.5	5.1	4.3	4.7	5.0	5.9	5.3	4.7	6.1	4.4
200	0.7	5.3	5.2	4.7	4.4	5.8	5.9	5.2	5.2	5.5

[a]GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

Table 4. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Trivariate Time Series of Lengths $n \in \{50, 100, 200\}$, Which Are Generated by the Trivariate Pair-Copula Under the Null Hypothesis[a]

					CLR								
		CvM			GH			Fr			Cl		
n	τ	GH	Fr	Cl	θ^{12}	θ^{13}	$\theta^{23\|1}$	θ^{12}	θ^{13}	$\theta^{23\|1}$	θ^{12}	θ^{13}	$\theta^{23\|1}$
50	0.3	5.7	4.4	4.3	3.6	4.6	5.2	5.0	4.4	4.3	4.9	3.6	4.4
50	0.5	5.5	4.9	3.7	3.9	4.1	3.4	4.7	5.4	5.4	4.6	5.3	4.1
50	0.7	4.2	4.1	4.4	4.6	4.5	4.8	3.6	4.8	5.2	4.8	5.6	4.1
100	0.3	5.6	5.6	4.9	5.3	4.7	4.5	6.0	4.7	4.6	3.1	4.3	3.4
100	0.5	5.1	4.8	5.2	3.8	5.9	4.4	4.9	5.0	3.6	4.0	4.1	5.3
100	0.7	5.0	5.9	5.6	4.4	4.3	4.3	4.7	3.9	5.2	4.5	5.2	5.4
200	0.3	5.9	6.1	4.7	3.7	4.8	4.0	3.7	4.9	4.2	5.1	5.9	5.1
200	0.5	6.3	5.7	6.0	4.1	4.3	4.1	4.3	4.9	4.3	4.6	4.4	4.8
200	0.7	6.1	5.6	6.0	3.7	3.5	4.4	4.4	4.1	3.7	4.5	4.4	4.3

[a]GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

Table 5. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Bivariate Time Series of Lengths $n \in \{50, 100, 200\}$, Which Are Generated Under the Alternative Hypothesis That There is a Change-Point in Dependence Structure[a]

			CvM			CLR		
n	CP	τ	GH	Fr	Cl	GH	Fr	Cl
200	100	(0.3, 0.5)	2.1	2.4	3.3	42.3	38.5	66.8
200	100	(0.3, 0.7)	9.9	9.3	9.2	100.0	100.0	100.0
200	100	(0.5, 0.7)	2.6	2.1	1.5	88.8	87.9	96.8
200	50	(0.3, 0.5)	0.9	0.5	0.6	35.8	31.0	52.5
200	50	(0.3, 0.7)	2.4	2.0	2.6	99.0	99.9	100.0
200	50	(0.5, 0.7)	0.5	0.5	0.4	77.6	74.8	90.3
200	20	(0.3, 0.5)	0.0	0.0	0.0	21.9	20.8	27.2
200	20	(0.3, 0.7)	0.0	0.0	0.0	91.2	90.5	97.4
200	20	(0.5, 0.7)	0.0	0.0	0.0	39.6	39.4	51.4
100	50	(0.3, 0.5)	2.3	2.1	1.5	20.5	22.2	29.6
100	50	(0.3, 0.7)	5.3	3.9	4.4	97.9	97.1	99.7
100	50	(0.5, 0.7)	1.7	2.0	1.7	50.9	47.6	67.0
100	25	(0.3, 0.5)	0.4	0.2	0.2	16.8	16.5	27.1
100	25	(0.3, 0.7)	1.1	0.5	1.1	93.4	92.9	98.2
100	25	(0.5, 0.7)	0.1	0.4	0.1	41.2	37.2	53.7
100	10	(0.3, 0.5)	0.0	0.0	0.0	16.2	17.0	18.2
100	10	(0.3, 0.7)	0.0	0.0	0.0	61.0	58.1	76.1
100	10	(0.5, 0.7)	0.0	0.0	0.0	22.1	23.5	28.0
50	25	(0.3, 0.5)	1.0	2.4	1.7	7.0	6.3	9.6
50	25	(0.3, 0.7)	2.9	2.3	3.2	76.9	71.7	88.2
50	25	(0.5, 0.7)	1.6	1.5	1.4	19.5	18.7	31.9
50	13	(0.3, 0.5)	0.3	0.2	0.1	4.5	5.6	12.1
50	13	(0.3, 0.7)	0.6	0.2	0.6	64.7	59.1	81.0
50	13	(0.5, 0.7)	0.1	0.2	0.3	17.1	17.7	29.3
50	5	(0.3, 0.5)	0.0	0.0	0.0	2.5	2.5	4.6
50	5	(0.3, 0.7)	0.0	0.0	0.0	32.6	27.6	43.3
50	5	(0.5, 0.7)	0.0	0.0	0.0	7.2	7.6	10.1

[a]CP is the location of the given change-point. GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

who found that the CvM method performs much better in detecting the change-point in marginal distribution than that in dependence structure. Actually, the test statistic in the CvM method (i.e., $\hat{S}_n$ in equation (4)) relies on the distance between empirical (nonparametric) copulas, therefore the CvM method is intrinsically affected by higher uncertainty, which leads to the poor performance in detecting the change-point in dependence structure. However, the test statistic in the CLR method (i.e., Z_n in equation (11)) is based on theoretical copulas, so the CLR method is much more robust.

As displayed in Tables (5–8), some factors such as sample length, change-point location, change range of dependence strength, sample dimension, and copula type have varying influences on the power of the CvM method or CLR method in detecting the change-point in dependence structure. In particular, the power of the both methods is an increasing function of the sample length. Both the CvM and CLR methods are more powerful when the change-point located in the middle of the samples, and perform poorly to detect the change-point located toward the extremes. In fact, as $n \to \infty$ and after suitable standardizations, both the test statistics $\hat{S}_n$ in the CvM method and Z_n in the CLR method converge to the supremum of a Brownian bridge [*Holmes et al.*, 2013; *Dias and Embrechts*, 2004], which is similar to the Mann-Whitney type statistics such as Pettitt test statistics [*Pettitt*, 1979; *Serinaldi and Kilsby*, 2015a]. It is known that the test such as the Mann-Whitney type is preferable if the real change-point occurs in the middle [*Gurevich*, 2009], which is consistent with what we have found.

For the both methods, the change-point with a larger change range in dependence strength is more easily to be detected. On the other hand, with the equal change range of dependence strength measured by τ, i.e., (0.3, 0.5) and (0.5, 0.7), the CLR method has a better performance for the samples with the higher dependence strength, i.e., in the range of (0.5, 0.7), while the CvM method is not sensitive to this difference in dependence strength. As for the influence from sample dimension, the CvM method has a larger rejection rate of the null hypothesis for the trivariate samples than that for the bivariate samples. While for the CLR method, the rejection rate of the null hypothesis for the trivariate samples generated by symmetric copula is larger than both the bivariate samples, and the trivariate samples generated by asymmetric copula

Table 6. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Trivariate Time Series of Lengths $n \in \{50, 100, 200\}$, Which Are Generated by the Trivariate Symmetric Copula Under the Alternative Hypothesis That There is a Change-Point in Dependence Structure[a]

			CvM			CLR		
n	CP	τ	GH	Fr	Cl	GH	Fr	Cl
200	100	(0.3, 0.5)	8.6	6.3	7.6	87.9	85.1	98.6
200	100	(0.3, 0.7)	39.1	33.5	28.3	100.0	100.0	100.0
200	100	(0.5, 0.7)	6.0	5.0	5.2	100.0	100.0	100.0
200	50	(0.3, 0.5)	1.5	1.6	0.8	73.7	69.2	92.8
200	50	(0.3, 0.7)	10.8	11.3	8.6	100.0	100.0	100.0
200	50	(0.5, 0.7)	1.6	1.6	1.0	98.5	98.2	99.9
200	20	(0.3, 0.5)	0.0	0.0	0.0	36.1	35.1	59.7
200	20	(0.3, 0.7)	0.0	0.0	0.0	99.8	99.7	100.0
200	20	(0.5, 0.7)	0.0	0.0	0.0	77.6	75.3	90.2
100	50	(0.3, 0.5)	4.3	4.5	3.4	51.9	45.0	76.4
100	50	(0.3, 0.7)	16.3	16.7	12.1	100.0	100.0	100.0
100	50	(0.5, 0.7)	3.1	3.8	3.1	92.2	90.7	98.0
100	25	(0.3, 0.5)	0.9	0.8	0.4	41.4	33.5	59.4
100	25	(0.3, 0.7)	2.1	3.7	2.1	99.9	99.8	100.0
100	25	(0.5, 0.7)	1.0	0.7	0.7	81.0	77.7	94.4
100	10	(0.3, 0.5)	0.0	0.0	0.0	18.8	21.3	33.7
100	10	(0.3, 0.7)	0.0	0.0	0.0	93.6	93.0	98.2
100	10	(0.5, 0.7)	0.0	0.0	0.0	43.5	42.6	61.1
50	25	(0.3, 0.5)	1.7	2.5	1.5	20.4	17.1	35.6
50	25	(0.3, 0.7)	7.5	7.2	6.7	99.1	99.2	99.9
50	25	(0.5, 0.7)	2.1	2.2	1.6	56.4	55.3	75.5
50	13	(0.3, 0.5)	0.3	0.8	0.3	17.3	13.0	29.0
50	13	(0.3, 0.7)	1.3	0.7	0.4	96.5	93.7	99.4
50	13	(0.5, 0.7)	0.3	0.7	0.5	46.0	45.0	63.3
50	5	(0.3, 0.5)	0.0	0.0	0.0	6.6	6.0	11.6
50	5	(0.3, 0.7)	0.0	0.0	0.0	68.2	64.0	81.6
50	5	(0.5, 0.7)	0.0	0.0	0.0	20.5	18.1	31.7

[a]CP is the location of the given change-point. GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

and pair-copula. The copula type seems to only affect the power of the CLR method. In general, the rejection rate for the samples generated by Clayton copula is larger than that for the samples generated by both GH and Frank copulas.

4. Application to the Trivariate Flood Series of the Upper Hanjiang River

4.1. Study Area and Data Set

The Hanjiang River with a catchment area of 159,000 km^2 is the largest tributary of the Yangtze River, China. For the purposes of electricity generation and flood control, several large-scale reservoirs with the capacity over 10^8 m^3 [*Ministry of Water Resources of People's Republic of China*, 1996] have been built in this basin in recent decades. It is found that the flow regime of the river has been influenced by these reservoirs [*Jiang et al.*, 2015].

In this study, we would like to perform a change-point analysis for the trivariate flood series composed of AMDD (Q_1), annual maxima 3 day flood volume (V_3), and annual maxima 15 day flood volume (V_{15}) observed at the Ankang hydrological station at the Upper Hanjiang River during the period of 1950–2011. The Ankang hydrological station has a catchment area of 38,600 km^2, where two large-scale reservoirs named Ankang and Shiquan have been built (see Figure 2). The Shiquan Reservoir, located about 200 km upstream the Ankang hydrological station, controls a catchment of 23,400 km^2. The construction of the Shiquan Reservoir was began from 1971, and finished in 1975. The Ankang Reservoir is located about 30 km upstream the Ankang hydrological station and controls a catchment of 35,700 km^2, which occupies more than 90% of the catchment area controlled by the Ankang hydrological station. The Ankang Reservoir was built from 1982, began to store water in 1989, and was completed in 1992. The capacity of the Ankang Reservoir is 3.21×10^9 m^3, which is far larger than that of the Shiquan Reservoir of 0.566×10^9 m^3. According to the reservoir index, which is defined by *López and Francés* [2013] and quantifies the relative effect of different reservoirs on the river flow according to the catchment areas and capacities of reservoirs, the flow regime of the Upper Hanjiang River observed at the Ankang hydrological station is mainly influenced by the Ankang reservoir [*Jiang et al.*, 2015].

Table 7. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Trivariate Time Series of Lengths $n \in \{50, 100, 200\}$, Which Are Generated by the Trivariate Asymmetric Copula Under the Alternative Hypothesis That There is a Change-Point in Dependence Structure[a]

						CLR					
			CvM			GH		Fr		Cl	
n	CP	τ	GH	Fr	Cl	θ^1	θ^2	θ^1	θ^2	θ^1	θ^2
200	100	(0.3, 0.5)	8.5	6.7	6.9	46.8	46.3	38.0	43.0	68.3	80.3
200	100	(0.3, 0.7)	38.2	33.5	28.9	100.0	100.0	100.0	100.0	100.0	100.0
200	100	(0.5, 0.7)	6.5	5.5	4.1	90.3	91.2	88.1	87.4	97.5	98.9
200	50	(0.3, 0.5)	1.1	1.5	1.1	35.1	35.1	30.7	34.0	53.1	65.5
200	50	(0.3, 0.7)	12.3	9.5	8.4	100.0	100.0	100.0	99.8	100.0	100.0
200	50	(0.5, 0.7)	2.0	0.9	0.9	76.1	79.3	74.4	76.1	90.3	93.7
200	20	(0.3, 0.5)	0.0	0.0	0.0	23.1	23.6	22.4	22.9	28.6	35.2
200	20	(0.3, 0.7)	0.0	0.0	0.0	93.1	93.7	89.3	91.4	96.9	99.6
200	20	(0.5, 0.7)	0.0	0.0	0.0	39.2	41.2	38.2	41.9	54.3	61.0
100	50	(0.3, 0.5)	3.7	4.0	3.3	20.3	21.4	18.7	19.0	29.8	40.3
100	50	(0.3, 0.7)	15.8	15.0	12.4	98.6	98.5	97.2	98.2	99.9	100.0
100	50	(0.5, 0.7)	3.6	3.4	3.2	48.6	52.7	46.9	49.3	66.9	76.1
100	25	(0.3, 0.5)	0.6	0.2	0.8	18.0	18.7	19.5	17.1	23.7	30.1
100	25	(0.3, 0.7)	2.4	3.5	2.5	95.2	94.8	90.5	93.3	98.4	99.7
100	25	(0.5, 0.7)	0.1	0.7	0.7	39.2	42.8	37.8	38.9	54.6	61.2
100	10	(0.3, 0.5)	0.0	0.0	0.0	15.4	16.2	17.2	17.2	20.3	20.5
100	10	(0.3, 0.7)	0.0	0.0	0.0	62.0	64.8	59.5	60.9	76.0	86.4
100	10	(0.5, 0.7)	0.0	0.0	0.0	23.7	24.1	22.2	23.9	30.4	33.1
50	25	(0.3, 0.5)	2.0	2.2	1.6	7.9	6.9	5.2	5.2	12.5	13.7
50	25	(0.3, 0.7)	7.5	5.7	5.6	75.5	76.1	69.7	74.1	90.6	94.5
50	25	(0.5, 0.7)	2.2	1.9	2.8	19.6	19.4	20.1	20.3	31.5	34.7
50	13	(0.3, 0.5)	0.2	0.2	0.3	5.2	4.7	5.5	5.9	9.4	14.7
50	13	(0.3, 0.7)	1.2	1.9	0.3	64.2	66.1	61.4	64.9	80.9	89.0
50	13	(0.5, 0.7)	0.4	0.2	0.6	15.6	17.6	16.3	17.1	26.3	30.3
50	5	(0.3, 0.5)	0.0	0.0	0.0	2.4	2.5	1.3	1.6	2.7	4.5
50	5	(0.3, 0.7)	0.0	0.0	0.0	32.0	31.2	30.6	32.9	45.8	56.1
50	5	(0.5, 0.7)	0.0	0.0	0.0	6.1	8.5	9.5	7.6	10.6	13.4

[a]CP is the location of the given change-point. GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

4.2. Autocorrelation and Long-Term Persistence Analysis for the Individual Flood Series

Table 9 displays the autocorrelations of lag-1 through lag-4 for each individual flood series. It can be found that the autocorrelations for all three individual flood series cannot pass the significance test at the 5% level, thus the autocorrelations in the three individual series should be considered as minor and can be ignored.

The long-term persistence of each individual flood series is tested by the Hurst exponent H [*Hurst*, 1951]. As done by *Villarini et al.* [2009], the aggregated variance method [*Montanari et al.*, 1999] is applied to estimate H. The results of the computed H together with the corresponding p value estimated by the bootstrap method [*Villarini et al.*, 2009] are also presented in Table 9. It can be seen that all Hurst exponents H vary between 0.847 and 0.870, which indicates that all individual flood series present a strong long-term persistence. It is interesting to note that short-term dependence (i.e., autocorrelation) is absent for all flood series of the case study, while significant long-term dependence (i.e., Hurst exponent) is present for all flood series. A similar finding was also found by *Lye and Lin* [1994] when they investigated the annual peak flows of 90 Canadian rivers for both short-term and long-term dependence.

4.3. Change-Point Analysis for the Individual Flood Series

The results of change-point detection for the three individual flood series by using the CvM method are presented in Table 10 and Figure 3. It can be found that all three individual flood series show significant change-points at the 5% significance level. The AMDD series Q_1 changed abruptly in 1987, and the change-points of both flood volume series V_3 and V_{15} occurred at the year of 1985. Obviously, the change-points in the individual flood series are closely linked with the time of the construction of the Ankang Reservoir. As shown in Table 10, both the mean and coefficient of variance (Cv) of each individual flood series have obvious changes. Particularly, all means present a decrease over 30%, while all Cvs have an obvious upward jump.

Table 8. Percentage of Rejection of the Null Hypothesis That Dependence Structure has No Change, Out of 1000 Samples of Trivariate Time Series of Lengths $n \in \{50, 100, 200\}$, which Are Generated by the Trivariate Pair-Copula Under the Alternative Hypothesis That There is a Change-Point in Dependence Structure[a]

						CLR								
			CvM			GH			Fr			Cl		
n	CP	τ	GH	Fr	Cl	θ^{12}	θ^{13}	$\theta^{23\|1}$	θ^{12}	θ^{13}	$\theta^{23\|1}$	θ^{12}	θ^{13}	$\theta^{23\|1}$
200	100	(0.3, 0.5)	7.3	8.4	7.5	44.7	45.5	44.8	37.9	43.0	40.2	64.2	64.9	65.1
200	100	(0.3, 0.7)	36.3	35.5	32.2	100.0	100.0	100.0	100.0	99.9	100.0	100.0	100.0	100.0
200	100	(0.5, 0.7)	3.9	5.1	5.8	88.2	88.0	89.8	87.0	86.1	88.3	96.3	96.0	97.4
200	50	(0.3, 0.5)	1.5	2.0	0.9	31.2	31.5	33.0	31.8	31.3	30.1	53.5	53.3	52.9
200	50	(0.3, 0.7)	11.0	12.5	10.6	99.9	99.9	100.0	99.9	99.9	99.8	100.0	100.0	100.0
200	50	(0.5, 0.7)	0.3	1.5	1.6	77.7	77.5	75.2	73.0	74.7	72.4	90.5	88.3	89.6
200	20	(0.3, 0.5)	0.0	0.0	0.0	23.0	21.3	21.8	1.7	21.2	22.1	24.9	24.6	26.8
200	20	(0.3, 0.7)	0.0	0.0	0.0	91.3	92.1	91.5	89.2	89.1	88.7	96.9	97.1	98.0
200	20	(0.5, 0.7)	0.0	0.0	0.0	38.7	40.0	41.0	32.4	35.4	37.2	54.9	53.8	54.1
100	50	(0.3, 0.5)	3.4	2.5	4.7	20.7	19.1	19.6	16.9	18.1	18.4	28.5	28.2	30.0
100	50	(0.3, 0.7)	15.6	16.7	15.2	95.3	98.4	98.2	97.1	97.5	98.4	99.9	99.8	99.5
100	50	(0.5, 0.7)	3.8	2.6	2.7	49.5	50.4	48.4	45.3	47.2	46.6	68.6	68.1	66.7
100	25	(0.3, 0.5)	1.0	0.2	0.4	21.5	17.8	18.6	17.5	18.0	19.0	27.8	26.2	25.6
100	25	(0.3, 0.7)	3.3	3.5	3.9	93.2	92.9	93.4	90.8	90.1	91.9	97.9	98.5	98.2
100	25	(0.5, 0.7)	0.2	0.5	0.5	42.2	41.7	41.9	39.0	38.4	40.6	55.7	53.7	56.8
100	10	(0.3, 0.5)	0.0	0.0	0.0	18.4	17.1	16.4	19.2	15.7	18.8	16.3	17.7	18.8
100	10	(0.3, 0.7)	0.0	0.0	0.0	61.9	63.5	60.6	59.2	58.6	57.1	78.3	76.1	75.9
100	10	(0.5, 0.7)	0.0	0.0	0.0	20.7	22.7	23.2	21.6	18.9	22.5	29.6	28.4	29.4
50	25	(0.3, 0.5)	2.0	1.8	2.3	6.0	7.2	6.8	7.0	6.6	5.6	11.3	10.0	11.3
50	25	(0.3, 0.7)	5.0	6.4	6.5	73.9	75.7	74.5	71.1	72.0	71.3	89.3	89.6	88.8
50	25	(0.5, 0.7)	2.8	2.0	1.9	20.8	21.6	24.8	17.6	16.8	17.3	32.8	32.2	31.1
50	13	(0.3, 0.5)	0.3	0.3	0.1	6.0	5.7	6.3	4.9	3.8	5.2	11.2	11.3	6.3
50	13	(0.3, 0.7)	1.4	0.7	0.0	64.3	64.9	66.6	60.2	61.8	56.7	87.3	86.3	86.3
50	13	(0.5, 0.7)	0.3	0.5	0.6	17.2	15.5	17.8	15.4	17.1	14.6	26.2	27.0	24.6
50	5	(0.3, 0.5)	0.0	0.0	0.0	2.3	1.2	2.4	1.7	1.8	1.9	4.8	4.2	4.8
50	5	(0.3, 0.7)	0.0	0.0	0.0	30.6	27.9	31.3	30.3	27.3	29.5	47.0	45.9	44.0
50	5	(0.5, 0.7)	0.0	0.0	0.0	7.7	6.8	5.4	7.5	7.0	8.0	12.6	10.8	10.7

[a]CP is the location of the given change-point. GH, Fr, and Cl stand for Gumbel-Hougaard, Frank, and Clayton copulas, respectively.

4.4. Change-Point Analysis for Dependence Structure

According to the results of change-point detection for the univariate flood series, the marginal distributions are estimated by equation (7). Then the change-point detection for the dependence structure of the trivariate flood series is performed by the CLR method, which has been proved to be more powerful than the CvM method according to the power test displayed in section 3.

First, the dependences of the pairs (Q_1, V_3), (Q_1, V_{15}), and (V_3, V_{15}) are evaluated by Kendall's tau τ, which is widely used as measure of dependence strength for bivariate variables [*Nelsen*, 2006]. The values of τ for (Q_1, V_3), (Q_1, V_{15}), and (V_3, V_{15}) are 0.852, 0.677, and 0.782, respectively.

In this paper, GH copula, which accounts for upper tail dependence and is suitable to capture the dependence structure of multivariate flood series [*Zhang and Singh*, 2007], is chosen to construct the dependence structure of the trivariate flood series (Q_1, V_3, V_{15}) of the Upper Hanjiang River. Three copula models, i.e., symmetric copula, asymmetric copula, and pair-copula are taken into consideration to build the joint distribution of the trivariate flood series.

The expression of the symmetric trivariate GH copula is given as follows:

$$\begin{aligned} &C\{F_{Q_1}(Q_1), F_{V_3}(V_3), F_{V_{15}}(V_{15})|\theta\} = C(u_{Q_1}, u_{V_3}, u_{V_{15}}|\theta) \\ &= \exp\left\{-\left[(-\ln u_{Q_1})^\theta + (-\ln u_{V_3})^\theta + (-\ln u_{V_{15}})^\theta\right]^{1/\theta}\right\}, \end{aligned} \tag{22}$$

where $u_{Q_1} = F_{Q_1}(Q_1)$, $u_{V_3} = F_{V_3}(V_3)$, and $u_{V_{15}} = F_{V_{15}}(V_{15})$ are the marginal probabilities of (Q_1, V_3, V_{15}). According to the degrees of dependence of pairs (Q_1, V_3), (Q_1, V_{15}), and (V_3, V_{15}) measured by Kendall's tau τ, the asymmetric trivariate GH copula for (Q_1, V_3, V_{15}) is given as follows:

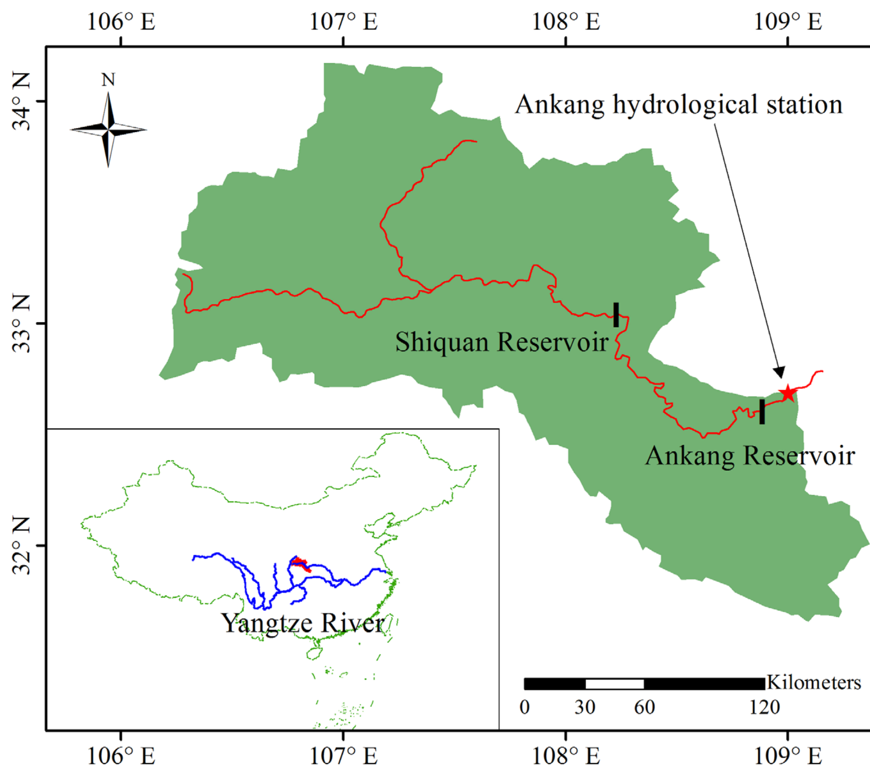

Figure 2. The map of the Hanjiang basin upstream the Ankang hydrological station.

$$\begin{aligned} C\{F_{Q_1}(Q_1), F_{V_3}(V_3), F_{V_{15}}(V_{15})|\theta^1, \theta^2\} &= C(u_{Q_1}, u_{V_3}, u_{V_{15}}|\theta^1, \theta^2) \\ &= C^{GH}\{u_{V_{15}}, C^{GH}(u_{Q_1}, u_{V_3}|\theta^1)|\theta^2\}, \end{aligned} \tag{23}$$

where $C^{GH}(\cdot)$ stands for bivariate GH copula.

In this paper, the peak discharge Q_1, which is related to the both flood volume variables V_3 and V_{15}, is regarded as the most important factor among the three flood variables Q_1, V_3, and V_{15}. Thus, with Q_1 being chosen as the key variable that governs the interactions in (Q_1, V_3, V_{15}), the form of the pair-copula for the joint distribution of (Q_1, V_3, V_{15}) is given as follows:

$$\begin{aligned} f\{F_{Q_1}(Q_1), F_{V_3}(V_3), F_{V_{15}}(V_{15})|\theta^{12}, \theta^{13}, \theta^{23|1}\} &= f\left(u_{Q_1}, u_{V_3}, u_{V_{15}}|\theta^{12}, \theta^{13}, \theta^{23|1}\right) \\ &= c_{12}^{GH}(u_{Q_1}, u_{V_3}|\theta^{12}) \cdot c_{13}^{GH}(u_{Q_1}, u_{V_{15}}|\theta^{13}) \cdot c_{23|1}^{GH}\{F(u_{V_3}|u_{Q_1}), F(u_{V_{15}}|u_{Q_1})|\theta^{23|1}\}, \end{aligned} \tag{24}$$

where $F(u_{V_3}|u_{Q_1})$ and $F(u_{V_{15}}|u_{Q_1})$ are estimated by equations (20) and (21). In equation (24), the dependence structure of (Q_1, V_3, V_{15}) is constructed by a pair-copula with three parameters, i.e., θ^{12}, θ^{13}, and $\theta^{23|1}$. Obviously, θ^{12} and θ^{13} measures the dependence strength of the pairs (Q_1, V_3) and (Q_1, V_{15}), respectively, and $\theta^{23|1}$ quantifies the dependence strength of the pair (V_3, V_{15}) conditioned on Q_1.

The results of change-point detection for dependence structure of (Q_1, V_3, V_{15}) by using the CLR method based on symmetric copula, asymmetric copula, and pair-copula are displayed in Table 11. The PIT test for

Table 9. Autocorrelation and Long-Term Persistence Analysis for Three Univariate Flood Series of the Upper Hanjiang River

	Autocorrelation					
Series	lag1	lag2	lag3	lag4	H	p Value of H
Q_1	−0.026	0.194	0.044	0.142	0.847	0.960
V_3	0.008	0.187	0.008	0.108	0.896	0.982
V_{15}	0.162	0.077	0.073	0.029	0.870	0.963

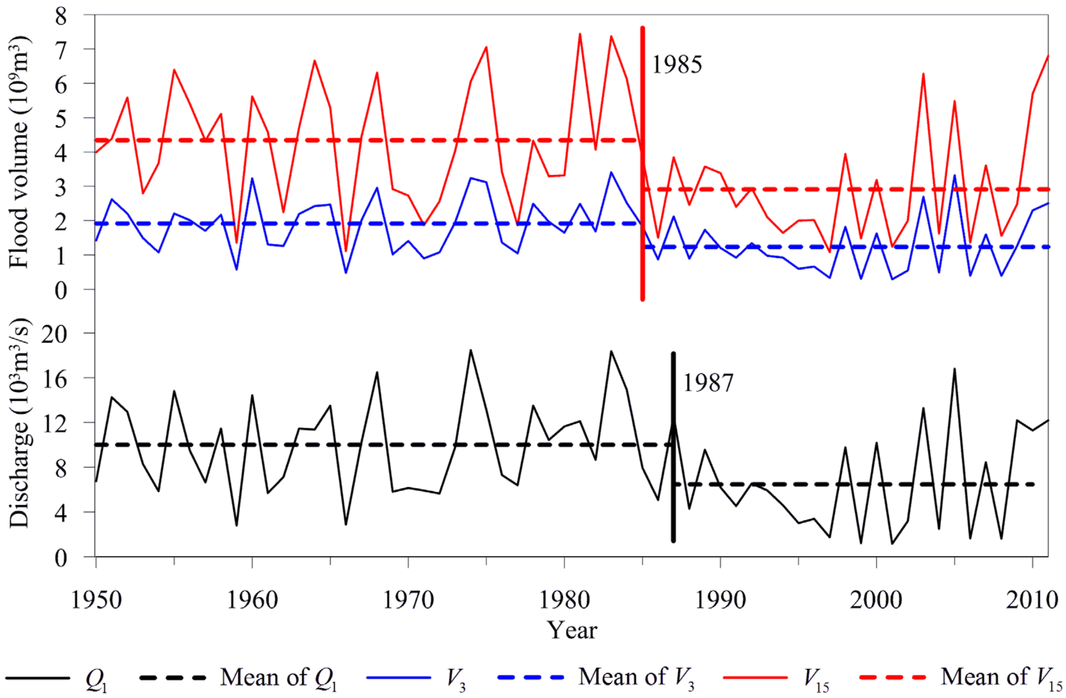

Figure 3. Change-points in three individual flood series of the Upper Hanjiang River.

the three copulas indicates that only asymmetric copula and pair-copula pass the goodness-of-fit test at the 5% significance level. Since the mutual dependence structures among (Q_1, V_3, V_{15}) are not identical, symmetric copula fails to capture the nature of the dependence structure. In terms of AIC values, pair-copula seems to be the best model to capture the dependence structure of (Q_1, V_3, V_{15}). The results based on pair-copula show that the dependence structure of the trivariate flood series presents an abrupt change in 1987,

Table 10. Change-Point Detection for Three Univariate Flood Series of the Upper Hanjiang River Based on CvM Method[a]

Series	CP	Mean				Cv		p Value
		Before CP	After CP	Absolute Change	Relative Change (%)	Before CP	After CP	
Q_1 (10^3 m^3/s)	1987	10.017	6.475	−3.542	−35.4	0.410	0.695	0.024
V_3 (10^9 m^3)	1985	1.915	1.235	−0.680	−35.5	0.399	0.672	0.009
V_{15} (10^9 m^3)	1985	4.342	2.911	−1.431	−33.0	0.393	0.556	0.006

[a]CP stands for the location of detected change-point, and Cv means coefficient of variance.

Table 11. Change-Point Detection for the Dependence Structure of the Trivariate Flood Series of the Upper Hanjiang River[a]

Construction of Distribution	Copula Parameter	Stationary Dependence	Dependence With Change-Point			p_{CP}	p_{PIT}	AIC
			CP	Before CP	After CP			
Symmetric copula	θ	2.888	1981	2.428	3.713	0.083	0.017	−173.88
Asymmetric copula	θ^1	4.323	1987	3.493	7.685	0.019	0.429	−203.98
	θ^2	2.622	1981	2.133	3.525	0.229		
Pair-copula	θ^{12}	4.323	1987	3.493	7.685	0.019	0.161	−213.24
	θ^{13}	2.363	1981	1.947	3.100	0.313		
	$\theta^{23\|1}$	1.652	1990	1.446	2.366	0.322		

[a]CP stands for the location of detected change-point. p_{CP} is the p value of the significance level of change-point, and if $p_{CP} < 0.05$ the copula parameter should have a significant change-point. p_{PIT} is the p value of PIT test for the copula constructing the dependence structure of the trivariate flood series, and a value of p_{PIT} bigger than 5% indicates that the copula can pass the goodness-of-fit test at the 5% significance level.

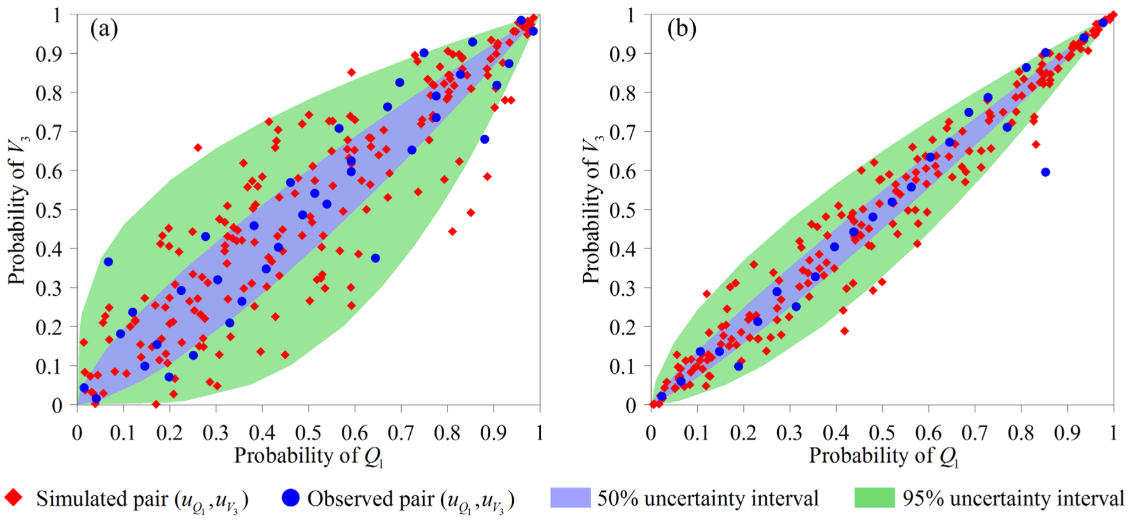

Figure 4. Comparison between scatterplots of pair (u_{Q_1}, u_{V_3}) before and after 1987. (a) Before 1987 and (b) after 1987.

when the copula parameter θ^{12}, which measures the dependence strength of (Q_1, V_3), jumps from 3.493 to 7.685. This finding means that it is the change in the relationship between Q_1 and V_3 that leads to the change in the dependence structure of (Q_1, V_3, V_{15}). Consistent with the change-points in marginal distributions, this change-point in dependence structure agrees with the construction time of the Ankang Reservoir. Therefore, the construction of the Ankang reservoir has also changed the dependence structure of the trivariate flood series.

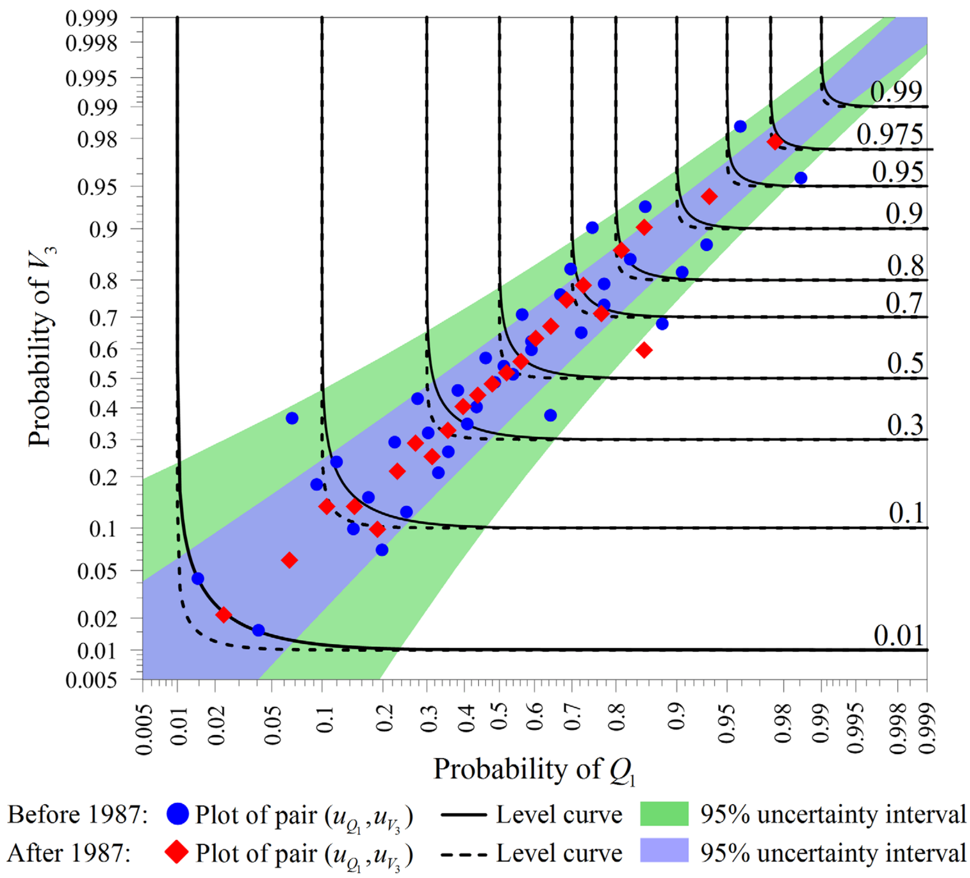

Figure 5. Comparison between the level curves of pair (u_{Q_1}, u_{V_3}) before and after 1987.

The Kendall's tau τ for (Q_1, V_3) before and after the change-point are calculated. Agreeing with the change in copula parameter θ^{12}, before 1987 the Kendall's tau τ for (Q_1, V_3) is 0.816, while after 1987 τ reaches up to 0.916. This finding means that the Ankang Reservoir enhances the dependence between Q_1 and V_3. As shown in Figure 4, both the observed and simulated pairs of (u_{Q_1}, u_{V_3}) are much more concentrated around the 45° line after the change-point. Using the method proposed by *Volpi and Fiori* [2012], the 50 and 95% uncertainty intervals of the distribution of (u_{Q_1}, u_{V_3}) are estimated. As shown by Figure 4, with the copula parameter increasing from 3.493 to 7.685, both uncertainty intervals become much narrower after 1987 than before. This indicates that the occurrences of Q_1 and V_3 are more inclined to have the same probability. Figure 5 presents the changes of the level curves corresponding to different given joint probabilities. It can be seen that the corner of each curve with the given joint probability after the change-point is sharper than that before the change-point due to the strengthening in the dependence between Q_1 and V_3. As a result, the joint probabilities for the given pairs of (u_{Q_1}, u_{V_3}) especially for those located around the 45° line get larger after the change-point.

5. Conclusions and Discussion

In this paper, a three-step framework is proposed for detecting change-point in multivariate hydrological series through making a clear distinction between the change-points in both marginal distributions and dependence structure. As the key aspect of change-point detection for multivariate hydrological series, the change-point in dependence structure is detected using both the CvM and CLR methods. For the CLR method, symmetric copula, asymmetric copula and pair-copula are employed to construct the dependence structure of multivariate hydrological series. This framework is applied to the trivariate flood series composed of annual maxima daily discharge, annual maxima 3 day flood volume, and annual maxima 15 day flood volume of the Upper Hanjiang River. The main findings are as follows:

1. Through large-scale Monte Carlo experiments, both the CvM and CLR methods perform well in holding the nominal level, while the CLR method appears to be far more powerful than the CvM method in detecting the change-point in dependence structure. This finding reveals the drawback of the CvM method, i.e., it is unsuitable for the multivariate series with change-point only hidden in dependence structure. It is also found that some characters of multivariate series such as sample length, change-point location, change range of dependence strength, sample dimension, and copula type (i.e., shape of dependence structure) can affect the ability of the CvM method or CLR method in detecting the change-point in dependence structure.
2. All univariate flood series Q_1, V_3, and V_{15} of the Upper Hanjiang River present significant change-points because of the construction of the Ankang Reservoir. For each flood series, the mean has a decrease over 30%, while the coefficient of variance has an obvious upward jump.
3. Pair-copula performs better than both symmetric and asymmetric copulas in describing the dependence structure of the trivariate flood series (Q_1, V_3, V_{15}), and a significant change-point in the dependence structure is found in 1987 because of the strengthening in the dependence between Q_1 and V_3. It is the construction of the Ankang Reservoir that leads to the change in the dependence structure of (Q_1, V_3, V_{15}).

In this paper, we perform a tentative study to detect the change-point in multivariate hydrological series, and there are two comments to be made as follows:

1. The first comment is the definition of the conception of nonstationarity in multivariate hydrological series. The nonstationarity in univariate hydrological series is generally defined as the variations of distribution parameters or moments over a certain time period [*Salas and Obeysekera*, 2014]. In this paper, the nonstationarity in a multivariate hydrological series is defined as the time-variations in both marginal distribution of each univariate series consisting of the multivariate series, and dependence structure among these univariate series. It is worth noting that we should be more careful when using the concept of nonstationarity to talk about the time series of hydrological processes, when there is no a clear cause-effect dynamics defined [*Serinaldi and Kilsby*, 2015b]. Since the observed period of hydrological records is very short compared with the whole period of the hydrological process, the representativeness of the observed series may not be sufficient. A so-called nonstationary series may be just a fragment in a long-term stationary process [*Koutsoyiannis*, 2006; *Koutsoyiannis and Montanari*, 2015]. Change does not necessarily imply nonstationarity and stationarity does not imply at all unchanging process state [*Montanari*

and Koutsoyiannis, 2014]. Consequently, the nonstationarity in the hydrological series cannot be judged simply from the result of a statistical test for the series itself, but should be supported by a well-defined cause-effect analysis.

2. The CLR method detects the change-point in dependence structure of multivariate hydrological series via measuring the change in the parameter(s) of a given copula, while the type of copula is assumed to be constant. Obviously, the CLR method is only able to detect changes in the strength of dependence structure, but not able to detect changes in the shape of dependence structure. Given that the shape of the dependence structure such as the tail dependence is important in estimating the risk of multivariate hydrological extremes [*Poulin et al.*, 2007], the change-point detection for the shape of dependence structure appears to be very necessary. Since the shape of dependence structure seems to be naturally linked to the type of copula, the possible change in the type of copula should be investigated in future researches of nonstationary hydrological frequency analysis.

Acknowledgments
This research is supported by the National Natural Science Foundation of China (grants 51190094, 51479139), which are greatly appreciated. Great thanks are due to the Associate Editor and reviewers for their constructive comments and suggestions that have helped the improvement of the manuscript. The data used in this paper can be requested by contacting the corresponding author L. Xiong at xionglh@whu.edu.cn.

References

Aas, K., and D. Berg (2009), Models for construction of multivariate dependence: A comparison study, *Eur. J. Finan.*, *15*(7–8), 639–659, doi:10.1080/13518470802588767.

Aas, K., C. Czado, A. Frigessi, and H. Bakken (2009), Pair-copula constructions of multiple dependence, *Insurance Math. Econ.*, *44*(2), 182–198, doi:10.1016/j.insmatheco.2007.02.001.

Akaike, H. (1974), A new look at the statistical model identification, *IEEE Trans. Autom. Control*, *19*(6), 716–723.

Ben Aissia, M. A., F. Chebana, T. B. M. J. Ouarda, L. Roy, G. Desrochers, I. Chartier, and E. Robichaud (2012), Multivariate analysis of flood characteristics in a climate change context of the watershed of the Baskatong reservoir, Province of Québec, Canada, *Hydrol. Processes*, *26*, 130–142, doi:10.1002/hyp.8117.

Ben Aissia, M. A., F. Chebana, T. B. M. J. Ouarda, L. Roy, P. Bruneau, and M. Barbet (2014), Dependence evolution of hydrological characteristics, applied to floods in a climate change context in Québec, *J. Hydrol.*, *519*, 148–163, doi:10.1016/j.jhydrol.2014.06.042.

Bender, J., T. Wahl, and J. Jensen (2014), Multivariate design in the presence of non-stationarity, *J. Hydrol.*, *514*, 123–130, doi:10.1016/j.jhydrol.2014.04.017.

Boubaker, H., and N. Sghaier (2013), Instability and dependence structure between oil prices and GCC stock markets, *Energy Study Rev.*, *20*(3), 50–65, doi:10.15173/esr.v20i3.555.

Bouzebda, S., and A. Keziou (2013), A semiparametric maximum likelihood ratio test for the change point in copula models, *Stat. Methodol.*, *14*, 39–61, doi:10.1016/j.stamet.2013.02.003.

Breymann, W., A. Dias, and P. Embrechts (2003), Dependence structures for multivariate high frequency data in finance, *Quant. Finan.*, 1, 1–14, doi:10.1080/713666155.

Bücher, A., I. Kojadinovic, T. Rohmer, and J. Segers (2014), Detecting changes in cross-sectional dependence in multivariate time series, *J. Multivariate Anal.*, *132*, 111–128, doi:10.1016/j.jmva.2014.07.012.

Burn, D. H., and M. A. Hag Elnur (2002), Detection of hydrologic trends and variability, *J. Hydrol.*, *225*, 107–122, doi:10.1016/S0022-1694(01)00514-5.

Chebana, F., T. B. M. J. Ouarda, and T. C. Duong (2013), Testing for multivariate trends in hydrologic frequency analysis, *J. Hydrol.*, *486*, 519–530, doi:10.1016/j.jhydrol.2013.01.007.

Chiew, F. H. S., and T. A. McMahon (1993), Detection of trend or change in annual flow of Australian rivers, *Int. J. Climatol.*, *13*, 643–653, doi:10.1002/joc.3370130605.

Csörgő, M., and L. Horváth (1997), *Limit Theorems in Change-Point Analysis*, chapter 1, pp. 1–66, Wiley, N. Y.

Dias, A., and P. Embrechts (2004), Change-point analysis for dependence structures in finance and insurance, in *Risk Measures for the 21st Century*, chapter 16, pp. 321–335, Wiley, Chichester.

Douglas, E. M., R. M. Vogel, and C. N. Kroll (2000), Trends in floods and low flows in the United States: Impact of spatial correlation, *J. Hydrol.*, *240*, 90–105, doi:10.1016/S0022-1694(00)00336-X.

Du, T., L. Xiong, C.-Y. Xu, C. J. Gippel, S. Guo, and P. Liu (2015), Return period and risk analysis of nonstationary low-flow series under climate change, *J. Hydrol.*, *527*, 234–250, doi:10.1016/j.jhydrol.2015.04.041.

El Adlouni, S., T. B. M. J. Ouarda, X. Zhang, R. Roy, and B. Bobée (2007), Generalized maximum likelihood estimators for the nonstationary generalized extreme value model, *Water Resour. Res.*, *43*, W03410, doi:10.1029/2005WR004545.

Favre, A. C., S. El Adlouni, L. Perreault, N. Thiémonge, and B. Bobée (2004), Multivariate hydrological frequency analysis using copulas, *Water Resour. Res.*, *40*, W01101, doi:10.1029/2003WR002456.

Genest, C., and A. C. Favre (2007), Everything you always wanted to know about copula modeling but were afraid to ask, *J. Hydrol. Eng.*, *12*(4), 347–368, doi:10.1061/(ASCE)1084-0699(2007)12:4(347).

Gombay, E., and L. Horváth (1996), On the rate of approximations for maximum likelihood tests in change-point models, *J. Multivariate Anal.*, *56*, 120–152, doi:10.1006/jmva.1996.0007.

Gombay, E., and L. Horváth (1999), Change-points and bootstrap, *Environmetrics*, *10*(6), 725–736, doi:10.1002/(SICI)1099-095X(199911/12)10:6 < 725::AID-ENV387 > 3.0.CO;2-K.

Grimaldi, S., and F. Serinaldi (2006), Asymmetric copula in multivariate flood frequency analysis, *Adv. Water Resour.*, *298*, 1115–1167, doi:10.1016/j.advwatres.2005.09.005.

Gringorten, I. I. (1963), A plotting rule for extreme probability paper, *J. Geophys. Res.*, *68*(3), 813–814, doi:10.1029/JZ068i003p00813.

Guégan, D., and J. Zhang (2010), Change analysis of a dynamic copula for measuring dependence in multivariate financial data, *Quant. Finan.*, *10*(4), 421–430, doi:10.1080/14697680902933041.

Guo, S. (1990), A discussion on unbiased plotting positions for generalized extreme value distribution, *J. Hydrol.*, *121*, 33–44, doi:10.1016/0022-1694(90)90223-K.

Gurevich, G. (2009), Asymptotic distribution of Mann–Whitney type statistics for nonparametric change point problems, *Comput. Model New Technol.*, *13*(3), 18–26.

Holmes, M., I. Kojadinovic, and J. Quessy (2013), Nonparametric tests for change-point detection à la Gombay and Horváth, *J. Multivariate Anal.*, *115*, 16–32, doi:10.1016/j.jmva.2012.10.004.

Hurst, H. E. (1951), Long-term storage capacity of reservoirs, *Trans. Am. Soc. Civ. Eng.*, *116*, 770–779.

Jiang, C., L. Xiong, C.-Y. Xu, and S. Guo (2015), Bivariate frequency analysis of nonstationary low-flow series based on the time-varying copula, *Hydrol. Processes*, *29*, 1521–1534, doi:10.1002/hyp.10288.

Joe, H. (1997), *Multivariate Models and Dependence Concept*, Chapman and Hall, N. Y.

Kao, S. C., and R. S. Govindaraju (2008), Trivariate statistical analysis of extreme rainfall events via the Plackett family of copulas, *Water Resour. Res.*, *44*, W02415, doi:10.1029/2007WR006261.

Kendall, M. G. (1975), *Rank Correlation Measures*, Charles Griffin, London, U. K.

Khaliq, M. N., T. B. M. J. Ouarda, J. C. Ondo, P. Gachon, and B. Bobeé (2006), Frequency analysis of a sequence of dependent and/or non-stationary hydrometeorological observations: A review, *J. Hydrol.*, *329*, 534–552, doi:10.1016/j.jhydrol.2006.03.004.

Koutsoyiannis, D. (2006), Nonstationarity versus scaling in hydrology, *J. Hydrol.*, *324*, 239–254, doi:10.1016/j.jhydrol.2005.09.022.

Koutsoyiannis, D., and A. Montanari (2015), Negligent killing of scientific concepts: The stationarity case, *Hydrol. Sci. J.*, *60*(7–8), 1174–1183, doi:10.1080/02626667.2014.959959.

Li, T., S. Guo, L. Chen, and J. Guo (2013), Bivariate flood frequency analysis with historical information based on copula, *J. Hydrol. Eng.*, *18*, 1018–1030, doi:10.1061/(ASCE)HE.1943-5584.0000684.

López, J., and F. Francés (2013), Non-stationary flood frequency analysis in continental Spanish rivers, using climate and reservoir indices as external covariates, *Hydrol. Earth Syst. Sci.*, *17*, 3189–3203, doi:10.5194/hess-17-3189-2013.

Lye, L. M., and Y. Lin (1994), Long-term dependence in annual peak flows of Canadian rivers, *J. Hydrol.*, *160*, 89–103, doi:10.1016/0022-1694(94)90035-3.

Manner, H., and B. Candelon (2010), Testing for asset market linkages: A new approach based on time-varying copulas, *Pac. Econ. Rev.*, *15*(3), 364–384, doi:10.1111/j.1468-0106.2010.00508.x.

Mediero, L., A. Jiménez-Álvarez, and L. Garrote (2010), Design flood hydrographs from the relationship between flood peak and volume, *Hydrol. Earth Syst. Sci.*, *14*, 2495–2505, doi:10.5194/hess-14-2495-2010.

Ministry of Water Resources of People's Republic of China (1996), *Design Criterion of Reservoir Management* [in Chinese], Chin. Water Resour. and Hydropower Press, Beijing.

Montanari, A., and D. Koutsoyiannis (2014), Modeling and mitigating natural hazards: Stationarity is immortal!, *Water Resour. Res.*, *50*, 9748–9756, doi:10.1002/2014WR016092.

Montanari, A., M. S. Taqqu, and V. Teverovsky (1999), Estimating longrange dependence in the presence of periodicity: An empirical study, *Math. Comput. Modell.*, *29*, 217–228, doi:10.1016/S0895-7177(99)00104-1.

Montanari, A., et al. (2013), Panta Rhei—Everything flows: Change in hydrology and societye in the pscientige decade 2013–2022, *Hydrol. Sci. J.*, *58*(6), 1256–1275, doi:10.1080/02626667.2013.809088.

Moraes, J. M., H. Q. Pellegrino, M. V. Ballester, L. A. Martinelli, R. Victoria, and A. V. Krusche (1998), Trends in hydrological parameters of southern Brazilian watershed and its relation to human induced changes, *Water Resour. Manage.*, *12*, 295–311, doi:10.1023/A:1008048212420.

Nelsen, R. B. (2006), *An Introduction to Copulas*, Springer, N. Y.

Perreault, L., M. Haché, M. Slivitsky, and B. Bobée (1999), Detection of changes in precipitation and runoff over eastern Canada and US using a Bayesian approach, *Stochastic Environ. Res. Risk Assess.*, *13*, 201–216, doi:10.1007/s004770050039.

Pettitt, A. N. (1979), A non-parametric approach to the change-point problem, *Appl. Stat.*, *28*, 126–135.

Poulin, A., D. Huard, A. C. Favre, and S. Pugin (2007), Importance of tail dependence in bivariate frequency analysis, *J. Hydrol. Eng.*, *12*(4), 394–403, doi:10.1061/ASCE1084-0699200712:4394.

Quessy, J., M. Saïd, and A. C. Favre (2013), Multivariate Kendall's tau for change-point detection in copulas, *Can. J. Stat.*, *41*(1), 65–82, doi:10.1002/cjs.11150.

Rasmussen, P. (2001), Bayesian estimation of change points using the general linear model, *Water Resour. Res.*, *37*, 2723–2731, doi:10.1029/2001WR000311.

Salas, J. D., and J. Obeysekera (2014), Revisiting the concepts of return period and risk for nonstationary hydrologic extreme events, *J. Hydrol. Eng.*, *19*(3), 554–568, doi:10.1061/(ASCE)HE.1943-5584.0000820.

Salvadori, G., and C. De Michele (2004), Analytical calculation of storm volume statistics involving Pareto-like intensity-duration marginals, *Geophys. Res. Lett.*, *31*, L04502, doi:10.1029/2003GL018767.

Salvadori, G., and C. De Michele (2010), Multivariate multiparameter extreme value models and return periods: A copula approach, *Water Resour. Res.*, *46*, W10501, doi:10.1029/2009WR009040.

Salvadori, G., C. De Michele, N. T. Kottegoda, and R. Rosso (2007), *Extremes in Nature: An Approach Using Copulas, Water Sci. Technol. Libr. Ser. 56*, Springer, Dordrecht, Netherlands.

Schmocker-Fackel, P., and F. Naef (2010), More frequent flooding? Changes in flood frequency in Switzerland since 1850, *J. Hydrol.*, *381*, 1–8, doi:10.1016/j.jhydrol.2009.09.022.

Serinaldi, F., and S. Grimaldi (2007), Fully nested 3-copula: Procedure and application on hydrological data, *J. Hydrol. Eng.*, *12*(4), 420–430, doi:10.1061/ASCE1084-0699200712:4420.

Serinaldi, F., and C. G. Kilsby (2015a), The importance of prewhitening in change point analysis under persistence, *Stochastic Environ. Res. Risk Assess.*, doi:10.1007/s00477-015-1041-5, in press.

Serinaldi, F., and C. G. Kilsby (2015b), Stationarity is undead: Uncertainty dominates the distribution of extremes, *Adv. Water Resour.*, *77*, 17–36, doi:10.1016/j.advwatres.2014.12.013.

Strupczewski, W. G., and Z. Kaczmarek (2001), Non-stationary approach to at-site flood frequency modeling II. Weighted least squares estimation, *J. Hydrol.*, *248*, 143–151, doi:10.1016/S0022-1694(01)00398-5.

Strupczewski, W. G., V. P. Singh, and W. Feluch (2001a), Non-stationary approach to at-site flood frequency modeling I. Maximum likelihood estimation, *J. Hydrol.*, *248*, 123–142, doi:10.1016/S0022-1694(01)00397-3.

Strupczewski, W. G., V. P. Singh, and H. T. Mitosek (2001b), Nonstationary approach to at-site flood frequency modeling III. Flood analysis of Polish rivers, *J. Hydrol.*, *248*, 152–167, doi:10.1016/S0022-1694(01)00399-7.

Villarini, G., and F. Serinaldi (2012), Development of statistical models for at-site probabilistic seasonal rainfall forecast, *Int. J. Climatol.*, *32*, 2197–2212, doi:10.1002/joc.3393.

Villarini, G., F. Serinaldi, J. A. Smith, and W. F. Krajewski (2009), On the stationarity of annual flood peaks in the Continental United States during the 20th Century, *Water Resour. Res.*, *45*, W08417, doi:10.1029/2008WR007645.

Villarini, G., J. A. Smith, and F. Napolitano (2010), Nonstationary modeling of a long record of rainfall and temperature over Rome, *Adv. Water Resour.*, *33*, 1256–1267, doi:10.1016/j.advwatres.2010.03.013.

Volpi, E., and A. Fiori (2012), Design event selection in bivariate hydrological frequency analysis, *Hydrol. Sci. J.*, *57*(8), 1506–1515, doi:10.1080/02626667.2012.726357.

Wong, H., B. Q. Hu, W. C. Ip, and J. Xia (2006), Change-point analysis of hydrological time series using grey relational method, *J. Hydrol.*, *324*, 323–338, doi:10.1016/j.jhydrol.2005.10.007.

Xie, H., D. Li, and L. Xiong (2014), Exploring the ability of the Pettitt method for detecting change point by Monte Carlo simulation, *Stochastic Environ. Res. Risk Assess.*, *28*(7), 1643–1655, doi:10.1007/s00477-013-0814-y.

Xiong, L., and S. Guo (2004), Trend test and change-point detection for the annual discharge series of the Yangtze River at the Yichang hydrological station, *Hydrol. Sci. J.*, *49*(1), 99–112, doi:10.1623/hysj.49.1.99.53998.

Xiong, L., K. Yu, and L. Gottschalk (2014), Estimation of the distribution of annual runoff from climatic variables using copulas, *Water Resour. Res.*, *50*, 7134–7152, doi:10.1002/2013WR015159.

Ye, W., X. Liu, and B. Miao (2012), Measuring the subprime crisis contagion: Evidence of change point analysis of copula functions, *Eur. J. Oper. Res.*, *222*, 96–103, doi:10.1016/j.ejor.2012.04.004.

Yue, S., P. Pilon, and G. Cavadias (2002), Power of the Mann–Kendall and Sperman's rho tests for detecting monotonic trends in hydrological series, *J. Hydrol.*, *259*, 254–271, doi:10.1016/S0022-1694(01)00594-7.

Zhan, D., and S. Ye (2000), *Engineering Hydrology* [in Chinese], 3rd ed., China Waterpower Press, Beijing.

Zhang, L., and V. P. Singh (2007), Trivariate flood frequency analysis using the Gumbel–Hougaard copula, *J. Hydrol. Eng.*, *12*, 431–439, doi:10.1061/(ASCE)1084-0699(2007)12:4(431).