Cable Dome

3D Cable Nets. Hyperbolic Paraboloids.

The Dorton Arena in North Carolina has a roof supported by a net of cables. The cable net lies in the surface of a hyperbolic paraboloid. This is a prestressed system; that's how deflections are controlled.

We consider the statics of cable nets made of two, four, six and eight cables to build up an understanding of cable nets, and to see the characteristic saddle shape of hyperbolic paraboloids. Then we look at a real one: the Dorton arena in North Carolina.

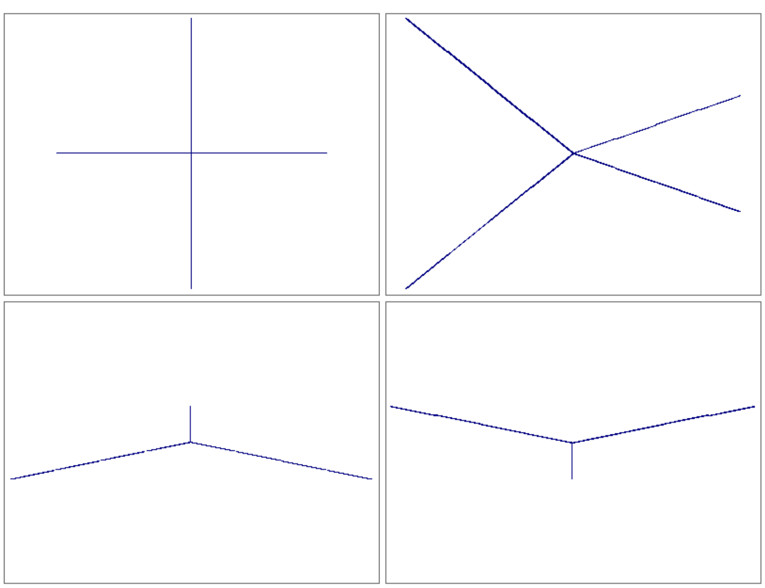
Two cables, single attachment point

The simplest 3D cable net has two cables. One is the load-carrying cable; called the main cable. The other is the prestressing cable; called the guy cable. The cables are in separate vertical planes; unlike the two-cable truss that we studied previously. Often the two cables are orthogonal.

For this first net, both cables span 200 ft. The main cable has sag equal to 20 ft. The guy cable has hog (upward sag) equal to 20 ft. The roofing is steel pan plus waterproofing. The deck weighs 12psf. Snow load is 20 psf. Deck weight and snow are applied as concentrated loads at the point where cables intersect.

The tributary area (on projection) for the cable intersection point is 10,000 SF. This gives permanent load equal to 120k, and transient load equal to 200k. Prestress is 80k.

Reference state is permanent load plus prestress.



Initial design

Cable	Href k	Htotal k	Tmax k	Tu tn	Select
Main	500	1000	1050	1154	4-1/2"
guy	200	0	210	231	2”

Force-displacement analysis

Final cable sizes after revision and re-analysis are shown.

Node Coordinates

Node	X ft	Y ft	Z ft	Sx	Sy	Sz
north	0	100	20	True	True	True
south	0	-100	20	True	True	True
east	100	0	-20	True	True	True
west	-100	0	-20	True	True	True
1	0	0	0			

Reference State

Cable	H k
main	509.523
guy	200

Deflection

Node	Dx ft	Dy ft	Dz ft
1	0	0	-0.96

Components

Cable	T max k	Tu tn	Select
main 0	1607	84	3-3/4”
guy 0	210	231	2”

Four cables, Four attachment points

A second version of the same roof is made of four cables. Two cables are load-carrying. Two cables are prestressing cables.

All cables span 200 ft. Intersection points along the main cables are on a parabola that has sag equal to 20 ft. Bur there are no nodes at mid span. There are nodes at L/3. So we see a vertical position of these nodes equal to 17.778 ft Guy cables have nodes on a similar parabola with hog (upward sag) equal to 20 ft. Here too, there are nodes at L/3, so we see a vertical position of nodes equal to 17.778 ft.

Roofing is steel pan plus waterproofing. The deck weighs 12psf. Snow load is 20 psf. Deck weight and snow are applied as concentrated loads at points where cables intersect.

The tributary area (on projection) for each cable intersection point is 4,444 SF. This gives permanent load equal to 53.3 k, and transient load equal to 88.9 k. Prestress is 35.6 k.

Reference state is permanent load plus prestress.

Note the geometry of a cable with equal concentrated loads at third points along the span.

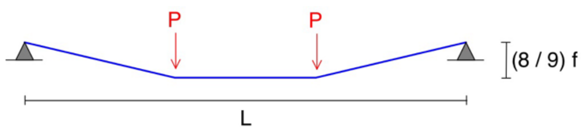
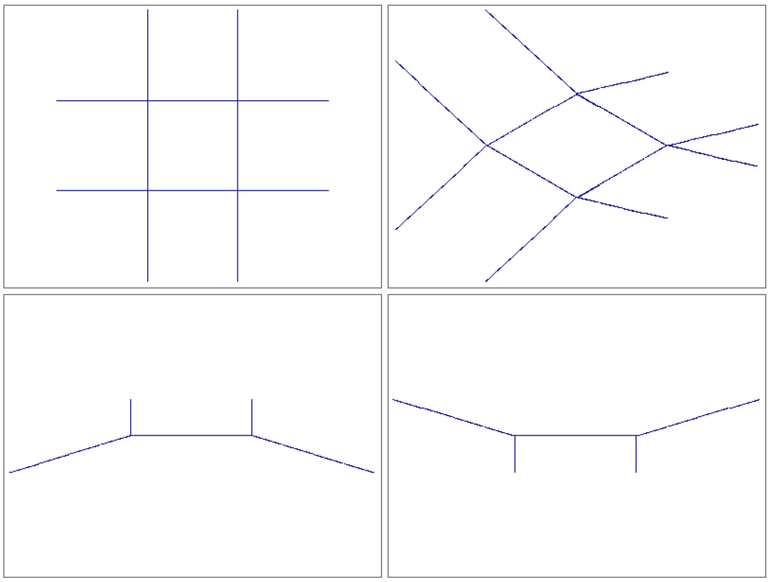


Figure 1

Statics for the cable with point loads:

H = P (15 / 4)

T = H sqrt(241 / 225)



Initial design

Cable	Href k	Htotal k	Tmax k	Tu tn	Select
Main	133	667	138	759	3-3/4”
guy	133	0	690	152	1-5/8”

Force-displacement analysis.

Final cable sizes after revision and re-analysis are shown.

Node Coordinates

Node	X ft	Y ft	Z ft	Sx	Sy	Sz
north 0	33.3	100	17.8	I True	TRUE	TRUE
north 1	-33.3	100	17.8	TRUE	TRUE	TRUE
south 0	33.3	-100	17.8	TRUE	TRUE	TRUE
south 1	-33.3	-100	17.8	TRUE	TRUE	TRUE
east 0	100	33.3	-17.8	TRUE	TRUE	TRUE
east 1	100	-33.3	-17.8	TRUE	TRUE	TRUE
west 0	-100	33.3	-17.8	TRUE	TRUE	TRUE
west 1	-100	-33.3	-17.8	TRUE	TRUE	TRUE
1	33.3	33.3	0			
2	-33.3	33.3	0			
3	-33.3	-33.3	0			
4	33.3	-33.3	0			

Reference State

Cable	H k
main	339.82
guy	133.33

Deflection

Node	Dx ft	Dy ft	Dz ft
1	-0.05	0.05	-0.57
2	0.05	0.05	-0.57
3	0.05	-0.05	-0.57
4	-0.05	-0.05	-0.57

Components

Cable	T max k	Tu tn	Select
main 0	1107	554	3-5/8”
guy 0	1265	138	1-5/8”

Six cables, Nine attachment points

A third version of the same roof is made of six cables. Three cables are load-carrying. Three cables are prestressing cables.

All cables span 200 ft. Main cables have sag equal to 20 ft. Guy cables have hog (upward sag) equal to 20 ft. The roofing is steel pan plus waterproofing. The deck weighs 12psf. Snow load is 20 psf. Deck weight and snow are applied as concentrated loads at points where cables intersect.

The tributary area (on projection) for each cable intersection point is 2,500 SF. This gives permanent load equal to 30 k, and transient load equal to 50 k. Prestress is 20 k.

Reference state is permanent load plus prestress.

Note the geometry of a cable with equal concentrated loads at quarter points along the span.

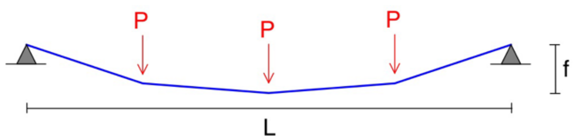


Figure 2

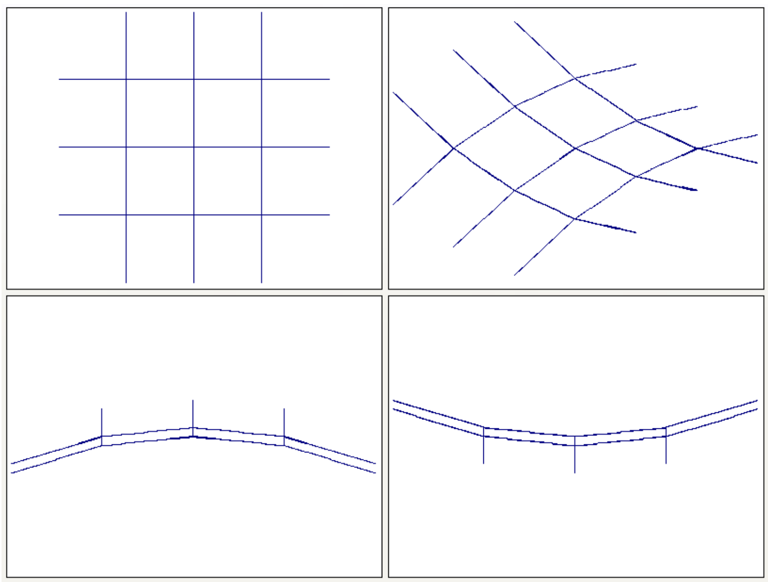
Statics for the cable with point loads:

H = 5 P

T = H sqrt(109 / 100)

All six cables have vertical positions of nodes placed on parabolas with span equal to 200 ft and sag or hog equal to 20 ft. For all six cables to have compatible z-coordinates at all intersection points, support points must be at different z-coordinates. Support points are placed on parabolas too.

Vertical positions of nodes must be adjusted to get the cable shape right to carry cable self weight. We set up joint equilibrium relations, use row reductions, etc. The outcomes for node coordinates are listed in the table.



Initial design

Cable	Href k	Htotal k	Tmax k	Tu tn	Select
Main	250	500	522	574	3-1/8”
guy	100	0	104	115	1-3/8”

Force-displacement analysis.

Final cable sizes after revision and re-analysis are shown.

Reference State

Cable	H k
main	254.857
guy	100

Node Coordinates

Node	X ft	Y ft	Z ft	Sx	Sy	Sz
north 0	50	100	15	True	True	True
north 1	0	100	20	True	True	True
north 2	-50	100	15	True	True	True

south 0	50	-100	15	True	True	True
south 1	0	-100	20	True	True	True
south 2	-50	-100	15	True	True	True
east 0	100	50	-15	True	True	True
east 1	100	0	-20	True	True	True
east 2	100	-50	-15	True	True	True
west 0	-100	50	-15	True	True	True
west 1	-100	0	-20	True	True	True
west 2	-100	-50	-15	True	True	True
1	50	50	-0.000686981			
2	50	0	-5.000400882			
3	50	-50	-0.000686981			
4	0	50	4.998991864			
5	0	0	0			
6	0	-50	4.998991864			
7	-50	50	-0.000686981			
8	-50	0	-5.000400882			
9	-50	-50	-0.000686981			

Deflection

Node	Dx ft	Dy ft	Dz ft
1	-0.08	0.08	-0.59
2	-0.1	0	-0.73
3	-0.08	-0.08	-0.59
4	0	0.08	-0.61
5	0	0	-0.75
6	0	-0.08	-0.61
7	0.08	0.08	-0.59
8	0.1	0	-0.73
9	0.08	-0.08	-0.59

Components

Cable	T max k	Tu tn	Select
main 0	460	341	2-3/8”
guy 0	104	115	1-1/4”

Eight cables, Sixteen attachment points

Another version. The net is expanded to eight cables. Four are main cables and four are guy cables. The tributary area for each intersection point is 1600 SF. At each intersection, roof deck load is 19.2 k and snow load is 32 k. Prestress is 12.8 k

All nodes are place on parabolas with 200 ft span and 20 ft sag or hog. Supports points also lie on parabolas. Note the statics for a cable

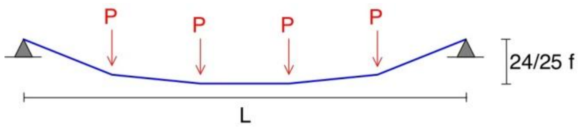
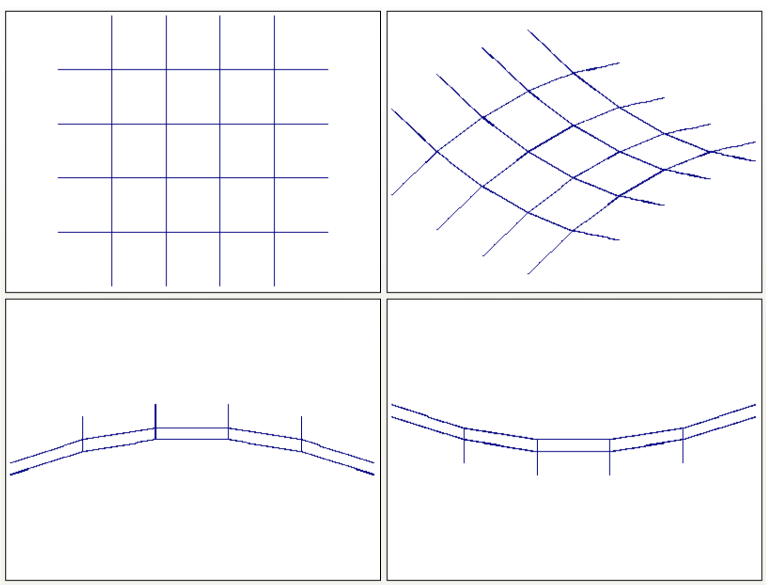


Figure 3

H = (25/4) P

T = H sqrt(689/625)

Reference state is permanent load plus prestress. Use joint equilibrium relations and row reduction to get z-coordinates with cable self weight in place.



Initial design

Cable	Href k	Htotal k	Tmax k	Tu tn	Select
Main	200	400	420	462	2-7/8”
guy	80	0	84	92	1-1/4”

Final design

after force-displacement analysis and selection of new main calbe

Node Coordinates

Node	X ft	Y ft	Z ft	Sx	Sy	Sz
north 0	60	100	12.8	True	True	True
north 1	20	100	19.2	True	True	True
north 2	-20	100	19.2	True	True	True
north 3	-60	100	12.8	True	True	True
south 0	60	-100	12.8	True	True	True
south 1	20	-100	19.2	True	True	True
south 2	-20	-100	19.2	True	True	True
south 3	-60	-100	12.8	True	True	True
east 0	100	60	-12.8	True	True	True
east 1	100	20	-19.2	True	True	True
east 2	100	-20	-19.2	True	True	True
east 3	100	-60	-12.8	True	True	True
west 0	-100	60	-12.8	True	True	True
west 1	-100	20	-19.2	True	True	True
west 2	-100	-20	-19.2	True	True	True
west 3	-100	-60	-12.8	True	True	True
1	60	60	-0.000716814			
2	60	20	-6.400580114			
3	60	-20	-6.400580114			
4	60	-60	-0.000716814			
5	20	60	6.399863301			
6	20	20	0.000716814			
7	20	-20	0.000716814			
8	20	-60	6.399863301			
9	-20	60	6.399863301			
10	-20	20	0.000716814			
11	-20	-20	0.000716814			
12	-20	-60	6.399863301			
13	-60	60	-0.000716814			
14	-60	20	-6.400580114			
15	-60	-20	-6.400580114			
16	-60	-60	-0.000716814			

Reference State

Cable	H k
main	203.793
guy	80

Deflection

Node	Dx ft	Dy ft	Dz ft
1	-0.09	0.08	-0.5
2	-0.12	0.04	-0.67
3	-0.12	-0.04	-0.67
4	-0.09	-0.08	-0.5
5	-0.03	0.08	-0.51
6	-0.04	0.04	-0.69
7	-0.04	-0.04	-0.69
8	-0.03	-0.08	-0.51

Components

Cable	T max k	Tu tn	Select
main 0	370	341	2-3/8”
guy 0	84.0	92.4	1-1/4”

9	0.03	0.08	-0.51		
10	0.04	0.04	-0.69		
11	0.04	-0.04	-0.69		
12	0.03	-0.08	-0.51		
13	0.09	0.08	-0.5		
14	0.12	0.04	-0.67		
15	0.12	-0.04	-0.67		
16	0.09	-0.08	-0.5		

Properties of hyperbolic paraboloids

The shape of the surface is

$$z = \alpha x^2 - \beta y^2 \qquad \text{Eq 1}$$

Introduce span L_x and sag f_x for the x-direction, and span L_y and hog f_y for the y-direction, Eq 1 becomes

$$z = \frac{4f_x}{{L_x}^2}x^2 - \frac{4f_y}{{L_y}^2}y^2 \qquad \text{Eq 2}$$

The surface is curved in some directions, and linear in others. Consider a trace on the surface such that

$$y = \sqrt{\frac{f_x}{{L_x}^2}\frac{{L_y}^2}{f_y}}x \qquad \text{Eq 3}$$

Substitute for y in Eq 2. We find that z is zero for all (x,y) pairs that satisfy Eq 3. The pairs (x, y) lie on a horizontal line.

$$z = 0 \qquad \text{Eq 4}$$

Any hypar has two linear, horizontal traces. The other trace is obtained when

$$y = -\sqrt{\frac{f_x}{{L_x}^2}\frac{{L_y}^2}{f_y}}x \qquad \text{Eq 5}$$

Both horizontal traces pass through the origin of the coordinate system.

Now consider a trace on the surface such that

$$y = \frac{L_y}{2} - x\frac{L_y}{L_x}\sqrt{\frac{f_x}{f_y}} \qquad \text{Eq 6}$$

Square this and substitute into Eq 2. This is a line with a z-direction slope.

$$z = -f_y + 4f_y\frac{x}{L_x}\sqrt{\frac{f_x}{f_y}} \qquad \text{Eq 7}$$

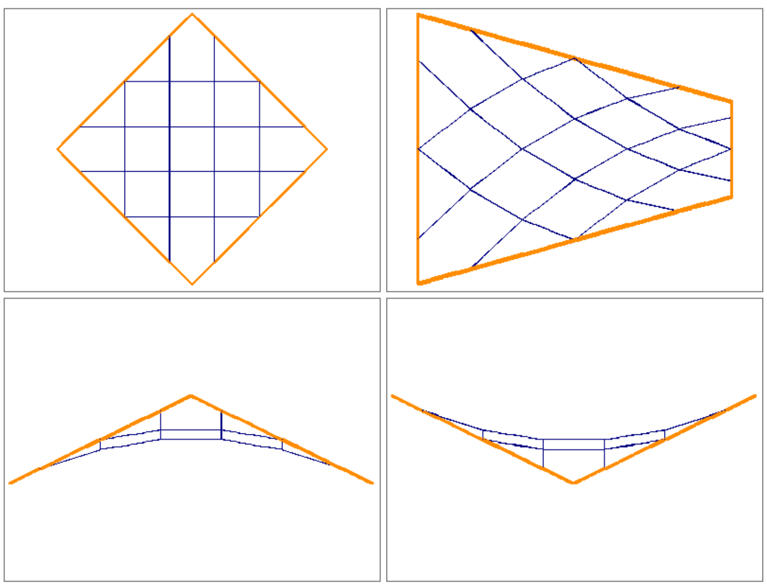
This is a single instance. A hyper has an infinite set of linear, sloped traces.

Sixteen Points with Support Rim

This cable net has eight cables. We place support nodes along linear traces on the surface of the hypar. In a real structure, these support nodes would be along a support beam or rim. That support rim is a linear element. That's easy to build. The geometry of the rim is not complex.

Cable geometry remains the same as the previous net with four cables each way. All nodes are placed along parabolas with 200 ft span and 20 ft sag or hog. For this arrangement of supports, we get portions of these parabolas; central portions.

Load information and prestress values are identical to our 4x4 cable net.



Final Design (adapted from 4x4 cable net)

Reference State

Cable	H k
main	203.778
guy	80

Node Coordinates

north 1	20	100	19.2	TRUE	TRUE	TRUE
north 2	-20	100	19.2	TRUE	TRUE	TRUE
south 1	20	-100	19.2	TRUE	TRUE	TRUE
south 2	-20	-100	19.2	TRUE	TRUE	TRUE
east 1	100	20	-19.2	TRUE	TRUE	TRUE
east 2	100	-20	-19.2	TRUE	TRUE	TRUE
west 1	-100	20	-19.2	TRUE	TRUE	TRUE
west 2	-100	-20	-19.2	TRUE	TRUE	TRUE

1	60	60	0	TRUE	TRUE	TRUE
2	60	20	-6.400622265			
3	60	-20	-6.400622265			
4	60	-60	0	TRUE	TRUE	TRUE
5	20	60	6.399212714			
6	20	20	0			
7	20	-20	0			
8	20	-60	6.399212714			
9	-20	60	6.399212714			
10	-20	20	0			
11	-20	-20	0			
12	-20	-60	6.399212714			
13	-60	60	0	TRUE	TRUE	TRUE
14	-60	20	-6.400622265			
15	-60	-20	-6.400622265			
16	-60	-60	0	TRUE	TRUE	TRUE
north	0	120	28.8	TRUE	TRUE	TRUE
south	0	-120	28.8	TRUE	TRUE	TRUE
east	120	0	-28.8	TRUE	TRUE	TRUE
west	-120	0	-28.8	TRUE	TRUE	TRUE

Deflection

Node	Dx ft	Dy ft	Dz ft
1	0	0	0
2	-0.09	0.03	-0.57
3	-0.09	-0.03	-0.57
4	0	0	0
5	-0.03	0.08	-0.51
6	-0.04	0.04	-0.7
7	-0.04	-0.04	-0.7
8	-0.03	-0.08	-0.51
9	0.03	0.08	-0.51
10	0.04	0.04	-0.7
11	0.04	-0.04	-0.7
12	0.03	-0.08	-0.51
13	0	0	0
14	0.09	0.03	-0.57
15	0.09	-0.03	-0.57
16	0	0	0

Components

Cable	T max k	Tu tn	Select
main 0	371	341	2-3/8”
guy 0	84.0	92.4	1-1/4”

Dorton Arena

That last hypar we set up, the one with the straight-line rims for support, is similar to the roof structure of the Dorton Arena. At our course calendar an article on the Dorton Arena linked at Jan 12. Read through the article.

Here is a photo of the arena (Figure 4). The support rim is in two parts and it is curved in plan. Each part lies in a plane; a plane inclined to the ground surface. Here is a photo near the end of construction.

Figure 4

Here is a more recent photo (Figure 5). Gotta keep folks from taking a stroll on the rims.

Figure 5

Here is another construction photo (Figure 6Figure 6). The near rim is complete. The far rim is still being poured. The vertical elements support the rim, and also support the glazing for the arena. Note the temporary stair on the rim for access by workers. Note the wooden spools in front of the building. That's how the strand was brought to the site. Zoom in and you'll just barely see "Bethlehem Wire Rope".

Figure 6

Another construction photo (Figure 7). The main cables are installed, but they are hanging limp. The prestressing cables are not in place yet. The seating area has been poured, and mullions are in place.

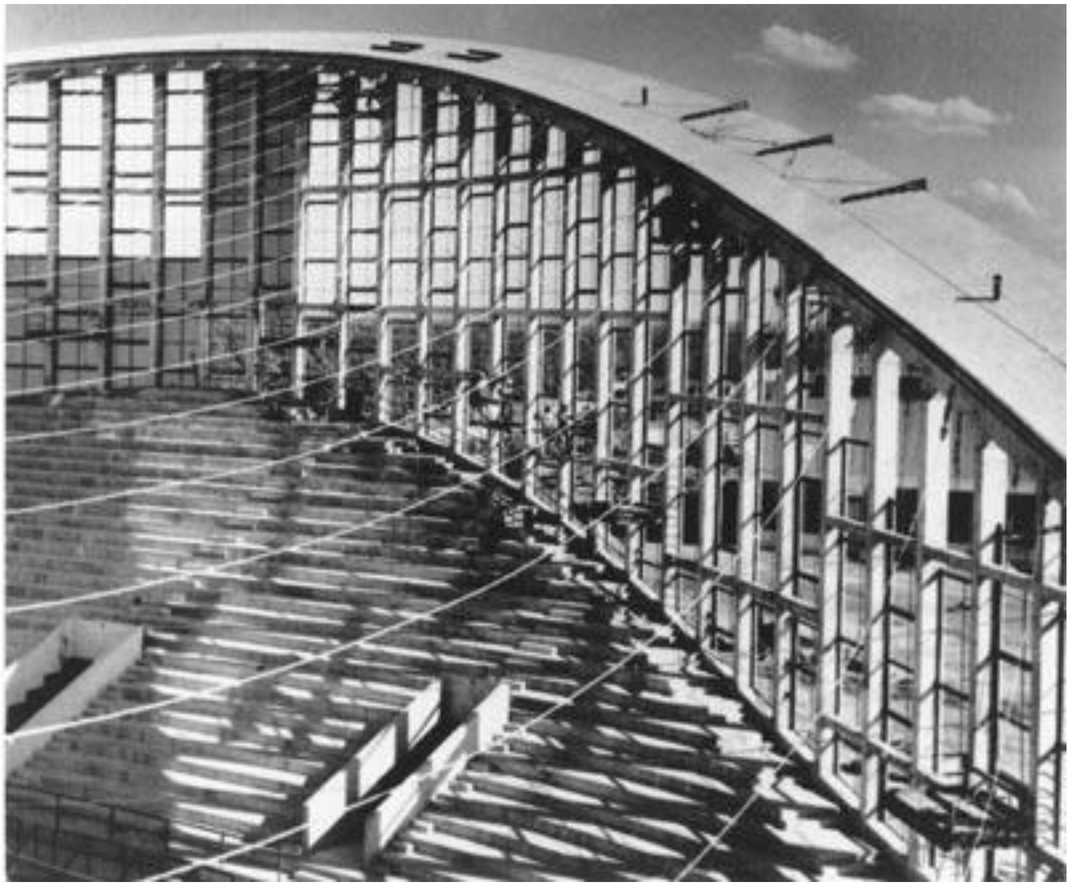


Figure 7

Cables are anchored *through*, not *at*, the rim (Figure 8). The anchor detail at the top left corner of the sheet. Pipes, some straight and some curved, are placed as sleeves in the concrete rim. Later, strands are threaded through and held with a threaded rod. Fred Severud was the structural engineer. He was also the engineer of record for the Madison Square Garden roof, though the design was the work of his assistant, Ron Mayrbaurl.

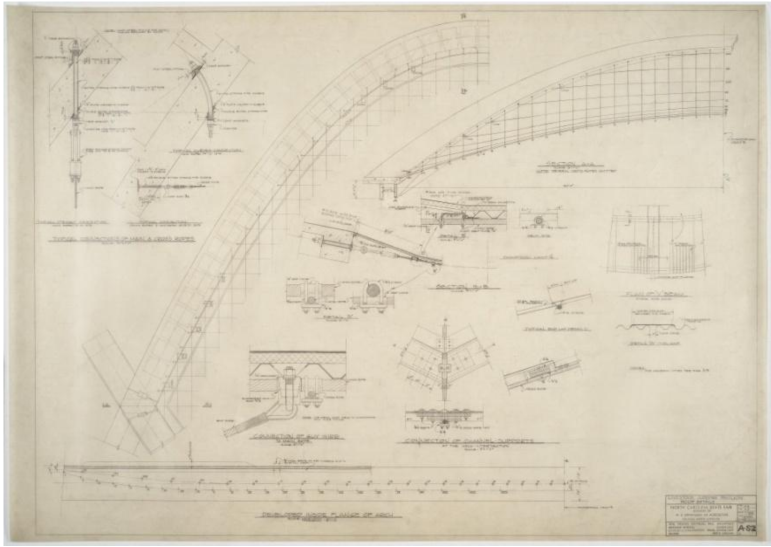


Figure 8

Another construction photo (Figure 9). All cables are in place and prestressing is complete. Now a cable clamp is installed at every intersection point. That's why that ironworker has a crane for a high-chair.

Figure 9

Another construction photo (Figure 10). Roofing is place. You can see the corrugated steel pan. There are turnbuckles a the ends of all prestressing cables.

Figure 10