Psychrometrics

(water in air mixtures)

AREN 2110

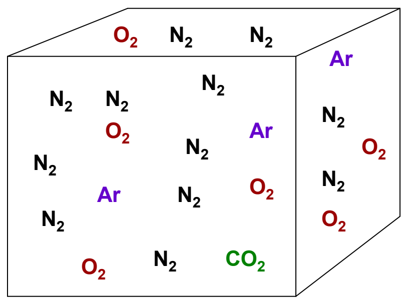
Introduction to Environmental Mixtures

Up until now, we have only modeled the behavior of pure substances

We will introduce the behavior of simple mixtures:
AIR + WATER

Where we assume the mixture ***is a GAS***, and is in a temperature/pressure range that follows the ideal gas law……(water vapor in air)

Atmospheric Gasses:
Air is a mixture of inert gasses



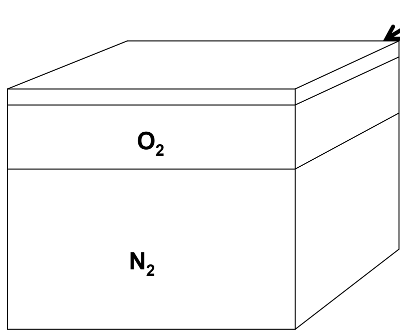
A m^3 of "air"

$$P_{total} = \frac{n_{total}}{V_{total}} R\,T$$

The total number of gas molecules per volume stays the same for a given P, T

Atmospheric Gasses
(predominantly O_2 & N_2)

Other: (Ar, CO_2, H_2O, CH_4)



"Air" 1 m ^{3}

Another way to look at a volume of air is to fractionate it into "pure" component volumes

Engineers define gas mixtures with 2 laws

Dalton's law (p)

The total **p** of any gas = the sum of the pressures of its components:

$$p_{Total} = \sum p_i$$

Amagat's law (v)

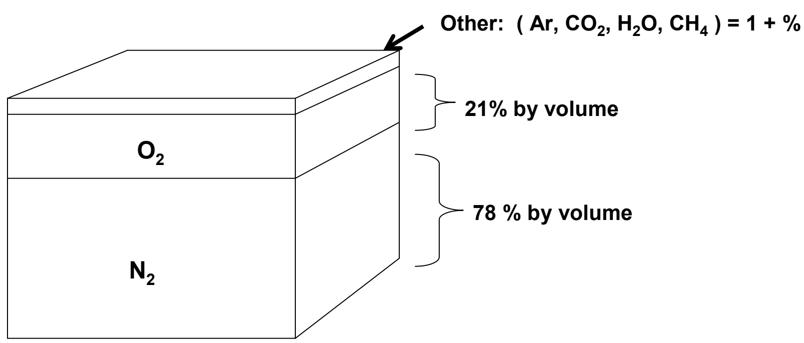
The total **V** of any gas = the sum of the volume of its components:

$$V_{Total} = \sum V_i$$

Where the components and mixtures are at the same temperature and pressure

Atmospheric Gasses

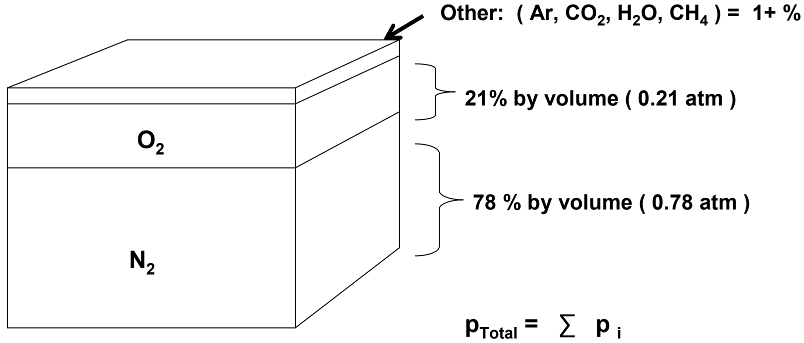
(predominantly O_2 & N_2)



"Air" 1 m ^{3}

Atmospheric Gasses

(predominantly O_2 & N_2)



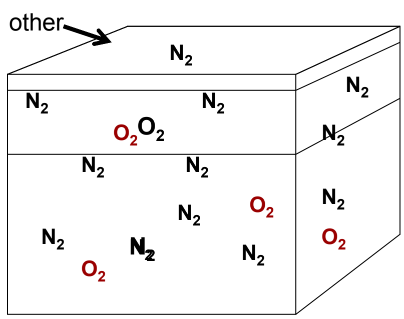
"Air" 1 m ^{3}

$$p_{Total} = \sum p_i$$

$$p_{air\ total} = p_N + p_o + p_{Ar} + p_{CO2} + p_{H2O}$$

Atmospheric Gasses

Restating the Ideal Gas Law for Mixtures



$$P_{total}\, V_{total} = n_{total}\ R\,T$$

(divide)

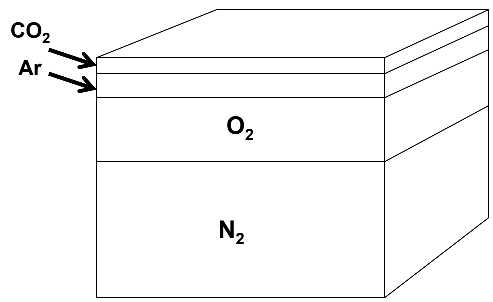
$$P_i\, V = n_i\ R\,T$$

" mole fraction" ⟶ $$\frac{P_i}{P_{total}} = \frac{n_i}{n_{total}} = Y_i$$

Effective Molecular Weight of DRY Air

is the average MW of its constituents weighted by their fractional abundance

$$MW_{air} = \overset{\overset{28}{\downarrow}}{N_2} \times 0.781 + \overset{\overset{32}{\downarrow}}{O_2} \times 0.209 + \overset{\overset{40}{\downarrow}}{Ar} \times 0.093 + \overset{\overset{44}{\downarrow}}{CO_2} \times 0.035$$



= 28.95 gram / mol

FOR DRY AIR

Wet air has a lower effective molecular weight than dry air

WHY?

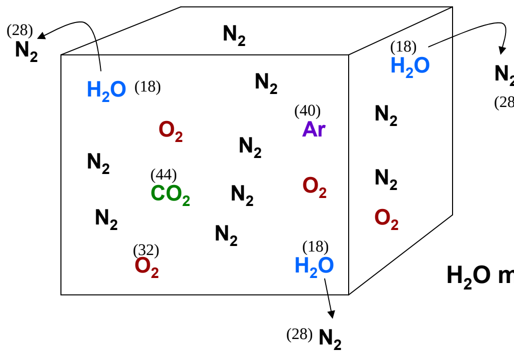
Because water has a lower MW than any constituents found in air !

N_2 = 28 g mol^{-1}; O_2 = 32 g mol^{-1}; Ar = 40 g mol^{-1}; CO_2 = 44 g mol^{-1}

$$\mathbf{MW_{air} = 28.95\ g\ mol^{-1}}$$

$$MW_{H2O} = 18.02\ g\ mol^{-1}$$

Atmospheric Gasses: when water dissolves in air



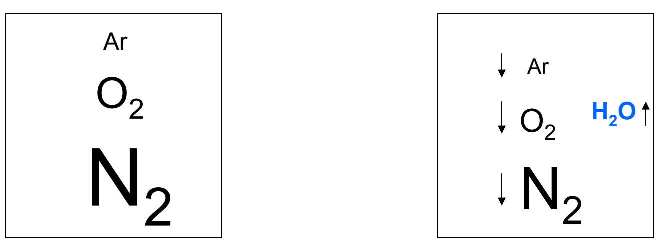
$$P_{total} = \frac{n_{total}}{V_{total}} R\,T$$

The total number of gas molecules per volume stays the same for a given P, T

H_2O molecules "replace" N_2, O_2, Ar… (literally 1:1)

1) **MW ↓**
2) **Density ↓**

Wet air vs. Dry Air



$MW_{wet\ air}$ =

$(18 \times Y_{H20}) + MW_{dry}(1 - Y_{H2O})$

Some other important properties change when water partitions into air

Ar
O_2
N_2

↓ Ar
↓ O_2 H_2O ↑
↓ N_2

$C_{p\ (dry\ air)} = 1\ \ J / g\ K$ $C_{p\ (water)} = 4\ \ J / g\ K$

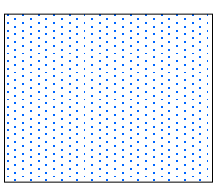
As the water content of air increases, the ability of air to hold and transfer energy increases markedly…..

The atmosphere can be modeled as two ideal gasses: dry air + water vapor

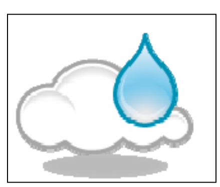
$$P_{sat} = f\ (Temp)\ \text{(table A-4)}$$

There is an upper limit on the amount of water vapor the atmosphere

Partial pressure of H_2O in air at maximum water vapor content = p_{sat}



If $p_{H2O} < p_{sat}$



If $p_{H2O} > p_{sat}$

Definition of Relative Humidity

Substitute Ideal Gas Law p = (n/v) RT

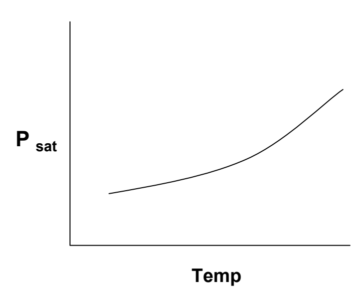
$$\Phi = \frac{p_{H2O\ (actual)}}{p_{sat}} = \frac{(n_{H2O} / V)\ RT}{(n_{sat} / V\ )\ RT}$$

$$= \frac{n_{H2O}}{n_{sat}} = \frac{\text{mass } H_2O_{\text{ actual}}}{\text{mass } H_2O_{\text{ saturation}}}$$

Relative Humidity and Thermal Energy Capacity of Air

$$\Phi = \frac{p_{H2O\ (actual)}}{p_{sat}} = \frac{\text{mass } H_2O_{\text{ actual}}}{\text{mass } H_2O_{\text{ saturation}}} \uparrow$$

Temp C	P sat (kPa)
0	0.6
5	0.9
10	1.2
15	1.7
20	2.4
25	3.1
30	4.2
35	5.6

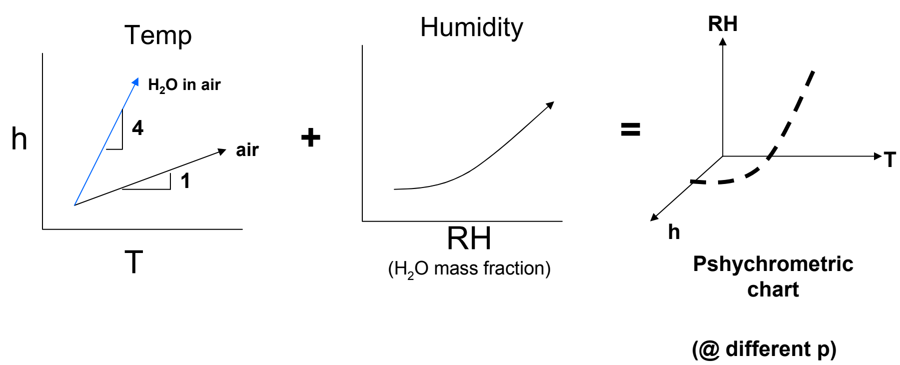


"Warm air can hold more water than cold air"

Energy Content of Air is a function of temperature AND relative humidity

$$h = C_p\ T$$

$$h\ [J/g] = C_p\ [J/g\ K]\ \ T\ [K]$$

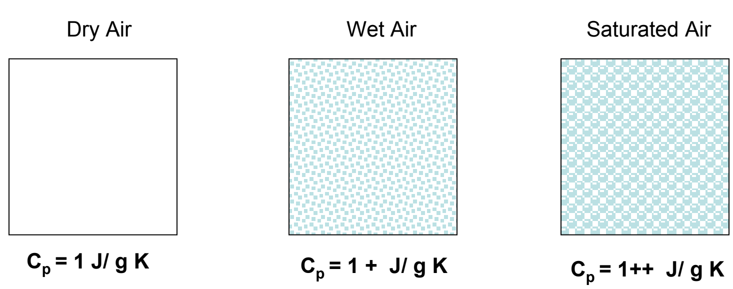


Relative Humidity and Thermal Energy Capacity of Air

Humidity varies with temperature and atmospheric pressure.

In general, the heat capacity of humid air is significantly greater than dry air; and,

the increase in enthalpy of air is a non-linear function of humidity (atmospheric pressure, T).

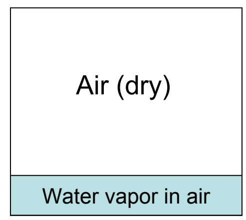


Water vapor acts like an ideal gas under most environmental conditions

$$p_{atm} = p_{\text{dry air}} + p_{\text{H2O vapor}}$$

Think of $p_{\text{H2O Vapor}}$ as pressure exerted by water vapor on its own, at the T and v of atmospheric air.

For most environmental conditions (< 50C) water vapor ***acts like an ideal gas.***



A reminder about the many different forms of the ideal gas law

(humidity calculations use a specific form (table A-1 and A-2))

$$\mathbf{P\ V = n\ R\ T}$$

mass = (n x MW_{stuff})

mole x (g / mol)

R

Depends on P, T reported ….and on Molecular Wt.

$$P\ V = \frac{m}{MW}\ R\ T$$

$$P\ V = m\ R_{stuff}\ T$$

Energy content of Air (h)

$$H_{wet\ air} = H_{dry\ air} + H_{water\ vapor} = h_{da}m_{da} + h_{wv}m_{wv}$$

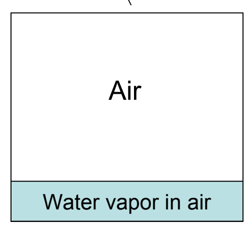
$$h_{system} = \frac{H_{total}}{m_{dry\ air}} = h_{da} + \frac{m_{wv}}{m_{da}} h_{wv}$$

"absolute humidity"

The enthalpy of air is expressed in terms of ***dry air*** and water vapor.

The amount of dry air remains constant, but the amount of water vapor changes.

So: ***h*** is expressed per unit mass of dry air



Enthalpy of Atmospheric Air depends primarily on humidity (water vapor content)

$$\text{Absolute Humidity} = \frac{\text{mass water vapor}}{\text{mass dry air}} = \boldsymbol{\omega}$$

$$\text{Relative Humidity} = \frac{\text{mass water vapor}}{\text{mass water vapor at sat}} = \boldsymbol{\Phi}$$

Energy content of Air (h)

$$H_{total} = H_{dry\ air} + H_{waterv} = h_{da}m_{da} + h_{wv}m_{wv}$$

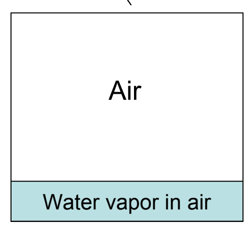
$$h_{system} = \frac{H_{total}}{m_{dry\ air}} = h_{da} + \frac{m_{wv}}{m_{da}} h_{wv}$$

"absolute humidity"

The enthalpy of air is expressed in terms of ***dry air*** and water vapor.

$$h_{air\ system} = h_{da} + \frac{m_{wv}}{m_{da}} h_{wv}$$

$$h_{air\ system} = h_{da} + \omega \ h_{wv}$$

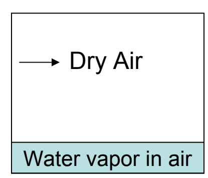


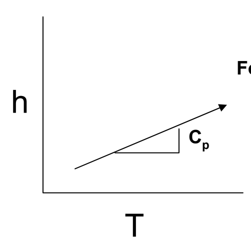
So how do we estimate the enthalpy of wet air?

(start with the dry air term)

$$\mathbf{h_{air\ system} = h_{da} + \omega \ h_{wv}}$$

$$h_{da} = C_p \ T$$





For Dry Air: C_p is constant

Linear between -20 to 50 C

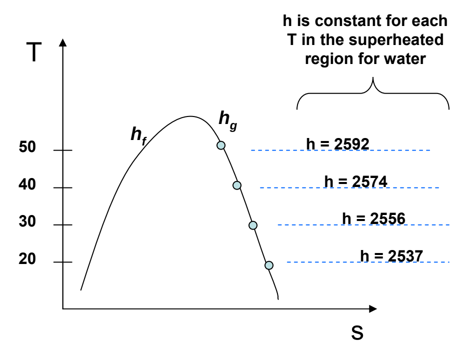
C_p dry air = 1.005 J/g

So how do we estimate the enthalpy of wet air?

(look at the water vapor term)

$$h_{air\ system} = h_{da} + \omega\ h_{wv}$$

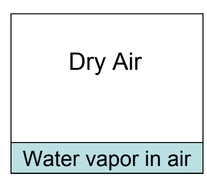
The enthalpy of water vapor in air, can be taken to be the same as h_g at the same T



So how do we estimate the enthalpy of wet air?

(look at the absolute humidity term)

$$h_{air\ system} = h_{da} + \omega\ h_{wv}$$



$$\omega = \frac{\text{water vapor mass}}{\text{dry air mass}} = \frac{m_{wv}}{m_{da}}$$

Substitute pV=nRT using molecular weight form for air and H_2O

$p\ V = m\ R_i\ T$

$R_{H2O} = 0.46$ kJ/kgK

$R_{dry\ air} = 0.29$ kJ/kgK

$$\omega = \frac{p_{wv}\ V / R_{wv}\ T}{p_{da}\ V / R_{da}\ T}$$

$$\omega = 0.62 \frac{p_{wv}}{p_{da}} = 0.62 \frac{p_{wv}}{(p_{atm} - p_{wv})}$$

Measure p_{atm}
Look up p_{wv} (T)

So how do we estimate the enthalpy of wet air?

(summary)

Dry Air

Water vapor in air

$$h_{air\ system} = h_{ad} + \omega \; h_{wv}$$

$C_{p(air)}\,T$

$h_g\,(T)$

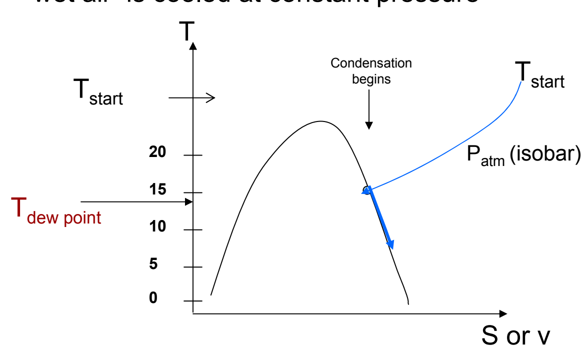
$$0.62 \frac{p_{wv}}{(p_{atm} - p_{wv})} = \frac{0.62\,(RH)\,p_{sat}}{(p_{atm} - (RH)\,p_{sat})}$$

relative humidity (RH)

$$\Phi = \frac{p_{wv}}{P_{sat}(T)}$$

Dew Point: very important atmospheric thermodynamic index

DEFN: the temperature at which condensation begins when wet air is cooled at constant pressure

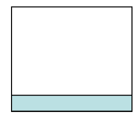


Dew Point: the temperature at which condensation begins when air is isobarically cooled

-or- the saturation temperature of water at P_{sat}

$T_{dew\ point} = T\ @\ p_{g\ (water\ vapor)}$

Where relative humidity is at 100 %



100% ! = $\Phi = \dfrac{p_{H2O\ (actual)}}{p_{sat} = f\ (temp)} = \dfrac{p_{H2O}\ (actual)}{\downarrow}$

As temperature drops, air has less ability to hold water: ($p_{sat}\downarrow$)

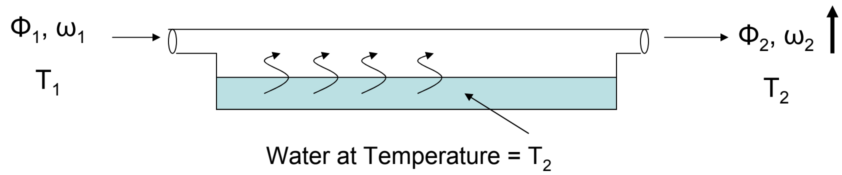
Adiabatic Saturation

Relative Humidity (Φ), and Absolute Humidity (ω) are frequently used in engineering practice:

Φ and ω are related to adiabatic saturation using the definition of……

$$\text{Absolute Humidity} = \frac{\text{mass water vapor}}{\text{mass dry air}} = \boldsymbol{\omega}$$

Adiabatic Saturation is determined by evaporating water into a stream of air to RH = 100%



Mass balance on air and water vapor

Dry air

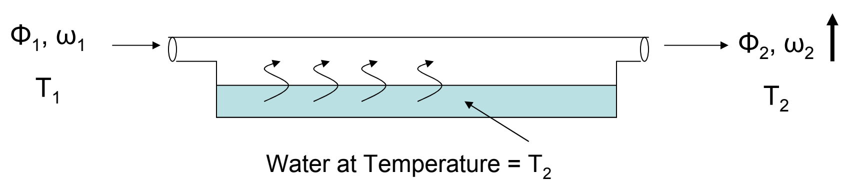
$$\dot{m}_{dryair\ (in)} = \dot{m}_{dryair\ (out)}$$

Water Vapor

$$\dot{m}_{water\ vapor\ (in)} + \underbrace{\text{evaporation rate}}_{\dot{m}_{dryair\ (in)}\,(\omega_2 - \omega_1)} = \dot{m}_{water\ vapor\ (out)}$$

$\omega_1\, m_{dryair\ (out)}$ $\qquad$ $\omega_2\, m_{dryair\ (out)}$

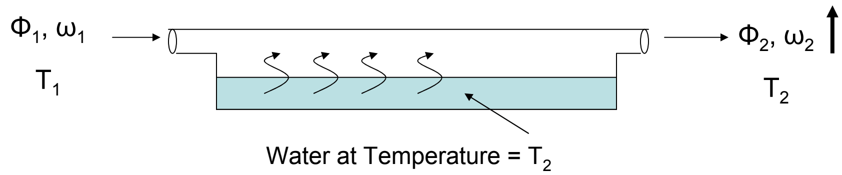
Adiabatic Saturation is determined by evaporating water into a stream of air to RH = 100%



Combined Energy balance on air and water vapor

$$H:\quad \underbrace{(\underbrace{C_p T_{in}}_{\text{Dry air}} + \underbrace{\omega_{in} h_{g\,(in)}}_{\text{Water vapor}})}_{\text{IN}} + \underbrace{h_{f\,(out)}\,(\omega_2 - \omega_1)}_{\text{Evaporating water}} = \underbrace{(\underbrace{C_p T_{out}}_{\text{Dry air}} + \underbrace{\omega_2\, h_{g\ out}}_{\text{Water vapor}})}_{\text{OUT}}$$

Adiabatic Saturation is determined by evaporating water into a stream of air to RH = 100%



$$\omega_1 = \frac{C_p\,(T_{out} - T_i) + \omega_2\, h_{fg\,(Temp\ out)}}{h_{g\,in} - h_{f\,(Temp\ out)}}$$

$$\omega_2 = \frac{0.622\ p_{g\ out\ (atm)}}{p_{\,(out)} - p_{g\ out\ (atm)}}$$

Psychrometric charts plot all critical atmospheric parameters as a function of temp and humidity